Multipair Two-Way Relay Networks with Very Large Antenna Arrays

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***Abstract*—We consider a multipair two-way relay network where multiple communication pairs simultaneously exchange information with the help of a single relay. Each terminal has only a single antenna, while the relay is equipped with a very large antenna array. We further assume that channel state information is available at the relay node. We investigate the power efficiency of this network when very simple signal processing, i.e., maximum ratio combining (MRC) or zero-forcing (ZF), is used at the relay. When the number of relay antennas grows infinite, the transmit power of each terminal or relay (or both) can be made inversely proportional to the number of relay antennas while maintaining a given quality-of-service. We show that with very large antenna arrays, the two-way relaying scheme outperforms both the orthogonal scheme and the one-way relaying scheme.**

I. Introduction

Multiple-input multiple-output (MIMO) time-division duplexing (TDD) system with large-scale transmit antenna arrays, so called massive MIMO system, is one of the key technologies for future wireless communications. The large size of the transmit antenna array relative to the number of receive terminals can average out thermal noise, fast channel fading, and some interference, based on the law of large numbers [1]–[3]. Very large multi-user MIMO (MU-MIMO) systems can substantially reduce the interference with simple signal processing techniques and achieve increased reliability and throughput [4]. On the other hand, relaying has been extensively explored to provide expanded coverage and high throughput [5]. In [6], the authors have done a large system analysis of a multipair one-way relay channel when the number of relay antennas grows into infinity. However, with one-way protocols, the half-duplex (HD) constraint at the relay imposes a pre-log factor 1/2 for the data rate and hence, limits the spectral efficiency. To overcome this spectral efficiency loss in the one-way relay channel, the multipair two-way relay channel has recently been considered. A distributed amplify-and-forward (AF) relaying scheme for multipair two-way relay channels is proposed in [7]. The asymptotic capacity increases without bound when each relay processes signals in a distributed manner. However, in the context of a single relay equipped with very large antenna arrays, the processing cannot be done distributedly at each antenna separately.

In this paper, we analyze the performance of a multipair two-way relay network which consists of K communication pairs and a single relay equipped with N antennas, where $N \gg K$. Here, we assume that the relay has perfect knowledge of channel state information (CSI). To the best of our knowledge, there is no reported work that analyzes the considered system. The fundamental basis of our analysis is that when the number of relay antennas is large, the channel vectors between the terminals and relay are pairwisely nearly orthogonal. The near-orthogonality assumption is also empirically supported in literature [1].

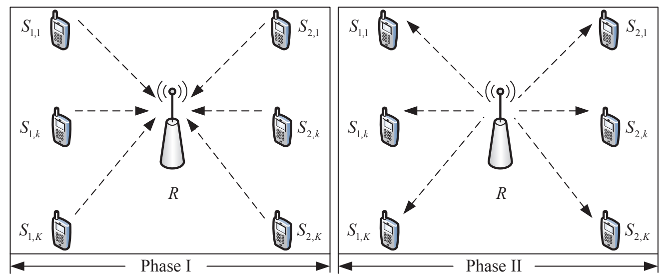

Fig. 1. Multipair two-way relaying network.

We have derived the asymptotic achievable rates when the number of relay antennas grows to infinity for cases when the transmit power of each terminal or/and the relay is made inversely proportional to N. The results show that the transmit power at each terminal or/and the relay can be scaled down proportionally to $1/N$ with no performance degradation. We show that with a very large antenna array, the two-way relaying scheme outperforms the one-way relaying scheme by virtue of the reduced pre-log penalty.

II. Multipair Two-Way Relay Channel Model

Consider a network in which K communication pairs ($S_{1,k}$, $S_{2,k}$), $k = 1, \ldots, K$, share the same time-frequency resource. Two terminals $S_{1,k}$ and $S_{2,k}$ exchange their information with the help of a single relay, R. Each terminal is equipped with a single antenna while the relay is equipped with N antennas. Typically, $K \ll N$. We assume that there are no direct links between $S_{1,k}$ and $S_{2,k}$ that can be exploited due to heavy shadowing and path loss. Transmission will take place in both directions (from the terminals to the relay and back) on the same frequency. We assume that the channels are reciprocal and the relay has full CSI. This assumption is reasonable in an environment with low or moderate mobility. The CSI at the relay node could be obtained by using training sequences transmitted from the terminals, at a cost of $2K$ symbols per coherence interval T. The communication occurs in two phases, as detailed next and in Fig. 1. We assume perfect time synchronization. The lack of synchronicity does not have much effect on the system performance.

A. Phase I

All terminals simultaneously broadcast their signals to the relay R. The received $N \times 1$ signal vector at R is given by

$$\mathbf{y}_{\mathrm{R}} = \sqrt{P_{\mathrm{t}}}\mathbf{G}_1\mathbf{x}_1 + \sqrt{P_{\mathrm{t}}}\mathbf{G}_2\mathbf{x}_2 + \mathbf{n}_{\mathrm{R}} \tag{1}$$

where $\mathbf{x}_i \triangleq [x_{i,1} \cdots x_{i,K}]^T$, $\sqrt{P_{\mathrm{t}}}x_{i,k}$ is the transmitted signal from $S_{i,k}$ with $\mathbb{E}\{\mathbf{x}_i\mathbf{x}_i^H\} = \mathbf{I}_K$ (the average transmit power of each terminal is P_{t}, $i = 1, 2$), $\mathbf{n}_{\mathrm{R}}$ is an $N \times 1$ additive white Gaussian noise (AWGN) vector at the relay node with $\mathbb{E}\{\mathbf{n}_{\mathrm{R}}\mathbf{n}_{\mathrm{R}}^H\} = \sigma_n^2\mathbf{I}_N$. The $N \times K$ channel matrix between the terminals S_i and R is expressed as $\mathbf{G}_i = \mathbf{H}_i\mathbf{D}_i^{1/2}$ where $\mathbf{H}_i$ contains independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$ entries and $\mathbf{D}_i$ is a $K \times K$ diagonal matrix with $[\mathbf{D}_i]_{k,k} = \eta_{i,k}$. Note that $\mathbf{H}_1$ and $\mathbf{H}_2$ represent fasting fading (small-scale fading), while $\mathbf{D}_1$ and $\mathbf{D}_2$ represent path-loss attenuation and log-normal shadow fading (large-scale fading). The assumption of i.i.d. fasting fading is sufficiently realistic for systems where the antennas are sufficiently well separated.

Note that the received signal $\mathbf{y}_{\mathrm{R}}$ can be rewritten as a more compact form, given by

$$\mathbf{y}_{\mathrm{R}} = \sqrt{P_{\mathrm{t}}}\mathbf{G}\mathbf{x} + \mathbf{n}_{\mathrm{R}} \tag{2}$$

where $\mathbf{G} \triangleq [\mathbf{G}_1\ \mathbf{G}_2]$ is $N \times 2K$ composite channel matrix, and $\mathbf{x} \triangleq [\mathbf{x}_1^T\ \mathbf{x}_2^T]^T$ is the transmitted signal vector from all terminals.

B. Phase II

The relay broadcast a scaled and linearly transformed version of the received signal to all terminals. Consider the two-way relay channel with K pairs as a one-way relay channel with $2K$ pairs where the groups of sources and destinations are the same. Then, we apply the relaying scheme for one-way relay channel with slight modification. We propose to let R transmit the following transformed version of the received signal:

$$\tilde{\mathbf{y}}_{\mathrm{R}} = \mathbf{W}\mathbf{y}_{\mathrm{R}} \tag{3}$$

where $\mathbf{W}$ is the $N \times N$ transformation matrix which will be described in detail in the next section. Let $n_{2,k}$ be the $\mathcal{CN}(0, \sigma_n^2)$ noise at $S_{2,k}$. Then, the received signal at $S_{2,k}$ is

$$\begin{aligned} y_{2,k} = &\underbrace{\sqrt{P_{\mathrm{t}}}\mathbf{g}_{2,k}^H\mathbf{W}\mathbf{g}_{1,k}x_{1,k}}_{\mathcal{L}_1} \\ &+ \underbrace{\sqrt{P_{\mathrm{t}}}\sum_{i=1,i\neq k}^{K}\mathbf{g}_{2,k}^H\mathbf{W}\mathbf{g}_{1,i}x_{1,i} + \sqrt{P_{\mathrm{t}}}\sum_{i=1}^{K}\mathbf{g}_{2,k}^H\mathbf{W}\mathbf{g}_{2,i}x_{2,i}}_{\mathcal{L}_2} \\ &+ \underbrace{\mathbf{g}_{2,k}^H\mathbf{W}\mathbf{n}_{\mathrm{R}} + n_{2,k}}_{\mathcal{L}_3} \end{aligned} \tag{4}$$

where $\mathbf{g}_{1,i}$ is the i-th column of $\mathbf{G}_1$, $\mathbf{g}_{2,i}$ is the i-th column of $\mathbf{G}_2$. Here, $\mathcal{L}_1$, $\mathcal{L}_2$, and $\mathcal{L}_3$ represent the desired signal, multi-terminal interference, and noise effects, respectively.

When the number of relay antennas N is large, the received signal at $S_{2,k}$ is dominated by the desired signal part (which includes $x_{1,k}$). As a result, we can obtain interference-free communication links when N grows without bound.

The associated instantaneous end-to-end (e2e) signal-to-interference-noise ratio (SINR) at $S_{2,k}$ is given by (5) on the top of the next page.

III. Amplify-and-Forward Transmission Scheme

In this section, we consider the transformation matrix $\mathbf{W}$ which is normalized to satisfy a total power constraint, P_{r}, at the relay as $\mathrm{Tr}\left(\mathbb{E}\left\{\tilde{\mathbf{y}}_{\mathrm{R}}\tilde{\mathbf{y}}_{\mathrm{R}}^H\right\}\right) = P_{\mathrm{r}}$.

A. Maximum-Ratio Combining at the Relay

When CSI is available at the relay for one-way relay channels, it is natural to apply a transformation based on the MRC/MRT principle [6]. Hence, the relay transformation matrix is given by $\mathbf{W} = a_{\mathrm{mrc}}\mathbf{G}_2\mathbf{G}_1^H$. In order to apply the relaying scheme for one-way relay channels, we propose to modify the transformation matrix as follows:

$$\mathbf{W} = a_{\mathrm{mrc}}(\mathbf{G}_2\mathbf{G}_1^H + \mathbf{G}_1\mathbf{G}_2^H) \tag{6}$$

Let $\mathbf{D} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{I}_K \\ \mathbf{I}_K & \mathbf{0} \end{bmatrix}$, recalling the definition of $\mathbf{G}$ in (2), the transformation matrix can be rewritten as

$$\mathbf{W} = a_{\mathrm{mrc}}\mathbf{G}\mathbf{D}\mathbf{G}^H \tag{7}$$

The normalization factor a_{mrc} controls the transmit power at the relay. Hence,

$$a_{\mathrm{mrc}} = \sqrt{\frac{P_{\mathrm{r}}}{\mathrm{Tr}\left(P_{\mathrm{t}}\mathbf{D}(\mathbf{G}^H\mathbf{G})^2\mathbf{D}(\mathbf{G}^H\mathbf{G}) + \sigma_n^2(\mathbf{D}\mathbf{G}^H\mathbf{G})^2\right)}} \tag{8}$$

B. Zero-Forcing at the Relay

We now consider the use of ZF receivers and precoders at the relay. With ZF processing, the transformation matrix can be expressed as

$$\begin{aligned} \mathbf{W} = &a_{\mathrm{zf}}\left(\mathbf{G}_2\left(\mathbf{G}_2^H\mathbf{G}_2\right)^{-1}\left(\mathbf{G}_1^H\mathbf{G}_1\right)^{-1}\mathbf{G}_1^H\right) \\ &+ a_{\mathrm{zf}}\left(\mathbf{G}_1\left(\mathbf{G}_1^H\mathbf{G}_1\right)^{-1}\left(\mathbf{G}_2^H\mathbf{G}_2\right)^{-1}\mathbf{G}_2^H\right) \end{aligned} \tag{9}$$

For notational simplicity, we will rewrite (9) as

$$\mathbf{W} = a_{\mathrm{zf}}\mathbf{H}\mathbf{D}\mathbf{H}^H \tag{10}$$

where $\mathbf{H} \triangleq \left[\ \mathbf{G}_1\left(\mathbf{G}_1^H\mathbf{G}_1\right)^{-1}\quad \mathbf{G}_2\left(\mathbf{G}_2^H\mathbf{G}_2\right)^{-1}\ \right]$. In this case, to meet the power constraint at the relay, we have

$$a_{\mathrm{zf}} = \sqrt{\frac{P_{\mathrm{r}}}{\mathrm{Tr}\left(P_{\mathrm{t}}\mathbf{D}\mathbf{H}^H\mathbf{G}\mathbf{G}^H\mathbf{H}\mathbf{D}\mathbf{H}^H\mathbf{H} + \sigma_n^2(\mathbf{D}\mathbf{H}^H\mathbf{H})^2\right)}} \tag{11}$$

$$\gamma_k = \frac{P_{\rm t}|\mathbf{g}_{2,k}^H \mathbf{W} \mathbf{g}_{1,k}|^2}{P_{\rm t}\sum_{i=1,i\neq k}^{K} |\mathbf{g}_{2,k}^H \mathbf{W} \mathbf{g}_{1,i}|^2 + P_{\rm t}\sum_{i=1}^{K} |\mathbf{g}_{2,k}^H \mathbf{W} \mathbf{g}_{2,i}|^2 + \|\mathbf{g}_{2,k}^H \mathbf{W}\|^2\sigma_n^2 + \sigma_n^2} \tag{5}$$

IV. Asymptotic ($N \to \infty$) Analysis

In this section, we further simplify the e2e SINR expression (5) in the very large N regime. These new expressions illuminate several aspects of the achievable rate vs. power efficiency performance in the considered two-way relay network.

We first review some related limit results for random vectors [8] which will be required for the asymptotic performance analysis. Let $\mathbf{p} \triangleq [p_1, \ldots, p_N]^T$ and $\mathbf{q} \triangleq [q_1, \ldots, q_N]^T$ be mutually independent $N \times 1$ vectors whose elements are i.i.d. zero-mean random variables (RVs) with $\mathbb{E}\left\{|p_i|^2\right\} = \sigma_p^2$, and $\mathbb{E}\left\{|q_i|^2\right\} = \sigma_q^2$, $i = 1, \ldots, N$. Then from the law of large numbers, we have

$$\frac{1}{N}\mathbf{p}^H\mathbf{p} \xrightarrow{a.s.} \sigma_p^2, \text{ and } \frac{1}{N}\mathbf{p}^H\mathbf{q} \xrightarrow{a.s.} 0, \text{ as } N \to \infty \tag{12}$$

where $\xrightarrow{a.s.}$ denotes the almost sure convergence. Also, from the Lindeberg-Lévy central limit theorem, we have

$$\frac{1}{\sqrt{N}}\mathbf{p}^H\mathbf{q} \xrightarrow{d} \mathcal{CN}(0, \sigma_p^2\sigma_q^2), \text{ as } N \to \infty \tag{13}$$

where $\xrightarrow{d}$ denotes convergence in distribution.

In the following analysis, we will consider three cases: namely, Case I) $P_{\rm t} = \frac{E_{\rm t}}{N}$, where $E_{\rm t}$ is fixed; Case II) $P_{\rm r} = \frac{E_{\rm r}}{N}$, where $E_{\rm r}$ is fixed; Case III) $P_{\rm t} = \frac{E_{\rm t}}{N}$, $P_{\rm r} = \frac{E_{\rm r}}{N}$, where $E_{\rm t}$ and $E_{\rm r}$ are fixed.

A. Maximum-Ratio Combining at the Relay

1) Case I: From (4) and (7) we have

$$\begin{aligned}\frac{y_{2,k}}{\sqrt{N}} &= \frac{a_{\rm mrc}\sqrt{E_{\rm t}}\mathbf{g}_{2,k}^H\mathbf{G}\mathbf{D}\mathbf{G}^H\mathbf{g}_{1,k}x_{1,k}}{N} \\ &+ \frac{a_{\rm mrc}\sqrt{E_{\rm t}}\sum_{i=1,i\neq k}^{K}\mathbf{g}_{2,k}^H\mathbf{G}\mathbf{D}\mathbf{G}^H\mathbf{g}_{1,i}x_{1,i}}{N} \\ &+ \frac{a_{\rm mrc}\sqrt{E_{\rm t}}\sum_{i=1}^{K}\mathbf{g}_{2,k}^H\mathbf{G}\mathbf{D}\mathbf{G}^H\mathbf{g}_{2,i}x_{2,i}}{N} \\ &+ \frac{a_{\rm mrc}\mathbf{g}_{2,k}^H\mathbf{G}\mathbf{D}\mathbf{G}^H\mathbf{n}_{\rm R}}{\sqrt{N}} + \frac{n_{2,k}}{\sqrt{N}}\end{aligned} \tag{14}$$

In the very large N regime, we apply the law of large numbers in (12) and obtain

$$\frac{\mathbf{G}_1^H\mathbf{G}_1}{N} \to \mathbf{D}_1, \ \frac{\mathbf{G}_2^H\mathbf{G}_2}{N} \to \mathbf{D}_2, \ \frac{\mathbf{G}_1^H\mathbf{G}_2}{N} \to \mathbf{0} \tag{15}$$

Thus,

$$\frac{\mathbf{G}^H\mathbf{G}}{N} \to \underbrace{\begin{bmatrix}\mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2\end{bmatrix}}_{\triangleq \mathbf{\Lambda}} \tag{16}$$

Note that in this case,

$$N a_{\rm mrc} \to \sqrt{\frac{P_{\rm r}}{\mathrm{Tr}\left(E_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2\right)}} \tag{17}$$

After some straightforward manipulations, when N goes to infinity, the second and third terms in (14) converge almost sure to 0. Therefore,

$$\begin{aligned}\frac{y_{2,k}}{\sqrt{N}} &\to \sqrt{\frac{P_{\rm r}E_{\rm t}}{\mathrm{Tr}\left(E_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2\right)}}\eta_{1,k}\eta_{2,k}x_{1,k} \\ &+ \sqrt{\frac{P_{\rm r}}{\mathrm{Tr}\left(E_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2\right)}}\eta_{2,k}\tilde{n}_{\rm R}\end{aligned} \tag{18}$$

where $\tilde{n}_{\rm R} \sim \mathcal{CN}(0, \eta_{1,k}\sigma_n^2)$. Now from (18) we obtain

$$\gamma_k^{\rm mrc} \to \frac{E_{\rm t}\eta_{1,k}}{\sigma_n^2} \tag{19}$$

2) Case II: In this case, we observe that

$$N^2 a_{\rm mrc} \to \sqrt{\frac{E_{\rm r}}{\mathrm{Tr}\left(P_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda}\right)}} \tag{20}$$

Similarly, when N grows without bound, the second, third and fourth terms in (4) tends to 0. Thus,

$$y_{2,k} \to \sqrt{\frac{E_{\rm r}}{\mathrm{Tr}\left(\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda}\right)}}\eta_{1,k}\eta_{2,k}x_{1,k} + n_{2,k} \tag{21}$$

From (21), we have

$$\gamma_k^{\rm mrc} \to \frac{E_{\rm r}\eta_{1,k}^2\eta_{2,k}^2}{\mathrm{Tr}(\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda})\sigma_n^2} \tag{22}$$

In the special case where $\eta_{1,k} = \eta_1$ and $\eta_{2,k} = \eta_2$ for $k = 1, \ldots, K$, we have $\gamma_k^{\rm mrc} \to \frac{1}{K}\frac{E_{\rm r}\eta_1\eta_2}{(\eta_1+\eta_2)\sigma_n^2}$.

3) Case III: In this case, using a similar approach as above we can show that

$$N\sqrt{N}a_{\rm mrc} \to \sqrt{\frac{E_{\rm r}}{\mathrm{Tr}\left(E_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2\right)}} \tag{23}$$

and

$$\begin{aligned}y_{2,k} &\to \sqrt{\frac{E_{\rm r}E_{\rm t}}{\mathrm{Tr}\left(E_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2\right)}}\eta_{1,k}\eta_{2,k}x_{1,k} \\ &+ \sqrt{\frac{E_{\rm r}}{\mathrm{Tr}\left(E_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2\right)}}\eta_{2,k}\tilde{n}_{\rm R} + n_{2,k}\end{aligned} \tag{24}$$

From (24), we have

$$\gamma_k^{\rm mrc} \to \frac{\frac{E_{\rm t}\eta_{1,k}}{\sigma_n^2}}{1 + \frac{\mathrm{Tr}(E_{\rm t}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2)}{E_{\rm r}\eta_{1,k}\eta_{2,k}^2}} \tag{25}$$

In the special case where $\eta_{1,k} = \eta_1$ and $\eta_{2,k} = \eta_2$ for $k = 1, \ldots, K$, we have $\gamma_k^{\rm mrc} \to \frac{\frac{E_{\rm t}\eta_1}{\sigma_n^2}}{1+\frac{K}{\frac{E_{\rm r}\eta_2}{\sigma_n^2}}\left(2+\frac{E_{\rm t}(\eta_1+\eta_2)}{\sigma_n^2}\right)}$.

B. Zero-Forcing at the Relay

By following a similar derivation as in the case of MRC, we can obtain the same power scaling laws as follows.

1) Case I: In the very large N regime we first note that

$$\frac{a_{\mathrm{zf}}}{N} \rightarrow \sqrt{\frac{P_{\mathrm{r}}}{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{\Lambda}^{-1} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2\right)}} \tag{26}$$

Then from (4) and (10) we have

$$\begin{aligned}\frac{y_{2,k}}{\sqrt{N}} \rightarrow & \sqrt{\frac{P_{\mathrm{r}}E_{\mathrm{t}}}{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{\Lambda}^{-1} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2\right)}}x_{1,k} \\ & + \sqrt{\frac{P_{\mathrm{r}}}{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{\Lambda}^{-1} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2\right)\eta_{1,k}^2}}\tilde{n}_{\mathrm{R}}\end{aligned} \tag{27}$$

Hence, when N grows without bound, we obtain

$$\gamma_k^{\mathrm{zf}} \rightarrow \frac{E_{\mathrm{t}}\eta_{1,k}}{\sigma_n^2} \tag{28}$$

2) Case II: In this case for very large N, a_{zf} tends to

$$a_{\mathrm{zf}} \rightarrow \sqrt{\frac{E_{\mathrm{r}}}{\operatorname{Tr}\left(P_{\mathrm{t}}\mathbf{\Lambda}^{-1}\right)}} \tag{29}$$

$y_{2,k}$ tends to

$$y_{2,k} \rightarrow \sqrt{\frac{E_{\mathrm{r}}}{\operatorname{Tr}\left(\mathbf{\Lambda}^{-1}\right)}}x_{1,k} + n_{2,k} \tag{30}$$

Therefore,

$$\gamma_k^{\mathrm{zf}} \rightarrow \frac{E_{\mathrm{r}}}{\operatorname{Tr}\left(\mathbf{\Lambda}^{-1}\right)\sigma_n^2} \tag{31}$$

3) Case III: In this case, when N tends to infinity, we obtain

$$\frac{a_{\mathrm{zf}}}{\sqrt{N}} \rightarrow \sqrt{\frac{E_{\mathrm{r}}}{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{\Lambda}^{-1} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2\right)}} \tag{32}$$

The received signal at $S_{2,k}$ tends to

$$\begin{aligned}y_{2,k} \rightarrow & \sqrt{\frac{E_{\mathrm{t}}E_{\mathrm{r}}}{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{\Lambda}^{-1} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2\right)}}x_{1,k} \\ & + \sqrt{\frac{E_{\mathrm{r}}}{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{\Lambda}^{-1} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2\right)\eta_{1,k}^2}}\tilde{n}_{\mathrm{R}} + n_{2,k}\end{aligned} \tag{33}$$

Therefore,

$$\gamma_k^{\mathrm{zf}} \rightarrow \frac{\frac{E_{\mathrm{t}}\eta_{1,k}}{\sigma_n^2}}{1 + \frac{\operatorname{Tr}(E_{\mathrm{t}}\mathbf{\Lambda}^{-1} + \sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2)\eta_{1,k}}{E_{\mathrm{r}}}} \tag{34}$$

In the special case where $\eta_{1,k} = \eta_1$ and $\eta_{2,k} = \eta_2$ for $k = 1, \ldots, K$, we have $\gamma_k^{\mathrm{zf}} \rightarrow \frac{\frac{E_{\mathrm{t}}\eta_1}{\sigma_n^2}}{1 + \frac{K}{\frac{E_{\mathrm{r}}\eta_2}{\sigma_n^2}}(2 + \frac{E_{\mathrm{t}}(\eta_1+\eta_2)}{\sigma_n^2})}$.

Table I summarizes the presented analytical asymptotic results in multipair two-way relay networks.

Remark 1: In all the cases, as N goes to infinity, the effects of inter-terminal interference and fast fading at $S_{2,k}$ disappear, thus we obtain an interference-free communication system.

TABLE I
THE E2E SNR IN MULTIPAIR TWO-WAY RELAY NETWORKS ($N \rightarrow \infty$)

	Case I	Case II	Case III
MRC	$\frac{E_{\mathrm{t}}\eta_{1,k}}{\sigma_n^2}$	$\frac{E_{\mathrm{r}}\eta_{1,k}^2\eta_{2,k}^2}{\operatorname{Tr}(\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda})\sigma_n^2}$	$\frac{\frac{E_{\mathrm{t}}\eta_{1,k}}{\sigma_n^2}}{1+\frac{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{D}\mathbf{\Lambda}^2\mathbf{D}\mathbf{\Lambda}+\sigma_n^2(\mathbf{D}\mathbf{\Lambda})^2\right)}{E_{\mathrm{r}}\eta_{1,k}\eta_{2,k}^2}}$
ZF	$\frac{E_{\mathrm{t}}\eta_{1,k}}{\sigma_n^2}$	$\frac{E_{\mathrm{r}}}{\operatorname{Tr}\left(\mathbf{\Lambda}^{-1}\right)\sigma_n^2}$	$\frac{\frac{E_{\mathrm{t}}\eta_{1,k}}{\sigma_n^2}}{1+\frac{\operatorname{Tr}\left(E_{\mathrm{t}}\mathbf{\Lambda}^{-1}+\sigma_n^2(\mathbf{D}\mathbf{\Lambda}^{-1})^2\right)\eta_{1,k}}{E_{\mathrm{r}}}}$

Remark 2: By using large antenna arrays at the relay, we can cut the transmit power at each terminal or/and the relay by a factor of $1/N$ with no reduction in performance. This result was originally established in [4] for the case when $N \gg K \gg 1$ whereas herein, we have generalized this result to the multi-pair two-way relay network where $N \gg 1$.

Remark 3: When $E_{\mathrm{t}} \rightarrow \infty$, the above SINR for case III coincides with the result for case II, and when $E_{\mathrm{r}} \rightarrow \infty$, the above SINR coincides with the result for case I. Note that this result is the same as that in [6] where one-way relay channel is considered.

V. NUMERICAL RESULTS AND DISCUSSION

In this section, we examine the sum rate of our proposed relaying scheme. For comparison, we also consider the sum rate of multipair one-way relaying scheme proposed in [6], and the sum rate of the conventional orthogonal scheme where the transmission of each pair is assigned different time slots or frequency bands. The achievable sum rate of the link $S_{1,k} \rightarrow R \rightarrow S_{2,k}$, $k = 1, \ldots, K$ is $C_{\mathrm{sum}}^{*} = \frac{1}{2}\mathbb{E}\left\{\sum_{k=1}^{K}\log_2(1+\gamma_k^{*})\right\}$ with $* = \{\mathrm{mrc}, \mathrm{zf}\}$. We choose $K = 5$, $\mathbf{D}_1 = \mathbf{D}_2 = \mathbf{I}_K$. For fair comparison, the total transmit powers of all schemes are the same.

Fig. 2 shows the simulated sum rate versus the number of relay antennas and the presented analytical asymptotic results for Case I. We can see that the number of relay antennas has a very strong impact on the performance. The sum rate increases significantly when we increase N. Interestingly, ZF processing at the relay performs better than the MRC counterpart on the way to the same asymptotic constant. Fig. 3 and Fig. 4 show results for the second and third power scaling laws respectively. Similar trends in results as in Fig. 2 can be observed.

Fig. 5 shows the sum rate versus the number of relay antennas for the different transmission schemes. For small N, owing to inter-terminal interference, our proposed scheme performs worse than the orthogonal scheme. However, as N increases, the effect of inter-terminal interference and noise dramatically reduces and hence, our proposed scheme outperforms the orthogonal scheme. Compared with the one-way relaying proposed in [6], our two-way relaying scheme is better when N is very large, and the advantage increases when N increases. However, our scheme suffers from more interference (see $\mathcal{L}_2$ in (4)) and therefore, the gain is somewhat less than a doubling. When N is small, the channels are

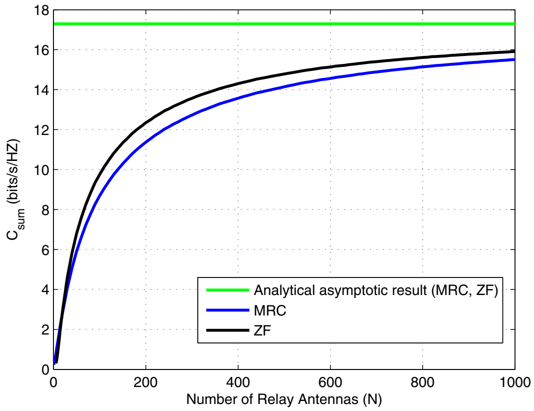

Fig. 2. *Case I)*: Sum rate versus the number of relay antennas ($E_{\rm t} = 10$ dB, $P_{\rm r} = 1$).

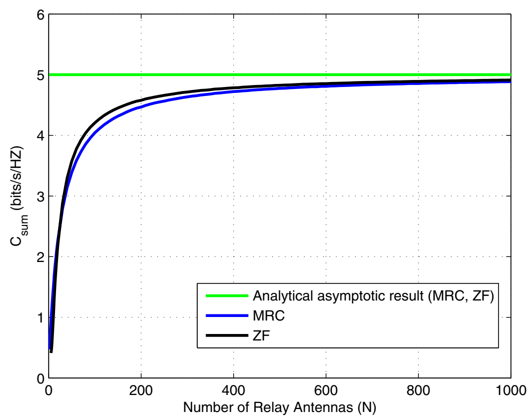

Fig. 3. *Case II)*: Sum rate versus the number of relay antennas ($P_{\rm t} = 1$, $E_{\rm r} = 10$ dB).

no longer nearly pairwise orthogonal and the inter-terminal interference cannot be notably canceled out and hence, our scheme is not better (even worse for ZF when $N < 200$) than the one-way relaying scheme.

VI. Conclusion

In this paper, we have shown that relay systems can benefit significantly from the use of very large antenna arrays. We have proposed two relay transformation matrices and derived asymptotic sum rates of a multi-pair two-way relay network for three different power scaling laws. The numerical results corroborate the validity of the derived asymptotic approximation.

Acknowledgment

The research is supported by National Natural Science Foundation of China under Grant No. 61171105, 61322110, and Program for New Century Excellent Talents in University of Ministry of Education of China under Grant No. NCET-11-0598.

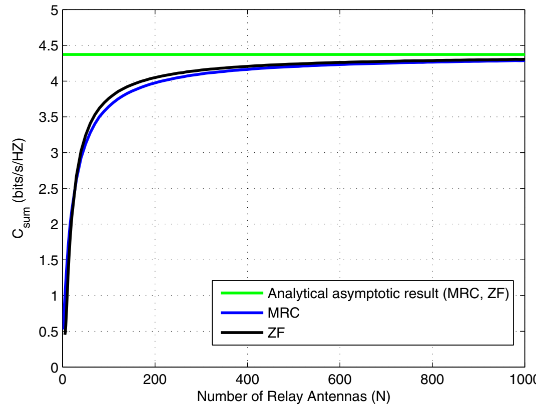

Fig. 4. *Case III)*: Sum rate versus the number of relay antennas ($E_{\rm t} = 10$ dB, $E_{\rm r} = 10$ dB).

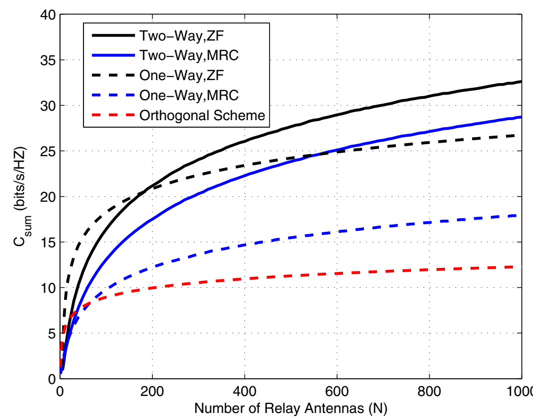

Fig. 5. Sum rate versus the number of relay antennas ($P_{\rm t} = 10$ dB, $P_{\rm r} = 10$ dB).

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