Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
The Journal of China Universities of Posts and Telecommunications
October 2013, 20(5): 51–56
www.sciencedirect.com/science/journal/10058885
http://jcupt.xsw.bupt.cn

Mutual information assessment of LOS MIMO systems with reconfigurable antenna arrays

GAO Jun-jun[1](✉), ZHANG Jian-hua[2,3], TAO Xiao-feng[2,3]

1. Science and Technology on Information Systems Engineering Laboratory, Beijing 100038, China
2. Wireless Technology Innovation Institute, Beijing University of Posts and Telecommunications, Beijing 100876, China
3. Key Laboratory of Universal Wireless Communication, Ministry of Education, Beijing University of Posts and Telecommunications, Beijing 100876, China

Abstract

In presence of line-of-sight (LOS) propagation, multiple-input multiple-output (MIMO) systems can achieve maximum capacity over the whole signal-to-noise ratio (SNR) region by deploying reconfigurable antenna arrays. In this paper, the moment generating function (MGF) formulas of the mutual information (MI) is obtained for LOS MIMO systems with reconfigurable arrays. Then the exact expression of the mean MI is derived in an easily evaluated form. The results show an excellent match between the theoretical curves and the Monte-Carlo simulations.

Keywords multiple-input multiple-output, line-of-sight, mutual information, reconfigurable antenna array, Wishart

1 Introduction

MIMO systems have aroused a lot of academia and industry interest due to the promising channel capacity without the spending of additional bandwidth and transmit power [1]. Assuming independent and identically distributed (i.i.d.) Rayleigh fading channel, the capacity will linearly increase with the minimum amount of the transmit and receive antennas in theory. The existence of dominated line-of-sight (LOS) propagation is often unavoidable in practice, and the channel between each antenna pair can be commonly modeled as Ricean distribution. Due to the high spatial correlation of the received signal between different antenna pairs, LOS propagation is usually a great limitation to MIMO technique, which results in rank-deficient channel matrix.

An attractive scheme, known as the optimized LOS MIMO systems, is proposed in Ref. [2] to break through the LOS propagation limitation. The idea is that full-rank LOS channel is constructed by smart configuration of the transmit and receive antenna arrays, which will lead to LOS rays orthogonality. The performance of the optimized LOS MIMO system and its sensitivity to array displacement and orientation were experimentally evaluated through channel measurements [3]. Exact and high signal-to-noise ratio (SNR) asymptotic expressions for the mutual information (MI) statistics of the optimized LOS MIMO system were derived in [4]. Other approaches such as using reflectarray [5] and repeaters [6] to improve LOS MIMO channel capacity have appeared recently.

However, the optimized LOS MIMO systems only exhibit optimal performance in the high SNR region. To achieve maximum capacity at any given operating SNR, the reconfigurable antenna array scheme is proposed [7] that adaptively adjusting the transmit and receive array configuration as a function of the SNR. The MI statistics are crucial for MIMO performance assessment [8]. In this paper, the joint ordered eigenvalue probability density function (p.d.f.) is first derived for LOS MIMO systems with reconfigurable arrays based on the Wishart random matrix theory. Then the exact expression for the MI MGF is obtained. Based on the moment generating function (MGF),

Received date: 07-01-2013
Corresponding author: GAO Jun-jun, E-mail: gaojunjun1986@gmail.com
DOI: 10.1016/S1005-8885(13)60089-8

the expression for the mean MI (known as ergodic capacity) is derived in an easily evaluated form. The theoretical results are validated by Monte-Carlo simulations in the end. Note that the MGF and mean MI results of the optimized LOS MIMO systems in Ref. [4] are special cases of our proposed theory.

Notation: the (i,j) th element of the matrix $\boldsymbol{M}$ is $M_{i,j}$. The symbol $(\cdot)^\dagger$ represents the Hermitian transpose, and $\mathrm{tr}(\cdot)$ is the matrix trace. Both $\det(\cdot)$ and $|\cdot|$ denote the determinant.

2 System model

Considering a MIMO system with N_t transmit and N_r receive antennas, we define $s=\min(N_\mathrm{t},N_\mathrm{r})$ and $t=\max(N_\mathrm{t},N_\mathrm{r})$. In presence of LOS propagation, the channel $\boldsymbol{H}\in\mathbb{C}^{N_\mathrm{r}\times N_\mathrm{t}}$ can be modeled as

$$\boldsymbol{H}=\varepsilon\sqrt{K}\boldsymbol{H}_1+\varepsilon\boldsymbol{H}_\mathrm{n} \tag{1}$$

where $\boldsymbol{H}_1\in\mathbb{C}^{N_\mathrm{r}\times N_\mathrm{t}}$ is the arbitrary rank-L ($1\leqslant L\leqslant s$) deterministic matrix representing the LOS components, and $\boldsymbol{H}_\mathrm{n}\in\mathbb{C}^{N_\mathrm{r}\times N_\mathrm{t}}$ characterizes the random scattered components with i.i.d. entries modeled as $\mathcal{CN}(0,1)$. Hence, $\boldsymbol{H}$ has a non-zero central matrix $\boldsymbol{M}=\varepsilon\sqrt{K}\boldsymbol{H}_1$. The parameter K is the Ricean K factor expressing the power ratio between the LOS and scattered signal, and $\varepsilon=1/\sqrt{K+1}$ is a power normalization factor.

The MIMO correlation matrix is defined as

$$\boldsymbol{W}=\begin{cases}\boldsymbol{H}\boldsymbol{H}^\dagger; & N_\mathrm{t}\leqslant N_\mathrm{r}\\ \boldsymbol{H}^\dagger\boldsymbol{H}; & N_\mathrm{t}>N_\mathrm{r}\end{cases} \tag{2}$$

where $\boldsymbol{W}\in\mathbb{C}^{s\times s}$ follows a uncorrelated complex Wishart distribution, written as $\boldsymbol{W}\sim\boldsymbol{W}_s(t,\boldsymbol{\Sigma},\boldsymbol{\Omega})$, and

$$\boldsymbol{\Omega}=\begin{cases}\boldsymbol{\Sigma}^{-1}\boldsymbol{M}\boldsymbol{M}^\dagger; & N_\mathrm{t}\leqslant N_\mathrm{r}\\ \boldsymbol{\Sigma}^{-1}\boldsymbol{M}^\dagger\boldsymbol{M}; & N_\mathrm{t}>N_\mathrm{r}\end{cases} \tag{3}$$

is the arbitrary rank L central matrix with $\boldsymbol{\Sigma}=\varepsilon^2\boldsymbol{I}_s$. In following, we will consider a scaled release of $\boldsymbol{W}$, denoted as

$$\boldsymbol{S}=\boldsymbol{\Sigma}^{-1}\boldsymbol{W}\sim\boldsymbol{W}_s(t,\boldsymbol{I}_s,\boldsymbol{\Omega}) \tag{4}$$

and the ordered eigenvalues of $\boldsymbol{S}$ are written as $\phi_1>\phi_2>...\phi_s>0$.

It has been demonstrated in Ref. [9] that the rank of the capacity-maximizing input covariance matrix relies on the operating SNR. Based on the fact that the rank of the LOS matrix is interrelated with the array configuration, the idea of reconfigurable array is to reach a rank matching between the LOS channel and input covariance matrix. Three near-optimum rank-L ($L=1,\sqrt{s},s$) array configurations for uniform linear array (ULA) are summarized as follows

$$\left.\begin{aligned}&\text{rank-1}: d_\mathrm{T}=d_\mathrm{R}=\frac{\lambda}{8};\quad \rho<\rho_\mathrm{low}\\ &\text{rank-}\sqrt{s}: d_\mathrm{T}=d_\mathrm{R}=\frac{\sqrt{\lambda D}}{t};\quad \rho\in\left[\rho_\mathrm{low},\rho_\mathrm{high}\right]\\ &\text{rank-}s: d_\mathrm{T}=d_\mathrm{R}=\sqrt{\frac{\lambda D}{t}};\quad \rho>\rho_\mathrm{high}\end{aligned}\right\} \tag{5}$$

where d_T and d_R are the element spacings of the transmit and receive arrays respectively, and D is the distance between them. The wavelength of the carrier is λ. The approximate solutions of ρ_low and ρ_high can be found in Ref. [7]. The array configurations will result in identical non-zero central eigenvalues of the matrix $\boldsymbol{\Omega}$, represented as $\lambda_1=\lambda_2=...=\lambda_L=\lambda=Kst/L$. According to Ref. [7], the capacity-maximizing input only allocates power to the non-zero transmit dimension uniformly. Assuming $N_\mathrm{t}\leqslant N_\mathrm{r}$, thereby the input covariance matrix $\boldsymbol{Q}$ is

$$\boldsymbol{Q}=\frac{1}{L}\begin{bmatrix}\boldsymbol{I}_L & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\in\mathbb{C}^{N_\mathrm{t}\times N_\mathrm{t}} \tag{6}$$

The instantaneous MI is given by

$$I=\mathrm{lb}\det\left(\boldsymbol{I}_{N_\mathrm{t}}+\frac{1}{\sigma_\mathrm{n}^2}\boldsymbol{H}^\dagger\boldsymbol{Q}\boldsymbol{H}\right)=\sum_{l=1}^{L}\mathrm{lb}(1+\rho_\mathrm{c}\phi_l) \tag{7}$$

where $\rho_\mathrm{c}=\rho\varepsilon^2/L$, and ρ is the measured SNR at the receive side. The formula shows that the essence of the reconfigurable array is to achieve a tradeoff between the number of spatial data streams and corresponding received SNR.

3 MGF and MI

We are usually interested in the mean MI (ergodic capacity), which is the most important performance metric of MIMO systems. The joint eigenvalue p.d.f. is the fundmental to step into further MI analysis, and the p.d.f. of the maximum and minimum eigenvalue can also be derived, which is very important for the performance analysis of bemforming MIMO systems and others. So we first study the joint eigenvalue p.d.f. of LOS MIMO systems with reconfigurable arrays in the following theorem.

Theorem 1 The joint p.d.f. of the ordered eigenvalues $\phi_1 > \phi_2 > ...\phi_s > 0$ of the uncorrelated complex Wishart matrix in Eq. (4) with identical non-zero central eigenvalues is given by

$$f(\phi_1,\phi_2,...,\phi_s) = c|\boldsymbol{\Upsilon}|\prod_{i<j}^{s}(\phi_i-\phi_j)\prod_{k=1}^{s}\phi_k^{t-s}\mathrm{e}^{-\phi_k} \tag{8}$$

where

$$c = \frac{\mathrm{e}^{-\mathrm{tr}\boldsymbol{\Omega}}}{\Gamma_{s-L}(s-L)\Gamma_{s-L}(t-L)\Gamma_L(L)\lambda^{L(s-L)}} \tag{9}$$

and

$$\Gamma_m(n) = \prod_{i=1}^{m}(n-i)! \tag{10}$$

While the entry of the $s\times s$ matrix $\boldsymbol{\Upsilon}$ is given by

$$\Upsilon_{i,j} = \begin{cases} \dfrac{\phi_i^{L-j}\,{}_0F_1(t-s+L-j+1;\phi_i\lambda)}{(t-s+L-j)!}; \\ \qquad i=1,2,\ldots,s,\ j=1,2,\ldots,L \\ \phi_i^{s-j};\quad i=1,2,\ldots,s,\ j=L+1,2,\ldots,s \end{cases} \tag{11}$$

and ${}_0F_1(.;.)$ is the scalar Bessel-type hypergeometric function [10].

Proof The proof is based on the joint ordered eigenvalue p.d.f. of Wishart matrix given by Jin [11], where the central matrix $\boldsymbol{\Omega}$ has arbitrary rank with distinct non-zero central eigenvalues. For the case of uniform non-zero central eigenvalues, the key step is to calculate the following $0/0$ type determinant limit

$$\lim_{\lambda_1=\lambda_2=\ldots=\lambda_L=\lambda}\frac{|\boldsymbol{\Upsilon}|}{\prod_{i<j}^{L}(\lambda_i-\lambda_j)} \tag{12}$$

using l'Hôpital that taking the $(s-j)$th differential at $\lambda_j=\lambda$ across the jth column for both the numerator and denominator in Eq. (12). Some simple manipulations are performed to get the desired form.

Utilizing the eigenvalue p.d.f. expression in Theorem 1, we will derive the MGF expression of the MI in following. The MI MGF is a very efficient tool for the statistical analysis of the MI, and arbitrary moment of the MI can be easily obtained with derivation operations on the MI MGF expression.

Theorem 2 The MI MGF of the uncorrelated complex Wishart matrix in Eq. (4) with identical non-zero central eigenvalues is

$$G(\tau) = \mathrm{E}(\mathrm{e}^{\tau I}) = c\det\boldsymbol{\Psi}(\tau) \tag{13}$$

where the entry of the $s\times s$ matrix $\boldsymbol{\Psi}(\tau)$ is given in Eq. (14), $U(.;.;.)$ denotes the second kind confluent hypergeometric function [12], and ${}_1F_1(.;.;.)$ is the Kummer's hypergeometric function [12].

$$\psi_{ij}^{(\tau)} = \begin{cases} \displaystyle\sum_{k=0}^{\infty}\Big[\lambda^k(t+L+k-i-j)!U\Big(t+L+k-i-j+ \\ \quad 1,t+L+k-i-j+2+\frac{\tau}{\ln 2},\frac{1}{\rho_{\mathrm{c}}}\Big)\Big]\cdot \\ \quad \Big[k!(t-s+L+k-j)!\rho_{\mathrm{c}}^{t+L+k-i-j+1}\Big]^{-1}; \\ \qquad i=1,2,\ldots,L,\ j=1,2,\ldots,L \\ \Big[(t+s-i-j)!U\Big(t+s-i-j+1,t+s-i-j+ \\ \quad 2+\frac{\tau}{\ln 2},\frac{1}{\rho_{\mathrm{c}}}\Big)\Big]\Big(\rho_{\mathrm{c}}^{t+s-i-j+1}\Big)^{-1}; \\ \qquad i=1,2,\ldots,L,\ j=L+1,\ldots,s \\ [(t+L-i-j)!\,{}_1F_1(t+L-i-j+1;t-s+ \\ \quad L-j+1;\lambda)]\big[(t-s+L-j)!\big]^{-1}; \\ \qquad i=L+1,\ldots,s,\ j=1,2,\ldots,L \\ (t+s-i-j)!;\quad i=L+1,\ldots,s,\ j=L+1,\ldots,s \end{cases} \tag{14}$$

Proof Utilizing the definition of the MI MGF and Theorem 1, we have

$$\begin{aligned}\mathrm{E}(\mathrm{e}^{\tau I}) &= \int_D\prod_{k=1}^{s}(1+p_k\phi_k)^{\frac{\tau}{\ln 2}}f_{\phi_1,\ldots,\phi_s}(\phi_1,\ldots,\phi_s)\mathrm{d}\phi_1\ldots\mathrm{d}\phi_s = \\ &c\sum_{\mu}\mathrm{sgn}(\mu)\sum_{\sigma}\int_D\prod_{k=1}^{s}(1+p_{\sigma_k}\phi_{\sigma_k})^{\frac{\tau}{\ln 2}}\phi_{\sigma_k}^{t-k}\cdot \\ &\mathrm{e}^{-\phi_{\sigma_k}}\gamma_{\sigma_k,\mu_k}\mathrm{d}\phi_1\ldots\mathrm{d}\phi_s\end{aligned} \tag{15}$$

where $D=\{0<\phi_1<\ldots<\phi_s<\infty\}$ is the integral area, and p_k represents the power allocated to each spatial stream.

The solving of the Eq. (15) needs the following integral equation [13]

$$\sum\int_D f_{i_1}(x_1)f_{i_2}(x_2)\ldots f_{i_n}(x_n)\mathrm{d}x_1\ldots\mathrm{d}x_n = \prod_{i=1}^{n}\Big(\int_x^{\infty}f_i(y)\mathrm{d}y\Big) \tag{16}$$

Substituting into Eq. (15) and use the property of the matrix, we derive

$$\mathrm{E}(\mathrm{e}^{\tau I}) = c\det\boldsymbol{\psi}(\tau) \tag{17}$$

and the element of the matrix $\boldsymbol{\psi}(\tau)$ is given by

$$\psi(\tau)_{i,j}=\begin{cases}\int_0^{\infty}(1+\rho_c y)^{\frac{\tau}{\ln 2}}y^{t+L-i-j}\mathrm{e}^{-y}{}_0F_1(t-s+L-j+\\ \quad 1;y\lambda)\left[(t-s+L-j)!\right]^{-1}\mathrm{d}y;\\ \quad i=1,2,\ldots,L,\ j=1,2,\ldots,L\\ \int_0^{\infty}(1+\rho_c y)^{\frac{\tau}{\ln 2}}y^{t+s-i-j}\mathrm{e}^{-y}\mathrm{d}y;\\ \quad i=1,2,\ldots,L,\ j=L+1,\ldots,s\\ \int_0^{\infty}y^{t+L-i-j}\mathrm{e}^{-y}\dfrac{{}_0F_1(t-s+L-j+1;y\lambda)}{(t-s+L-j)!}\mathrm{d}y;\\ \quad i=L+1,\ldots,s,\ j=1,2,\ldots,L\\ \int_0^{\infty}y^{t+s-i-j}\mathrm{e}^{-y}\mathrm{d}y;\quad i=L+1,\ldots,s,\ j=L+1,\ldots,s\end{cases} \tag{18}$$

In following, we simplify the Eq. (18) using the property of integral functions. Utilizing the integral equation given in Ref. [10],

$$\int_0^{\infty}\mathrm{e}^{-\mu y}\left(1+\alpha y\right)^{-n}y^{m-1}\mathrm{d}y=\frac{\Gamma(m)}{\alpha^m}U\left(m,m+1-n,\frac{\mu}{\alpha}\right) \tag{19}$$

The first integral item in Eq. (18) can be simplified as

$$\int_0^{\infty}(1+\rho_c y)^{\frac{\tau}{\ln 2}}y^{t+L-i-j}\mathrm{e}^{-y}\frac{{}_0F_1(t-s+L-j+1;y\lambda)}{(t-s+L-j)!}\mathrm{d}y=\\ \sum_{k=0}^{\infty}\frac{\lambda^k}{k!(t-s+L+k-j)!}\frac{(t+L+k-i-j)!}{\rho_c^{t+L+k-i-j+1}}U\Big(t+\\ L+k-i-j+1,t+L+k-i-j+2+\frac{\tau}{\ln 2},\frac{1}{\rho_c}\Big) \tag{20}$$

and the second integral item can be directly derived.

Using the equation given in Ref. [10]

$$\int_0^{\infty}\mathrm{e}^{-x}x^{s-1}{}_pF_q\left(a_1,\ldots,a_p;b_1,\ldots,b_q;ax\right)\mathrm{d}x=\\ \Gamma(s)\,{}_{p+1}F_q\left(s,a_1,\ldots,a_p;b_1,\ldots,b_q;a\right) \tag{21}$$

The third integral item in Eq. (18) can be simplified as

$$\int_0^{\infty}y^{t+L-i-j}\mathrm{e}^{-y}\frac{{}_0F_1(t-s+L-j+1;y\lambda)}{(t-s+L-j)!}\mathrm{d}y=\frac{(t+L-i-j)!}{(t-s+L-j)!}\cdot{}_1F_1\left(t+L-i-j+1;t-s+L-j+1;\lambda\right) \tag{22}$$

Using the equation in Ref. [10], the last integral item in Eq. (18) can be simplified as

$$\int_0^{\infty}y^{t+s-i-j}\mathrm{e}^{-y}\mathrm{d}y=(t+s-i-j)! \tag{23}$$

The final form is completed after some simple combinations.

With the MGF in hand, the first-order mement of the MI can be derived to assess the ergodic capacity of MIMO systems. Here we present the exact expression for the mean MI (ergodic capacity) of the LOS MIMO systems with reconfigurable arrays.

Theorem 3 The mean MI of the uncorrelated complex Wishart matrix in Eq. (4) with identical non-zero central eigenvalues is

$$\mathrm{E}(I)=c\sum_{l=1}^{s}\det \boldsymbol{\Lambda}(l) \tag{24}$$

where the entry of the $s\times s$ matrix $\boldsymbol{\Lambda}(l)$ is given by

$$\Lambda_{i,j}(l)=\begin{cases}(t+L-i-j)!\,{}_1F_1(t+L-i-j+1;t-s+\\ \quad L-j+1;\lambda)\left[(t-s+L-j)!\right]^{-1};\\ \quad i=1,2,\ldots,s,\ j=1,2,\ldots,L,\ j\neq l\\ \dfrac{1}{\ln 2}\displaystyle\sum_{k=0}^{\infty}\Big\{(t+L+k-i-j)!\lambda^k\mathrm{e}^{\frac{1}{\rho_c}}\big[k!(t-s+L+\\ \quad k-j)!\big]^{-1}\Delta(t+L+k-i-j+1,\rho_c)\Big\};\\ \quad i=1,2,\ldots,L,\ j=1,2,\ldots,L,\ j=l\\ (t+s-i-j)!;\quad i=1,2,\ldots,s,\ j=L+1,\ldots,s,\ j\neq l\\ \dfrac{1}{\ln 2}(t+s-i-j)!\mathrm{e}^{\frac{1}{\rho_c}}\Delta(t+s-i-j,\rho_c);\\ \quad i=1,2,\ldots,L,\ j=L+1,\ldots,s,\ j=l\\ 0;\quad i=L+1,\ldots,s,\ j=1,\ldots,s,\ j=l\end{cases} \tag{25}$$

and

$$\Delta(m,\beta)=\sum_{k=1}^{m}\frac{\Gamma\left(-m+k,\frac{1}{\beta}\right)}{\beta^{m-k}}=\sum_{k=0}^{m-1}E_{k+1}\left(\frac{1}{\beta}\right) \tag{26}$$

with $\Gamma(a,x)=\int_x^{\infty}t^{a-1}\mathrm{e}^{-t}\mathrm{d}t$ representing the complementary incomplete gamma function [12], and $E_n(z)=\int_1^{\infty}t^{-n}\mathrm{e}^{-zt}\mathrm{d}t, n=0,1,2,\ldots,\mathrm{Re}\{z\}>0$ is the exponential integral function [12].

Proof The proof is based on the relationship between the n-th order moment of MI and its MGF

$$E\left[I^n\right]=\left.\frac{\mathrm{d}^n\left(G(\tau)\right)}{\mathrm{d}\tau^n}\right|_{\tau=0} \tag{27}$$

Since the mean MI (ergodic capacity) is the first order moment, setting n=1 in Eq. (27). The derivation of the determinant in Eq. (14) can be accomplished column by column using the product rule.

Utilizing the equation in Ref. [8]

$$\int_0^\infty \ln\left(1+ay\right) y^{n-1} \mathrm{e}^{-cy} \mathrm{d}y = \Gamma\left(n\right) \mathrm{e}^{\frac{c}{a}} \sum_{k=1}^{n} \frac{\Gamma\left(-n+k, \frac{c}{a}\right)}{c^k a^{n-k}} \tag{28}$$

The derivation with respect to τ in the first integral item in Eq. 错误！未找到引用源。and setting $\tau = 0$, we have

$$\begin{aligned}
&\frac{1}{\ln 2}\int_0^\infty \ln(1+\rho_\mathrm{c} y) y^{t+L-i-j} \mathrm{e}^{-y} \frac{{}_0F_1(t-s+L-j+1; y\lambda)}{\left(t-s+L-j\right)!} \mathrm{d}y = \\
&\quad \frac{1}{\ln 2}\int_0^\infty \frac{\ln(1+\rho_\mathrm{c} y) y^{t+L-i-j} \mathrm{e}^{-y}}{\left(t-s+L-j\right)!} \left(\sum_{k=0}^\infty \frac{1}{k!\left(t-s+L-j+1\right)_k}\cdot \right. \\
&\quad \left. \left(y\lambda\right)^k\right) \mathrm{d}y = \frac{1}{\ln 2}\sum_{k=0}^\infty \frac{\lambda^k}{k!\left(t-s+L+k-j\right)!}\cdot \\
&\quad \int_0^\infty \ln(1+\rho_\mathrm{c} y) y^{t+L+k-i-j} \mathrm{e}^{-y} \mathrm{d}y = \\
&\quad \frac{1}{\ln 2}\sum_{k=0}^\infty \frac{\lambda^k \mathrm{e}^{\frac{1}{\rho_\mathrm{c}}} \left(t+L+k-i-j\right)!}{k!\left(t-s+L+k-j\right)!}\cdot \\
&\quad \sum_{l=1}^{t+L+k-i-j+1} \frac{\Gamma\left(-\left(t+L+k-i-j+1\right)+l, \frac{1}{\rho_\mathrm{c}}\right)}{\rho_\mathrm{c}^{\,t+L+k-i-j+1-l}}
\end{aligned} \tag{29}$$

The derivation with respect to τ in the second integral item in Eq. (18) can be derived as

$$\begin{aligned}
&\frac{1}{\ln 2}\int_0^\infty \ln(1+\rho_\mathrm{c} y) y^{t+s-i-j} \mathrm{e}^{-y} \mathrm{d}y = \\
&\quad \frac{1}{\ln 2}\Gamma\left(t+s-i-j+1\right) \mathrm{e}^{\frac{1}{\rho_\mathrm{c}}} \cdot \\
&\quad \sum_{k=1}^{t+s-i-j+1} \frac{\Gamma\left(-\left(t+s-i-j+1\right)+k, \frac{1}{\rho_\mathrm{c}}\right)}{\rho_\mathrm{c}^{\,t+s-i-j+1-k}}
\end{aligned} \tag{30}$$

Finally, after the derivation with respect to the lth column in matrix $\boldsymbol{\psi}(\tau)$, we get the matrix $\boldsymbol{\Lambda}\left(l\right)$ and its element is given by

$$\boldsymbol{\Lambda}\left(l\right)_{i,j} = \begin{cases}
\dfrac{\left(t+L-i-j\right)!}{\left(t-s+L-j\right)!} {}_1F_1(t+L-i-j+1; t-s+L- \\
\quad j+1; \lambda);\quad i=1,2,\dots,L, j=1,2,\dots,L, j\neq l \\
\dfrac{1}{\ln 2}\displaystyle\sum_{k=0}^\infty \frac{\lambda^k \mathrm{e}^{\frac{1}{\rho_\mathrm{c}}} \left(t+L+k-i-j\right)!}{k!\left(t-s+L+k-j\right)!}\cdot \\
\quad \displaystyle\sum_{l=1}^{t+L+k-i-j+1} \frac{\Gamma\left(-\left(t+L+k-i-j+1\right)+l, \frac{1}{\rho_\mathrm{c}}\right)}{\left(\rho_\mathrm{c}\right)^{t+L+k-i-j+1-l}}; \\
\quad i=1,2,\dots,L, j=1,2,\dots,L, j=1 \\
\dfrac{\left(t+L-i-j\right)!}{\left(t-s+L-j\right)!} {}_1F_1(t+L-i-j+1; t-s+ \\
\quad L-j+1; \lambda);\quad i=L+1,\dots,s, \\
\quad j=1,2,\dots,L, j\neq l \\
\left(t+s-i-j\right)!;\quad i=1,2,\dots,L, j=L+1,\dots,s, j\neq l \\
\dfrac{1}{\ln 2}\left(t+s-i-j\right)! \mathrm{e}^{\frac{1}{\rho_\mathrm{c}}} \displaystyle\sum_{k=1}^{t+s-i-j+1} \left[\Gamma\left(-\left(t+s- \right.\right.\right. \\
\quad \left.\left.\left. i-j+1\right)+k, \frac{1}{\rho_\mathrm{c}}\right)\right] \rho_\mathrm{c}^{-(t+s-i-j+1-k)}; \\
\quad i=1,2,\dots,L, j=L+1,\dots,s, j=1 \\
\left(t+s-i-j\right)!;\quad i=L+1,\dots,s, j=L+1,\dots,s, j\neq l \\
0;\quad i=L+1,\dots,s, j=1
\end{cases} \tag{31}$$

Combining the repeated items in Eq. (31), we obtained the final form and completed the proof.

Note the exponential integral function in Eq. (23) can be efficiently calculated with the aid of the following iterative formula [12]

$$E_{n+1}\left(z\right) = \frac{1}{n}[\mathrm{e}^{-z} - zE_n\left(z\right)];\quad n=1,2,3,\dots \tag{32}$$

Remark 1 The theoretical MGF and mean MI expressions of the optimized LOS MIMO system proposed in Ref. [4] are special cases of our proposed theory for L=s.

4 Simulation results

The theoretical mean MI results are validated by Monte-Carlo simulations in this section. For three types of array configurations, 10 000 random channel realizations are generated according to Eq. (1) respectively. Ricean K factor is set to 7 dB which is a typical value in indoor hotspot scenario recommended by the ITU-R M.2135 channel model [14].

The theoretical curves and simulation results for three types of array configurations are plotted in Fig. 1 across the considered SNR scope. It is obvious that the theory and simulations match with each other perfectly. For relative low dimension MIMO array (e.g. $s=t=4$), the rank-$\sqrt{s}$ configuration outperforms the others in the medium SNR range but with very slight benefit. As the elements number of transmit and receive arrays increases (eg. $s=t=9$), the gain of the reconfigurable array become clearly remarkable. More elements can be mounted for a fixed size antenna array as the operating frequency going higher, for instance the wavelength is only 5 mm at 60 GHz which is much smaller than that at 2.4 GHz (12.5 cm). It suggests us that the reconfigurable array can provide attractive performance potential in several actual application scenarios, such as indoor wireless local area network (WLAN) and short-range communications [15]. Actually, the capacity gain of reconfigurable antenna arrays exists for any typical K values in practical LOS scenarios.

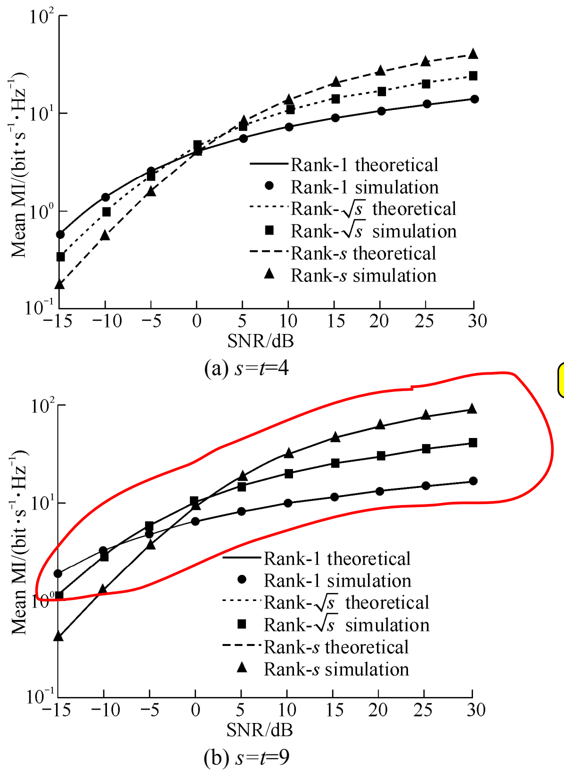

Fig. 1 Mean MI vs. SNR for the LOS MIMO systems with three types of array configurations (K=7 dB)

However, the channel degenerates into a pure i.i.d. Rayleigh channel as $K \to 0$, and the advantage of the reconfigurable antenna array vanishes.

5 Conclusions

LOS MIMO systems with reconfigurable arrays can achieve maximum capacity at any operating SNR by adaptively adjusting the array configurations. The eigenvalue p.d.f. of Wishart matrix with identical non-zero central eigenvalues is firstly obtained as the fundament of this paper. Then the exact expressions for the MGF and mean MI were derived. The proposed theory was validated by simulations. The numerical results indicate that the capacity benefit of the reconfigurable arrays is more distinct for high dimension antenna arrays.

Acknowledgements

The work was supported by the China Important National Science & Technology Specific Projects (2012ZX03001043-009, 2012ZX03006003-003), and the National Natural Science Foundation of China (61171105), and the Program for New Century Excellent Talents in University of Ministry of Education of China (NCET-11-0598), also by Telecommunications Science Technical Institute (Datang Telecommunication Technology and Industry Group).

References

1. Telatar E. Capacity of multi-antenna Gaussian channels. European Transactions on Telecommunications,1999, 10(6): 585–595
2. Bohagen F, Orten P, Oien G E. Construction and capacity analysis of high-rank line-of-sight MIMO channels. Proceedings of the 2005 IEEE Wireless Communications and Networking Conference: Vol 1, Mar 13–17, 2005, New Orleans, LA, USA. Piscataway, NJ, USA: IEEE, 2005: 432–437
3. Sarris I, Nix A R. Design and performance assessment of high-capacity MIMO architectures in the presence of a line-of-sight component. IEEE Transactions on Vehicular Technology, 2007, 56(4): 2194–2202
4. Matthaiou M, Pitarokoilis A, Nossek J A. Mutual information statistics of optimized LOS MIMO systems. Proceedings of the IEEE International Conference on Communications (ICC'10), May 23–27, Cape Town, South Africa. Piscataway, NJ, USA: IEEE, 2010: 5p
5. Jiyun S, Oda Y, Furuno T, et al. A novel approach for capacity improvement of 2×2 MIMO in LOS channel using reflectarray. Proceedings of the 73rd Vehicular Technology Conference (VTC-Spring'11), May 15–18, 2011, Budapest, Hungary. Piscataway, NJ, USA: IEEE, 2011: 5p
6. Walkenhorst B T, Ingram M A. Repeater-assisted capacity enhancement (RACE) for MIMO links in a line-of-sight environment. Proceedings of the IEEE International Conference on Communications (ICC'09), Jun 14–18, 2009, Dresden, Germany. Piscataway, NJ, USA: IEEE, 2009: 6p
7. Sayeed A M, Raghavan V. Maximizing MIMO capacity in sparse multipath with reconfigurable antenna arrays. IEEE Journal of Selected Topics in Signal Processing, 2007, 1(1): 156–166
8. Kang M, Alouini M S. Capacity of MIMO Rician channels. IEEE Transactions on Wireless Communications, 2006, 5(1): 112–122

To p. 70