Low Complexity Implementation of Block Diagonalization Algorithm in 3D MU-MIMO Systems

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***Abstract*—In three dimension multiple-input multiple-output (3D MIMO) systems, large number of antennas are arranged at base station (BS) to obtain high spectral efficiency and large throughput. But increase of BS antennas also enhances the computational complexity of the multiuser precoding algorithm such as block diagonalization (BD) significantly. For the dimension increase of channel matrix, various ways can be chosen to obtain the equivalent matrix which can eliminate the user interference completely in the regular BD algorithm. Motivated by this observation, a low complexity precoding method based BD is proposed in this paper, which can simplify the procedure of the two singular value decomposition (SVD) operations in the regular BD algorithm prominently with the optimal equivalent matrix. Also with the 3D channel measurement data we validate the 3D capacity performance of the proposed method. Simulation results show that the proposed method can achieve the same capacity as the regular BD algorithm.**

I. Introduction

It has been shown that three dimension multi-input multi-output (3D MIMO) systems, in which Multi-Element Arrays (MEAs) are used at both ends of the link, can in principle offer a linear increase in capacity [1] [2]. But the large number of base station (BS) antennas in 3D MIMO systems increase the computational complexity of the multiuser (MU) MIMO precoding algorithm such as block diagonalization (BD) significantly. BD algorithm [3] [4] is one of the key precoding techniques for downlink MU-MIMO systems, which can cancel the interference between data transmitted to mobile systems (MSs) completely. Though it can gain high spectral efficiency [5] and large throughput [6], the BD precoding requires heavy implementations of singular value decomposition (SVD) [7] of channel matrices. The main steps of the BD precoding algorithms are two SVD operations, which need to be implemented for each user. Therefore, the computational complexity of the regular BD precoding algorithm depends on the number of users and the dimensions of each user's channel matrix. For 3D MU-MIMO systems with a large number of users and multiple transmit antennas, this could result in a considerable computational cost.

Recent work on BD-type precoding algorithms has focused on how to equivalently implement the BD-type precoding algorithms with less computational complexity. A low complexity generalized zero forcing (ZF) channel inversion (GZI) method has been proposed in [8] to equivalently implement the first SVD operation of the original BD precoding, and a generalized MMSE channel inversion (GMI) method is also developed in [8] for the original regularized block diagonalization (RBD) precoding. In [9] the first SVD operation of the RBD precoding is replaced with a less complex QR decomposition [7]. For the second SVD operation, however, both the work in [8] and [9] employ it in a similar way as the conventional BD-type precoding algorithms to parallelize each user's streams. All the techniques above are solely low complexity equivalent implementations of the two SVD operations. Different with them, this paper simplifies the procedure of the two SVD operations with the proper choice of the equivalent matrix, which can be chose in different ways. Larger is the difference between the number of transmit and receive antennas, more ways we have to choose the equivalent matrix. The optimal equivalent matrix degrades the dimension of the matrices which will experience SVD operations. As a consquence the computational complexity of the precoding process degrades prominently with the proposed method.

In order to checkout whether complexity decrease influence the capacity performance. The capacity performance of the proposed method is investigated with the 3D channel measurement data. With the same number of antennas at the BS, the capacity performance of the proposed method with different BS antennas arrangement are compared to the regular BD algorithm. We not only verify that the proposed method can achieve the same capacity as the regular BD algorithm in different case, but also find the antennas arranged horizontally perform better than the ones arranged vertically.

The remainder of this paper is organized as follows. Section II provides the system model. The regular BD algorithm and the proposed method are shown in Section III. Section IV provides the channel measurement and results analysis. Conclusion is given in Section V.

II. System Model

In this section, we introduce the system model for downlink MU-MIMO with linear precoding. In this system with K users, N_t denotes the number of transmit antennas at the base station and N_r represents the number of receive antennas at each mobile user. The downlink channel between the base station and user k is described by the matrix $\mathbf{H}_k \in \mathcal{C}^{N_r \times N_t}$. Therefore, the aggregate downlink channel is described by $\mathbf{H}_s = \begin{bmatrix} \mathbf{H}_1^T & \mathbf{H}_2^T & \cdots & \mathbf{H}_K^T \end{bmatrix}^T$, where $(.)^T$ denotes the matrix transpose. Let $\mathbf{x} \in \mathcal{C}^{N_t \times 1}$ denote the transmitted signal vector from the base station antennas, and $\mathbf{y}_k \in \mathcal{C}^{N_r \times 1}$ be the received signal vector at the mobile receiver k. The noise at receiver k is represented by $\mathbf{z}_k \in \mathcal{C}^{N_r \times 1}$ and is assumed to be circularly symmetric zero-mean complex Gaussian noise ($\mathbf{z}_k \sim \mathcal{CN}(0, \sigma^2 I)$). The received signal for user k can be expressed as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{z}_k, k = 1, \cdots, K \tag{1}$$

The covariance matrix of the transmitted signal is $\mathbf{S}_x \stackrel{\Delta}{=} E\left[\mathbf{x}\mathbf{x}^H\right]$. The sum power constraint over the transmit antennas imposes $tr\{\mathbf{S}_x\} = P$, where P is the power limitation.

The transmitted symbol of user k is an N_r-dimensional vector $\mathbf{u}_k$ which is multiplied by a $N_t \times N_r$ precoding matrix $\mathbf{W}_k$ to produce the transmitted signal $\mathbf{x}_k$ intended for user k. Thus, the complex transmit antenna output vector $\mathbf{x}$ is composed of signals for all K users and is given by

$$\mathbf{x} = \sum_{k=1}^{K} \mathbf{x}_k = \sum_{k=1}^{K} \mathbf{W}_k \mathbf{u}_k \tag{2}$$

where $E\left[\mathbf{u}_k \mathbf{u}_k^H\right] = \mathbf{I}_{N_r}$. The aggregate precoding matrix is described by $\mathbf{W}_s = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 & \cdots & \mathbf{W}_K \end{bmatrix}$. The received signal for user k can be represented as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{W}_k \mathbf{u}_k + \sum_{j=1, j \neq k}^{K} \mathbf{H}_k \mathbf{W}_j \mathbf{u}_j + \mathbf{z}_k \tag{3}$$

III. BD Algorithm and the Proposed Method

A. BD Algorithm

This section outlines the procedure of the regular BD algorithm [6]. With a sum power constraint, the achievable throughput for the resulting block-diagonal system is

$$C_{BD} = \max_{\mathbf{W}_s, \mathbf{H}_j \mathbf{W}_j = 0, i \neq j} \log_2 \left| I + \frac{1}{\sigma_n^2} \mathbf{H}_s \mathbf{W}_s {\mathbf{W}_s}^* {\mathbf{H}_s}^* \right| \tag{4}$$

$$= \max_{\mathbf{H}_j \mathbf{W}_j = 0, i \neq j} \sum_{j=1}^{K} \log_2 \left| I + \frac{1}{\sigma_n^2} \mathbf{H}_j \mathbf{W}_j {\mathbf{W}_j}^* {\mathbf{H}_j}^* \right| \leq C_s \tag{5}$$

where C_s represents the sum capacity of the system, and $*$ indicates the Hermitian transpose. If we define $\tilde{\mathbf{H}}_j$ as

$$\tilde{\mathbf{H}}_j = \begin{bmatrix} \mathbf{H}_1^T & \cdots & \mathbf{H}_{j-1}^T & \mathbf{H}_{j+1}^T & \cdots & \mathbf{H}_K^T \end{bmatrix}^T \tag{6}$$

the zero-interference constraint forces $\mathbf{W}_j$ to lie in the null space of $\tilde{\mathbf{H}}_j$. This definition allows us to define the dimension condition necessary to guarantee that all users can be accomodated under the zero-interference constraint. Data can be transmitted to user j if the null space of $\tilde{\mathbf{H}}_j$ has a dimension greater than 0. This is satisfied when $rank(\tilde{\mathbf{H}}_j) < N_t$. So for any $\mathbf{H}_s$, block diagonalization is possible if $N_t > \max\left\{rank\left(\tilde{\mathbf{H}}_1\right), \cdots, rank\left(\tilde{\mathbf{H}}_K\right)\right\}$. Assuming the dimension condition is satisfied for all users, let $\tilde{L}_j = rank\left(\tilde{\mathbf{H}}_j\right) \leq (K-1)N_r$, and define SVD

$$\tilde{\mathbf{H}}_j = \tilde{\mathbf{U}}_j \tilde{\mathbf{\Sigma}}_j \begin{bmatrix} \tilde{\mathbf{V}}_j^{(1)} & \tilde{\mathbf{V}}_j^{(0)} \end{bmatrix}^* \tag{7}$$

where $\tilde{\mathbf{V}}_j^{(1)}$ holds the first $\tilde{L}_j$ right singular vectors, and $\tilde{\mathbf{V}}_j^{(0)}$ holds the last $(N_T - \tilde{L}_j)$ right singular vectors. Thus, $\tilde{\mathbf{V}}_j^{(0)}$ forms an orthogonal basis for the null space of $\tilde{\mathbf{H}}_j$, and its columns are, thus, candidates for the precoding matrix $\mathbf{W}_j$ of user j. Let $\bar{L}_j$ represent the rank of the product $\mathbf{H}_j \tilde{\mathbf{V}}_j^{(0)}$. In order for transmission to user j to take place under the zero-interference constraint, $\bar{L}_j > 1$ is necessary. In general, $\bar{L}_j$ is bounded by $L_j + \tilde{L}_j - n_T \leq \bar{L}_j \leq \min\{L_j, \tilde{L}_j\}$ [10]. We now define the matrix

$$\mathbf{H}_S' = \begin{bmatrix} \mathbf{H}_1 \tilde{\mathbf{V}}_1^{(0)} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{H}_K \tilde{\mathbf{V}}_K^{(0)} \end{bmatrix} \tag{8}$$

The system capacity under the zero-interference constraint can now be written as

$$C_{BD} = \max_{\mathbf{W}_s'} \log_2 \left| I + \frac{1}{\sigma_n^2} \mathbf{H}_s' \mathbf{W}_s' \mathbf{W}_s'^* \mathbf{H}_s'^* \right| \tag{9}$$

The problem is now to find a matrix $\mathbf{W}_s'$ that maximizes the determinant. This is now equivalent to the single-user MIMO capacity problem, and the solution is to let $\mathbf{W}_s'$ be the right singular vectors of $\mathbf{H}_s'$, weighted by water-filling on the corresponding singular values [11]. Thus, a solution for $\mathbf{W}_s'$ based on an SVD and water-filling is the solution that maximizes sum capacity for the system under the zero-interference constraint.

The block structure of $\mathbf{H}_s'$ allows the SVD to be determined individually for each user, rather than computing a single large SVD. Define the SVD

$$\mathbf{H}_j \tilde{\mathbf{V}}_j^{(0)} = \mathbf{U}_j \begin{bmatrix} \mathbf{\Sigma}_j & 0 \\ 0 & 0 \end{bmatrix} \left[\mathbf{V}_j^{(1)} \mathbf{V}_j^{(0)}\right]^* \tag{10}$$

where $\mathbf{\Sigma}_j$ is $\bar{L}_j \times \bar{L}_j$, and $\mathbf{V}_j^{(1)}$ represents the first $\bar{L}_j$ singular vectors. The product of $\tilde{\mathbf{V}}_j^{(0)}$ and $\mathbf{V}_j^{(1)}$ now produces an orthogonal basis of dimension $\bar{L}_j$ and represents the transmission vectors that maximize the information rate for user subject to producing zero interference. Thus, we define the precoding matrix as

$$\mathbf{W}_s = \begin{bmatrix} \tilde{\mathbf{V}}_1^{(0)} \mathbf{V}_1^{(1)} & \tilde{\mathbf{V}}_2^{(0)} \mathbf{V}_2^{(1)} & \cdots & \tilde{\mathbf{V}}_K^{(0)} \mathbf{V}_K^{(1)} \end{bmatrix} \mathbf{\Lambda}^{1/2} \tag{11}$$

where $\mathbf{\Lambda}$ is a diagonal matrix whose elements scale the power transmitted into each of the columns of $\mathbf{W}_s$. With $\mathbf{W}_s$ chosen as in (11), the capacity of the BD method in (4) becomes

$$C_{BD} = \max_{\mathbf{\Lambda}} \log_2 \left| \mathbf{I} + \frac{\mathbf{\Sigma}^2 \mathbf{\Lambda}}{\sigma_n^2} \right| \quad (12)$$

where

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_1 & & \\ & \ddots & \\ & & \mathbf{\Sigma}_{\mathrm{K}} \end{bmatrix} \quad (13)$$

The optimal power loading coefficients in $\mathbf{\Lambda}$ are then found using water-filling on the diagonal elements of $\mathbf{\Sigma}$, assuming a total power constraint P.

B. The Proposed Method

The main steps of the regular BD precoding algorithms are two SVD operations. The first SVD (7) is to obtain an equivalent matrix which can make users interference disappear completely. Assumes that the matrix for each user is full rank. In (7)

$$\tilde{\mathbf{V}}_j^{(0)} = \begin{bmatrix} \tilde{\mathbf{V}}_{j_1}^{(0)} & \tilde{\mathbf{V}}_{j_2}^{(0)} & \cdots & \tilde{\mathbf{V}}_{j_{N_t-(K-1)N_r}}^{(0)} \end{bmatrix} \quad (14)$$

In order to eliminate the user interference the channel matrix must multiply the sub matrix of the null space $\tilde{\mathbf{V}}_j^{(0)}$, which is noted as $\tilde{\mathbf{V}}_{j_{sub}}^{(0)}$. And $\begin{bmatrix} \tilde{\mathbf{V}}_j^{(1)} & \tilde{\mathbf{V}}_{j_{sub}}^{(0)} \end{bmatrix}^* = \tilde{\mathbf{V}}_{j_{sub}}^*$. The second SVD is to get the weight $\mathbf{W}_s$, and $\mathbf{V}_j^{(1)}$ in (10) is the equivalent weight.

The most important step of the regular BD algorithm is to obtain the equivalent matrix $\mathbf{H}_j \tilde{\mathbf{V}}_j^{(0)}$, in which $\tilde{\mathbf{V}}_j^{(0)}$ is the null space of $\tilde{\mathbf{H}}_j$. Only one vector in the null space is enough for the demand to eliminate the user interference. But its infeasible for two reasons. First, the precoding matrix of each user is a matrix with the dimension $N_t \times N_r$, the column number of the equivalent matrix can not less than N_r , so is the vector number in the sub null space. Second, less vector may not obtain the best capacity performance. Above all we assume the best capacity performance can be achieved with only M $(N_r \leq M \leq N_t - (K-1)N_r)$ vectors. The proposed method based BD precoding can be then summarized in Table I.

TABLE I
THE PROPOSED METHOD

Steps	Operations
1)	Compute the first SVD $\tilde{\mathbf{H}}_j$, but compute only M vectors in the null space, and obtain the sub null space $\tilde{\mathbf{V}}_{j_{sub}}^{(0)}$.
2)	Compute the product $\mathbf{H}_j \tilde{\mathbf{V}}_{j_{sub}}^{(0)}$.
3)	Compute the second SVD $\mathbf{H}_j \tilde{\mathbf{V}}_{j_{sub}}^{(0)} = \widehat{\mathbf{U}}_j \begin{bmatrix} \widehat{\mathbf{\Sigma}}_j & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{V}}_j^{(1)} \widehat{\mathbf{V}}_j^{(0)} \end{bmatrix}^*$
4)	Use water filling on the diagonal elements of $\widehat{\mathbf{\Sigma}}$ to determine the optimal power loading matrix $\mathbf{\Lambda}$ under power constraint P.
5)	Set $\mathbf{W}_s = \begin{bmatrix} \bar{\mathbf{V}}_1^{(0)} \widehat{\mathbf{V}}_1^{(1)} & \bar{\mathbf{V}}_2^{(0)} \widehat{\mathbf{V}}_2^{(1)} & \cdots & \bar{\mathbf{V}}_K^{(0)} \widehat{\mathbf{V}}_K^{(1)} \end{bmatrix} \mathbf{\Lambda}^{1/2}$

C. Complexity Analysis

In this section, we will analyze the complexity of the proposed method. In practice, the number of receiving antennas of the user equipment is usually limited due to size restraint. In the current evolution of 3G, two-receiving-antenna is a main antenna configuration of the downlink [12]. In LTE system, 8 antenna elements are selected at the transmitter. In order to explain the proposed method compactly, 8 antenna elements selected at the transmitter and 2 at the receiver are adopted in a simple two users MIMO system. The main complexity cost of the regular BD algorithm is the two SVD operations. Fitst is to do SVD of $\tilde{\mathbf{H}}_j$ and obtain $\bar{\mathbf{V}}_j^{(0)}$. Compared to the traditional BD algorithm, we do not need to calculate all the six vectors in the null space and only M vectors is enough. So the complexity of the first SVD is decreased. Second is to do SVD of $\mathbf{H}_j \bar{\mathbf{V}}_j^{(0)}$, the dimension of which is 2×6 , but in our proposed method, it is $2 \times M$. So the second SVD also has lower complexity than the regular BD algorithm. What's more the multiplication $\mathbf{H}_j \tilde{\mathbf{V}}_j^{(0)}$ is a 8×6 matrix in the regular BD algorithm, but is $8 \times M$ in our proposed method. So the complexity of the multiplication is also decreased. In conclusion, the proposed method has lower complexity than the regular BD algorithm in all the first three steps.

We make flops as the judgment criteria [13], though flop counting might be a little different from the actual computational complexity, it approximates elementally the computation load and is enough for the purpose of the complexity analysis. The complexity comparison between the two schemes is showed in Table II. In each step, first row is the flops of the regular algorithm and the second is that of the proposed method.

TABLE II
COMPLEXITY COMPARISON

Steps	Operations	Flops
1)	$\tilde{\mathbf{U}}_j \tilde{\mathbf{\Sigma}}_j \tilde{\mathbf{V}}_j^*$	$48{N_t}^2 + 192N_t + 432$
	$\tilde{\mathbf{U}}_j \tilde{\mathbf{\Sigma}}_j \tilde{\mathbf{V}}_{j_{sub}}^*$	$48(N_t - 6 + M)^2 + 192\,(N_t - 6 + M)$
2)	$\mathbf{H}_j \bar{\mathbf{V}}_j^{(0)}$	$16{N_t}^2 - 28N_t + 8$
	$\mathbf{H}_j \bar{\mathbf{V}}_j^{(0)}$	$16(N_t - 8 + M)^2 - 28\,(N_t - 8 + M) + 8$
3)	$\mathbf{U}_j \mathbf{\Sigma}_j \mathbf{V}_j^*$	$48[N_t - 2]^2 + 192\,[N_t - 2]$
	$\widehat{\mathbf{U}}_j \widehat{\mathbf{\Sigma}}_j \widehat{\mathbf{V}}_j^*$	$48M^2 + 192M + 432$

Complexity comparison between the regular BD algorithm and the proposed method is shown in Fig. 1. When $M = 6$ the proposed method is equivalent to the regular BD method. We can see, the complexity of the three steps are all lower than the regular BD algorithm, and so is the sum complexity. What's more the fewer are the vectors in the sub null space the lower computational complexity is the proposed method. On average, the complexity of the proposed method is about 56% of the regular BD algorithm.

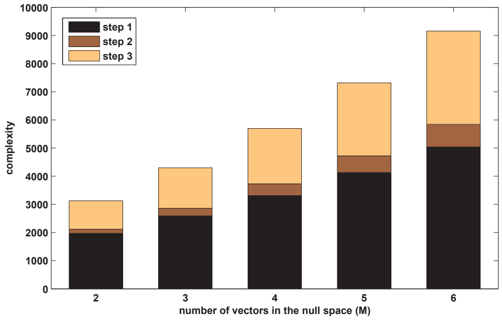
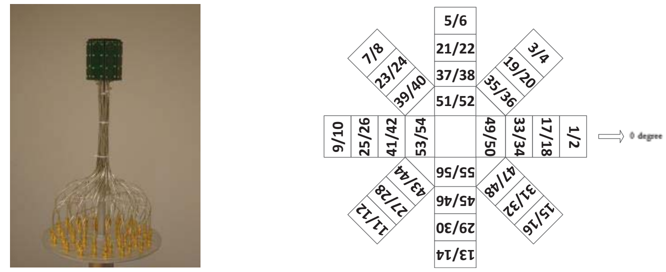


Fig. 1. Complexity comparison of the two BD schemes



(a) (b)

Fig. 2. The layout and schematic plot of the antenna arrays at Rx side

1/2	3/4	5/6	7/8
9/10	11/12	13/14	15/16
17/18	19/20	21/22	23/24
25/26	27/28	29/30	31/32

(a) (b)

Fig. 3. The layout and schematic plot of the antenna arrays at Tx side

IV. Channel Measurement and Capacity Validation

A. Channel Measurement

In order to verify the 3D capacity performance of the proposed method based BD, the channel response with 3D channel parameters are captured through the real channel measurement. The measurement antenna array equipped at both sides of a communication link are all full dimension. The dual-polarized omnidirectional array (ODA) consisting of 56 antenna elements is implemented at the Rx, and the dual-polarized uniform planar array (UPA) with 32 antenna elements is used at Tx. The layout and schematic plot of the antenna arrays at Rx side are illustrated in Fig. 2, and those of the antenna arrays at Tx side are shown in Fig. 3.

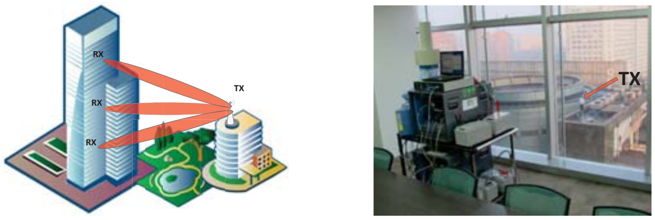
Fully utilizing both the horizontal and vertical dimensions, 3D MIMO is particularly suitable for scenarios with vertical user location distributions. One example is the dense urban area with lots of high-rise buildings. There is usually a huge demand for the mobile data capacity in these areas. Transmissions from outdoor BSs to users located on different floors can be better separated in their elevation angles and this scenario is called as O2I or high rise by 3GPP.

The layout of the O2I scenario is given in Fig. 4. The Tx antenna height is 46 m. The measurement spots are mainly planned in a 22-floor building near the Tx. This measurement was conducted on the 6th, 7th, 8th, 9th, 11th, 16th, 18th, 19th and 21st floor, the height of the mobile station are respectively 21.8 m, 25.6 m, 29.4 m, 33.2 m, 41 m, 52.8 m, 60.6 m, 64.6 m, and 72.4 m. Fig. 3 (b) shows the view of the Rx side in the 16th floor. In all three scenarios, the Rx is placed on the trolley with the height of 1.78 m.

B. Capacity Performance Validation

We choose 8 antennas from the UPA at the transmitter. Four different antenna arrangement methods namely row arranged, column arranged, and block arranged with 1.5 and 0.5 wavelength interval, are selected to make a comparison. The selected transmitter antenna element ports are shown in Fig. 5. The capacity performance of the the proposed method and the regular algorithm with different antenna arrangement are compared in Fig. 6. From the results, we can see the proposed method can achieve nearly the same capacity as the regular BD algorithm. For different antenna arrangement, we also find that the row arranged maximal aperture antenna sub-array can achieve higher channel capacity compared to other arrangement. And the column arranged antenna sub-array can not achieve good capacity performance. In the UPA, azimuth AS of AoD is larger than elevation AS of EoD and horizontal channel correlation is smaller than that of vertical antennas. As a consequence, row arranged antennas will obtain better capacity performance. So we can arrange the BS antennas to the horizontal dimension as many as possible to achieve better performance. Also the precoding and beamforming algorithm in 3D MIMO systems can also pay more attention to horizontal dimension than vertical dimension.

Through the simulation with numerous 3D channel impulse response (CIR) data, we find that less vectors in the null space can achieve the capacity which must be achieved by 6 in the regular BD algorithm. The proportion of the sub null space with different number of vectors that can achieve the best capacity performance is described in Fig. 7. From the simulation results, we can see for about 30% of the CIR, we don't need 6 vectors to achieve the best capacity. For 1% of the CIR just two vectors can achieve the capacity performance. When the receive antennas is two, only two columns are needed in $\mathbf{V}_j^{(1)}$. Equally, only two orthogonal vector are needed in $\tilde{\mathbf{V}}_j^{(0)}$ to form the equivalent matrix. Therefor not all the vectors in the null space should be calculated, two or more will just achieve the performance of the regular BD algorithm.



(a) (b)

Fig. 4. Measurement environment

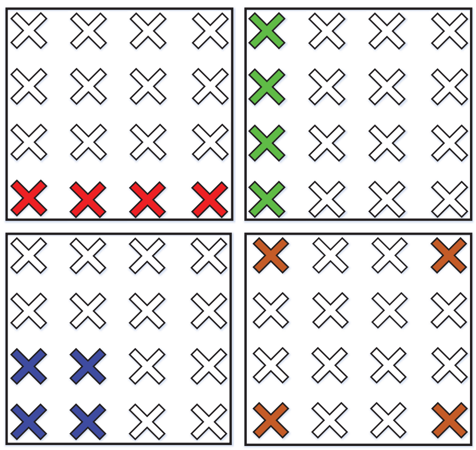


Fig. 5. The four arrangement plots of the 8 antennas at the BS

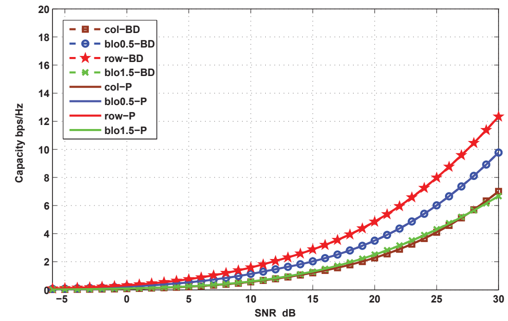


Fig. 6. Capacity performance of four different antenna arrangement

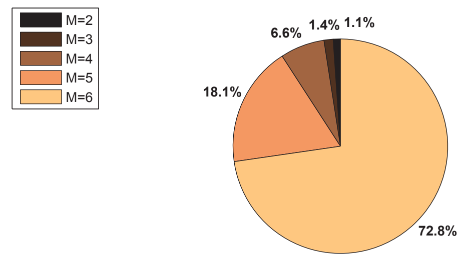


Fig. 7. Proportion of the optimal sub null space

V. Conclusion

In 3D MIMO systems, more antennas are arranged at BS. Increase of BS antennas enhances the computational complexity of the MU-MIMO precoding algorithm BD significantly. But at the same time, the channel matrix has redundancy to make different choices of the equivalent matrix which can eliminate the user interference completely. This paper proposed a new method based BD algorithm which degrades the complexity significantly. And we validate the capacity performance with different antenna arrangement at the BS. The simulation results show that with this method we can obtain the same capacity performance as the Regular BD algorithm. Also we find row arranged antenna array can obtain better capacity performance than the other arranged antennas.

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