# Energy-Efficient Power and Subcarrier Allocation in Multiuser OFDMA Networks

Yuning Wang*†‡, Jianhua Zhang*† and Ping Zhang*†
*Key Lab of Universal Wireless Communications, Ministry of Education
†Beijing University of Posts and Telecommunications, Beijing 100876, P. R. China
‡ yuning.wang11@gmail.com

***Abstract*—This paper addresses the energy-efficient resource allocation problem in downlink Orthogonal Frequency Division Multiple Access (OFDMA) networks with multiple users. The joint optimization of power and subcarrier allocation to maximize bits-per-Joule is formulated constrained by the maximum transmit power and the minimum required system data rate. Due to the prohibitive computational complexity, this paper proposes an iteration-based sub-optimal scheme with superlinear convergence based on the Dinkelbach method by relaxing the constraints and exploiting the properties of non-linear fractional programming. The analytical solutions for the power and subcarrier allocation in each iteration are derived by the subgradient method according to Karush-Kuhn-Tucker (KKT) conditions and Lagrange multipliers. Finally, the simulation results validate the convergence and energy efficiency performance of the proposed scheme.**

***Index Terms*—OFDMA, energy efficiency, power allocation, subcarrier allocation**

## I. Introduction

The next generation wireless communication promises higher data rates and diverse radio interfaces to provide all users with a continuous seamless connection. However, the battery capacity for the state-of-the-art handsets lags behind the surge in power requirements for these multimedia-rich and multi-standard services [1]. Meanwhile, greenhouse gas emissions leave a significant environmental footprint. Thus the energy-efficient communication, also well-known as green radios, has drawn much attention in recent years and prompted new waves of research and standard activities [2]-[3]. Green radios will not only be beneficial for the global environment but also make commercial sense to gain sustainable profits.

OFDMA has emerged as one of the prime multiple access schemes for the future broadband wireless networks, such as WiMAX and the 3GPP LTE. In OFDMA systems, resources such as subcarriers and power are allocated appropriately to different users to enhance the performance of the system. The resource allocation in OFDMA-based systems has been extensively studied. But previous research mainly focuses on maximizing throughput, i.e., rate adaptation (RA) [4], or minimizing total transmit power, i.e., margin adaptation (MA) [5]. But neither of them can guarantee the energy-efficient resource allocation if evaluated in the overall bits transmitted per Joule of energy.

Recently, more efforts have been spent in designing energy-efficient resource allocation schemes in Orthogonal Frequency Division Multiplexing (OFDM) networks. A fixed-point algorithm for finding the energy-efficient power allocation is proposed in [6] under frequency-selective parallel additive white Gaussian noise (AWGN) channels. The optimal power allocation scheme to minimize energy-per-goodbit for the parallel channels of an OFDM system is presented in [7]. But both of them limit to the single user case. The energy-efficient resource allocation schemes to optimize throughput per Joule in the uplink OFDMA network are developed under flat fading [8] and frequency selective fading channels [9]. An energy-efficient scheduler with low-complexity is further developed in [10]. But all of them are discussed in uplink scenarios or mobile terminal sides. There are limited works about the energy efficient resource allocation in downlink OFDMA networks. Only in [11]-[13] the issue about the energy-efficient power allocation is discussed. However, none of them covers the topic about the subcarrier allocation.

Motivated by this consideration, this paper addresses the joint power and subcarrier allocation problem in downlink OFDMA networks for the green design target. An optimization problem to maximize throughput per Joule is formulated constrained by the maximum transmit power and the minimum required data rate. Since the problem is non-convex, the constraints are relaxed and the properties of non-linear fractional programming are exploited. Then according to the Dinkelbach technique, an efficient iterative-based scheme with superlinear convergence is proposed. The analytical solutions for the power and subcarrier allocation in each iteration are derived by the subgradient method based on the Karush-Kuhn-Tucker (KKT) conditions and Lagrange multipliers. Finally, simulation results present the performance of the proposed scheme.

The rest of the paper is organized as follows. In Section II, the system model is described. In Section III, the problem to maximize bits-per-Joule constrained by the maximum transmit power and the minimum required data rate is formulated. Then the suboptimal joint power and subcarrier allocation scheme is presented in Section IV. Section V demonstrates the numerical results with detailed analysis. Section VI concludes the paper finally.

## II. System Model

Consider a multiuser OFDM downlink system consisting of one source $S$ and $M$ users with a total number of $K$

subcarriers. The system bandwidth is $B$. The received signal on the $k_{\rm th}$ subcarrier at the $i_{\rm th}$ user is denoted as

$$y_{i,k} = \sqrt{p_{i,k}} h_{i,k} x_{i,k} + n_{i,k}, \quad (1)$$

where the transmit signal is denoted by $x_{i,k}$ with unit energy, i.e., $E\left(|x_{i,k}|^2\right) = 1$. We denote the transmit power and the channel state information (CSI) from $S$ to the $i_{\rm th}$ user on the $k_{\rm th}$ subcarrier by $p_{i,k}$ and $h_{i,k}$, respectively. It is assumed that the channel is block fading. That is it remains constant during each frame and is independent from one to another. We denote the additive white Gaussian noise on the $k_{\rm th}$ subcarrier at the $i_{\rm th}$ user by $n_{i,k}$ with zero mean and variance $\sigma_z^2$. Assuming perfect CSIs at users, the data rate on the $k_{\rm th}$ subcarrier at the $i_{\rm th}$ user is formulated as

$$r_{i,k} = W \log_2 \left(1 + p_{i,k} \Gamma_{i,k}\right), \quad (2)$$

where the bandwidth per subcarrier is denoted by $W = \frac{B}{K}$. The channel to noise ratio on the $k_{\rm th}$ subcarrier at the $i_{\rm th}$ user is denoted by $\Gamma_{i,k} = \frac{|h_{i,k}|^2}{\sigma_z^2}$. Then the overall data rate is formulated as

$$R = \sum_{i=1}^{M} \sum_{k=1}^{K} \omega_{i,k} r_{i,k}, \quad (3)$$

where we denote the subcarrier allocation indicator by $\omega_{i,k}$. It satisfies $\omega_{i,k} = 1$ if subcarrier $k$ is allocated to the $i_{\rm th}$ user and $\omega_{i,k} = 0$, otherwise.

The total power consumption $P$ consists of two parts: transmit power $P_t$ and circuit power $P_c$. The total transmit power is represented as

$$P_t = \sum_{i=1}^{M} \sum_{k=1}^{K} \omega_{i,k} \varepsilon p_{i,k}, \quad (4)$$

where the peak-to-average power ratio divided by the drain efficiency of the power amplifier is denoted by $\varepsilon$. The circuit power $P_c$ includes all transmitter circuitry, which is incurred by the baseband processing and the radio frequency (RF) transceiver frontend. As stated in [14], if the clock frequency is dynamically scaled with the data rate, it is reasonable to model the circuit power consumption as a linear function of the data rate with a constant offset. The power dissipation in the RF frontend during transmission can be modeled as a rate-independent constant. Then based on the above analysis, the circuit power dissipation is mathematically represented as

$$P_c = \alpha + \beta \sum_{i=1}^{M} \sum_{k=1}^{K} \omega_{i,k} r_{i,k}. \quad (5)$$

## III. Problem Formulation

For energy-efficient communications, it is desirable to transmit as much data as possible for a given amount of energy. Thus the number of bits transmitted per Joule of energy is used as the energy efficiency metric, which is defined as

$$U = \frac{R}{P} = \frac{R}{P_t + P_c}. \quad (6)$$

Then the problem to maximize bits-per-Joule in the overall network by allocating power and subcarriers among users is formulated as

$$\max_{\mathbf{P},\mathbf{\Omega}} \; U\left(\mathbf{P}, \mathbf{\Omega}\right) = \frac{R\left(\mathbf{P}, \mathbf{\Omega}\right)}{P\left(\mathbf{P}, \mathbf{\Omega}\right)} \quad (7)$$

$$\text{s.t.} \;\; \omega_{i,k} \in \{0, 1\}, \forall i, k, \quad (8)$$

$$\sum_{i=1}^{M} \omega_{i,k} \le 1, \forall k, \quad (9)$$

$$\sum_{i=1}^{M} \sum_{k=1}^{K} \omega_{i,k} r_{i,k} \ge R_{\min}, \quad (10)$$

$$\sum_{i=1}^{M} \sum_{k=1}^{K} \omega_{i,k} p_{i,k} \le P_{\max}, \quad (11)$$

$$p_{i,k} \ge 0, \forall i, k, \quad (12)$$

where $\mathbf{P} = \{p_{i,k} \ge 0, \forall i, k\}$ and $\mathbf{\Omega} = \{\omega_{i,k} \in \{0, 1\}, \forall i, k\}$. Constraints (8) and (9) imply that each subcarrier can be only assigned to one user. Constraint (10) guarantees the minimum required data rate in the system. Constraint (11) limits the maximum transmit power at the source. Constraint (12) ensures that the allocated power is a non-negative value.

## IV. The Energy-Efficient Power and Subcarrier Allocation Scheme

It is observed from (7)-(12) that the optimization problem is a mix-integer non-linear fractional programming. The fractional characteristic results from the ratio of two functions. The combinatorial nature results from the integer constraint for the subcarrier allocation. Thus in order to solve the problem in (7)-(12), the first step is to transform the objective function using some standard techniques from non-linear fractional programming.

### A. Transformation of the Objective Function

The feasible solution set of the optimization problem in (7) is defined as $\mathbf{S}$ and $\{\mathbf{P}, \mathbf{\Omega}\} \in \mathbf{S}$. Since the transmit power is constrained by (11) and (12), $\mathbf{S}$ is a compact set. Then (7) can be transformed into a new parameterized concave function with a parameter $q$, which is mathematically represented as

$$\begin{array}{ll} \max\limits_{\mathbf{P},\mathbf{\Omega}} & F(\mathbf{P}, \mathbf{\Omega}, q) = R(\mathbf{P}, \mathbf{\Omega}) - qP(\mathbf{P}, \mathbf{\Omega}) \\ \text{s.t.} & (8), (9), (10), (11), (12) \end{array} . \quad (13)$$

Let $(\mathbf{P}^*, \mathbf{\Omega}^*)$ be the optimal solution set in (13) and $q^* = \frac{R(\mathbf{P}^*, \mathbf{\Omega}^*)}{P(\mathbf{P}^*, \mathbf{\Omega}^*)}$. According to [15], the following statements are equivalent

$$\begin{array}{l} F\left(\mathbf{P}^*, \mathbf{\Omega}^*, q\right) > 0 \Leftrightarrow q < q^*, \\ F\left(\mathbf{P}^*, \mathbf{\Omega}^*, q\right) = 0 \Leftrightarrow q = q^*, \\ F\left(\mathbf{P}^*, \mathbf{\Omega}^*, q\right) < 0 \Leftrightarrow q > q^*. \end{array} \quad (14)$$

Then solving (13) is equivalent to finding the root of the nonlinear equation $F\left(\mathbf{P}^*, \mathbf{\Omega}^*, q\right) = 0$. Then on the basis of the nonlinear fractional programming in [15], a theorem is stated as follows.

TABLE I
THE PROPOSED POWER AND SUBCARRIER ALLOCATION SCHEME

| Algorithm 1 Energy-efficient power and subcarrier allocation |
| --- |
| 1: **Initialization** the maximum tolerance $\Delta$ and the maximum number of iterations $T_{\max}$ |
| 2: **Set** $q_0 = 0$ and the iteration index $m = 0$ |
| 3: **Repeat** |
| 4: Solve the problem in (13) for a given $q_m$ to obtain $(\mathbf{P}_m^*, \mathbf{\Omega}_m^*)$ |
| 5: $q_{m+1} \leftarrow \frac{R(\mathbf{P}_m^*, \mathbf{\Omega}_m^*)}{P(\mathbf{P}_m^*, \mathbf{\Omega}_m^*)}$ |
| 6: $m \leftarrow m+1$ |
| 7: **until** $\lvert F(q_m)\rvert \le \Delta$ or $m = T_{\max}$ |
| 8: **Return** $(\mathbf{P}^*, \mathbf{\Omega}^*) \leftarrow (\mathbf{P}_m^*, \mathbf{\Omega}_m^*)$ |

**Theorem:** The maximum energy efficiency $q^* = \frac{R(\mathbf{P}^*, \mathbf{\Omega}*)}{P(\mathbf{P}^*, \mathbf{\Omega}^*)}$ is achieved if and only if $\max_{\mathbf{P},\mathbf{\Omega}} R(\mathbf{P}, \mathbf{\Omega}) - q^* P(\mathbf{P}, \mathbf{\Omega}) = R(\mathbf{P}^*, \mathbf{\Omega}^*) - q^* P(\mathbf{P}^*, \mathbf{\Omega}^*) = 0$.

This theorem indicates that for a fractional optimization problem, an equivalent optimization problem exists with an objective function in the subtractive form, i.e., $R(\mathbf{P}, \mathbf{\Omega}) - q^* P(\mathbf{P}, \mathbf{\Omega})$. Then the power and subcarrier allocation solution for the transformed problem in (13) at $q = q^*$ yields the optimal solution for the original problem in (7)-(12). Then the Dinkelbach method in [15] can be adopted to solve this parametric problem. The detailed algorithm is shown in Table I. The algorithm converges to the optimal opoint with a superlinear convergence rate. The detailed convergence analysis is described in [16].

### B. Dual Problem

The complexity of finding the optimal solution in (7)-(12) by exhaustive search is $O(M^K K)$. In order to reduce the computational complexity and make the problem tractable, the binary variable $\omega_{i,k}$ is relaxed to be a real number within interval $[0, 1]$. Then the considered parameterized problem is rewritten as

$$\max_{\mathbf{P},\mathbf{\Omega}} F(\mathbf{P}, \mathbf{\Omega}, q) = R(\mathbf{P}, \mathbf{\Omega}) - qP(\mathbf{P}, \mathbf{\Omega}) \tag{15}$$

$$\text{s.t.} \quad 0 \le \omega_{i,k} \le 1, \forall i, k, \tag{16}$$

$$\sum_{i=1}^{M} \omega_{i,k} \le 1, \forall k, \tag{17}$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} \omega_{i,k} r_{i,k} \ge R_{\min}, \tag{18}$$

$$\sum_{i=1}^{M}\sum_{k=1}^{K} \omega_{i,k} p_{i,k} \le P_{\max}, \tag{19}$$

$$p_{i,k} \ge 0, \forall i, k, \tag{20}$$

which is called the primal problem. This continuous relaxation permits time sharing of each subcarrier. For the formulation of the dual problem, the Lagrangian function corresponding to the primal problem is first defined as

$$\begin{aligned} &L(\mathbf{P}, \mathbf{\Omega}, \delta, \theta, \lambda) = \sum_{i=1}^{M}\sum_{k=1}^{K} \omega_{i,k} r_{i,k} - \\ &q\left[\sum_{i=1}^{M}\sum_{k=1}^{K} \omega_{i,k}\varepsilon p_{i,k} + \left(\alpha + \beta\sum_{i=1}^{M}\sum_{k=1}^{K} \omega_{i,k} r_{i,k}\right)\right] \\ &-\delta\left(\sum_{k=1}^{K} \omega_{i,k} - 1\right) - \theta\left(R_{\min} - \sum_{i=1}^{M}\sum_{k=1}^{K} \omega_{i,k} r_{i,k}\right) \\ &-\lambda\left(\sum_{i=1}^{M}\sum_{k=1}^{K} \omega_{i,k} p_{i,k} - P_{\max}\right) \end{aligned}, \tag{21}$$

where $\delta$, $\theta$ and $\lambda$ are the non-negative Lagrange multipliers that account for (17), (18) and (19), respectively. Then the dual objective function problem is formulated as

$$g(\delta, \theta, \lambda) = \begin{cases} \max_{\mathbf{P},\mathbf{\Omega}} & L(\mathbf{P}, \mathbf{\Omega}, \delta, \theta, \lambda) \\ \text{s.t.} & (16) - (20) \end{cases} \tag{22}$$

and the dual problem is given as

$$\begin{aligned} &\min_{\delta,\theta,\lambda} \; g(\delta, \theta, \lambda) \\ &\text{s.t.} \quad \delta \ge 0, \theta \ge 0, \lambda \ge 0 \end{aligned}. \tag{23}$$

### C. Solution for the dual problem

Since a strictly feasible point exists for (21), strong duality holds based on Slater's condition and the KKT conditions are necessary and sufficient for optimality. Then differentiating with respect to $p_{i,k}$ and setting it to zero, the closed-form solution for the power allocation on the $k_{\text{th}}$ subcarrier at the $i_{\text{th}}$ user for a given $q$ is derived as

$$p_{i,k}^* = \left[\frac{W(1 - q\beta + \theta)}{(\lambda + q\varepsilon)\ln 2} - \frac{1}{\Gamma_{i,k}}\right]^+, \tag{24}$$

where $[x]^+ = \max\{0, x\}$. It is observed from (24) that the power allocation solution has the form of water-filling to level $\frac{W(1-q\beta+\theta)}{(\lambda+q\varepsilon)\ln 2}$. Similarly, taking derivative of (21) with respect to $\omega_{i,k}$, we derive

$$\begin{aligned} Q_{i,k} &\triangleq \frac{\partial L(\mathbf{P}, \mathbf{\Omega}, \delta, \theta, \lambda)}{\partial \omega_{i,k}} \\ &= W\log_2\left(1 + p_{i,k}^*\Gamma_{i,k}\right)(1 - q\beta + \theta) \\ &\quad -p_{i,k}^*(q\varepsilon + \lambda) - \delta, \end{aligned} \tag{25}$$

where we call $Q_{i,k}$ as the marginal benefit function. The continuous relaxation of $\omega_{i,k}$ generally yields a fractional valued solution. Thus we should round $\omega_{i,k}$ to 0 or 1 to get an integer-valued solution. Thus the closed-form solution for the subcarrier allocation indicator is derived as

$$\omega_{i,k}^* = \begin{cases} 1 & Q_{i,k} > 0 \\ 0 & \text{otherwise} \end{cases}. \tag{26}$$

Considering that the dual problem is always convex, it is guaranteed that the gradient-type schemes converge to the global optimum. Thus the dual problem in (23) can be solved by the subgradient method. At the first step of this scheme, $\delta^{(0)}$, $\theta^{(0)}$ and $\lambda^{(0)}$ are initialized. Then given $\delta^{(n)}$, $\theta^{(n)}$ and $\lambda^{(n)}$, the dual objective function in (22) can be computed, where the iteration index is denoted by $n$.

TABLE II
THE PROPOSED SUBGRADIENT SCHEME

| Algorithm 2 Subgradient scheme |
|---|
| 1: **Initialization** $\delta^{(0)}$, $\theta^{(0)}$, $\lambda^{(0)}$, $n=0$ and $M_{\text{out}}=0$ |
| 2: **while** $n \leq I_{\max}$ **do** |
| 3: **Find** $p^*_{i,k}$, $\omega^*_{i,k}$ using (24), (25) and (26) |
| 4: **update** $\delta^{(n)}$, $\theta^{(n)}$, $\lambda^{(n)}$ according to (27), $n=n+1$ |
| 5: **end while** |
| 6: **Return** $p^*_{i,k}$, $\omega^*_{i,k}$ |

Substituting $p^*_{i,k}$ and $\omega^*_{i,k}$ into (22), $\delta^{(n)}$, $\theta^{(n)}$ and $\lambda^{(n)}$ can be derived by utilizing the subgradient method. The subgradient updating equations are given as

$$\begin{aligned}
\delta^{(n+1)} &= \left[\delta^{(n)} - \mu_1^{(n)}\left(\sum_{k=1}^{K}\omega_{i,k} - 1\right)\right]^+ \\
\theta^{(n+1)} &= \left[\theta^{(n)} - \mu_2^{(n)}\left(\sum_{i=1}^{M}\sum_{k=1}^{K}\omega_{i,k} r_{i,k} - R_{\min}\right)\right]^+ , \\
\lambda^{(n+1)} &= \left[\lambda^{(n)} - \mu_3^{(n)}\left(P_{\max} - \sum_{i=1}^{M}\sum_{k=1}^{K}\omega_{i,k} p_{i,k}\right)\right]^+
\end{aligned} \tag{27}$$

where $\mu_1^{(n)}$, $\mu_2^{(n)}$ and $\mu_3^{(n)}$ are sequences of positive step size designed properly [17]. Then the solution of (23) can be obtained by the subgradient method with a sufficiently large number of iterations $I_{\max}$, which is shown in Table II.

## V. NUMERICAL RESULTS AND ANALYSIS

This section evaluates the performance of the proposed energy-efficient power and subcarrier allocation scheme and presents the simulations results. A single cell with radius $R$ is considered. The users are randomly located within the cell. Each transmitted signal suffers from the large-scale path loss proportional to $d^{-2}$, where $d$ denotes the transmission distance from $S$ to each user. The subcarriers experience independent identically distributed Rayleigh fading with unit average power gain. Simulation parameters are listed in Table III.

TABLE III
SIMULATION PARAMETERS

| Terms | Values |
|---|---|
| $B$ | 960kHz |
| $K$ | 64 |
| $\sigma_z^2$ | $1.4 \times 10^{-5}$ |
| $P_{\max}$ | 50dBm |
| $\alpha$ | 10 |
| $\beta$ | $10^{-5}$ |
| $\varepsilon$ | 2.5 |

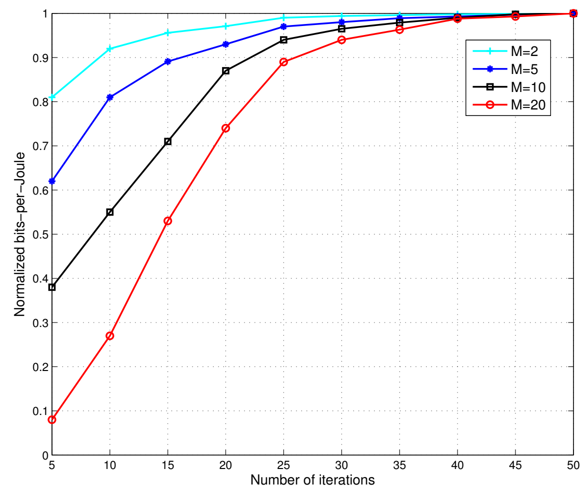


Fig. 1. The convergence of the proposed scheme

Fig. 1 illustrates the convergence of the proposed scheme under different number of users with $R = 100m$ and $R_{\min} = 1\text{kbps}$. The energy efficiency obtained by the proposed scheme is normalized by the optimal one. The optimal value is obtained by the numerical optimization. It can be observed that the energy efficiency of the proposed scheme converges to approximately 95% of the optimal solution within 35 iterations for all considered scenarios. Thus 35 iterations are adopted in the following simulations. Moreover, the fast convergence reveals that the proposed scheme is practically implemented.

Fig. 2 compares the energy efficiency performance of the proposed scheme with three other schemes as a function of the number of users under different minimum data rate requirements. The cell radius is 100m. The global optimum is obtained by the exhaustive search. The exhaustive search scheme chooses the optimal allocation pair of power and subcarrier combination which achieves the maximum energy efficiency for the overall network. The max-throughput scheme maximizes the system data rate constrained by (8)-(12). The equal power allocation (EPA) scheme allocates equal transmit power to each subcarrier and assigns each subcarrier to the destination with the maximum data rate. It can be observed that the proposed scheme achieves similar performance to the exhaustive search but with fast convergence as shown in Fig. 1. Moreover, the maximization of throughput cannot guarantee that the energy efficiency is maximal as indicated by the magenta lines. There is a trade-off between energy efficiency and spectral efficiency. The energy efficiency performance of EPA is the worst. It can be also seen from Fig. 2 that the energy efficiency is higher with the number of user increasing due to the multiuser diversity gain. But more energy is dissipated when the minimum required data rate is higher. The discrepancy becomes significant when a larger number of users exists in the network.

Fig. 3 depicts the energy efficiency performance of the above four schemes over the transmission distance between $S$ and users. Here, all users are located at the cell edge. The number of users is $M = 15$ and $R_{\min} = 1\text{kbps}$. In this simulation, the energy efficiency performance of the proposed scheme still approaches to the optimal solution and is better than max-throughput and EPA. As the cell radius increases, the energy efficiency decreases. More energy is required when users are far from the source.

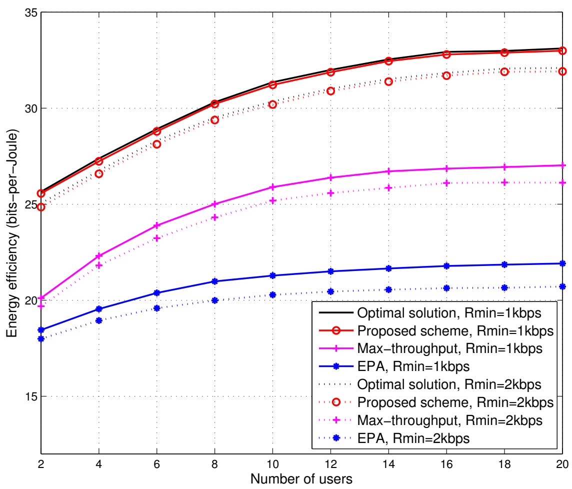


Fig. 2. The convergence of the proposed scheme

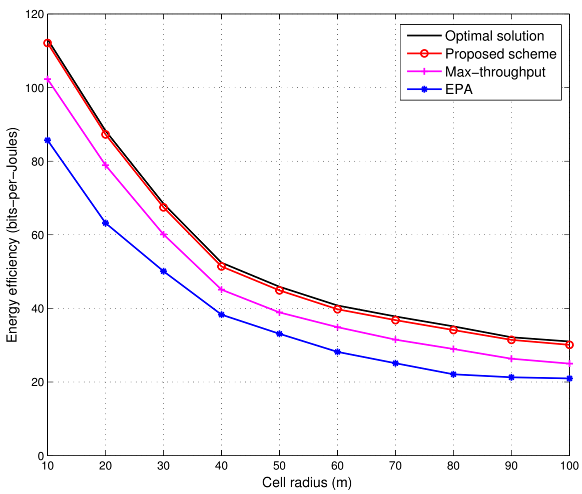


Fig. 3. The convergence of the proposed scheme

## VI. Conclusion

This paper has formulated the power and subcarrier allocation problem to maximize bits-per-Joule in downlink OFDMA networks with multiple users. Since the problem is non-convex, the constraints are relaxed and the properties of non-linear fractional programming are exploited. Then an iterative-based near-optimal scheme with superlinear convergence is proposed based on the Dinkelbach method. In each iteration, the closed-form solutions for the power and subcarrier allocation have be derived by the subgradient method according to the KKT conditions and Lagrange multipliers. The simulation results have shown that the proposed scheme has fast convergence. The energy efficiency performance of the proposed scheme approaches to the optimal solution. Moreover, the proposed scheme is more energy-efficient as either the number of users increases or the minimum required data rate decreases. But the proposed scheme is always more energy-efficient than the max-throughput scheme and the equal power allocation scheme.

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