Channel Estimation for Amplify-and-Forward Relay Networks with Both Time and Frequency Offsets

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***Abstract*—Cooperative multi-relay networks are asynchronous both in time and frequency inherently due to their spatial distributed nature. This paper addresses channel estimation for amplify-and-forward relay networks using training sequences in the presence of both time and frequency offsets. A practical training scheme is provided to reduce the complexity at the relay nodes and to combat the effects of time and frequency offsets at the destination. By exploiting the training scheme, a piecewise channel estimator with respect to signal-to-noise ratio (SNR) is proposed to reduce the computational complexity of the minimum mean square error (MMSE) method. Simulation results show that our method approaches the MMSE estimator and outperforms the linear least-square (LS) estimator.**

I. Introduction

Cooperative communication systems have recently attracted vast research attention due to their capability of increasing data throughput and robustness against signal fading in wireless networks [1]. The basic underlying concept of cooperative network is that the relay nodes share their antennas with source node to create a "virtual antennas". It has been pointed out that with proper cooperative strategies, the same benefits of multiple-input-multiple-output (MIMO) systems can be achieved in cooperative systems with only one antenna at each node. Several cooperative relaying protocols have been proposed and can be categorized into two principal classes: 1) amplify-and-forward (AF) [2], where the relay simply amplifies and forwards the source's signal to the destination terminal; 2) decode-and-forward (DF) [3], where the relay decodes, re-encodes and forwards the received signal to the destination. Compared with the DF scheme, the AF scheme is more attractive since it puts less processing burden on the relays. Therefore, we focus on the AF relay scheme in this paper.

To take advantage of all the benefits brought by the relay network, accurate channel state information (CSI) is required at the destination. One of the most widely used approaches to estimate the channel is training-based method which source node transmit known training sequences to the destination node. Recently, channel estimation for AF relay networks has been studied [4]-[7]. In [4], a training-based LS method and a suboptimal LMMSE channel estimator are proposed, while [5] provides an efficient method for a terminal to get all the CSI of the relay network. The optimal training for multiple AF relays network is addressed in [6], [7] and it is found that the optimal training can be achieved from a set of linear precoding matrices for all relays. However, all of these works mentioned above assume perfect synchronization of the received signal at the destination node.

Different from conventional MIMO systems, the relay nodes in distributed cooperative networks are spatially separated with their own oscillators and independent propagation delays to the destination, which results in multiple time and frequency Offsets. Moreover, the signal transmitted from different relays are overlapped with each other, so it is difficult to carry out resynchronization at the destination, since the alignment of one specific relay would misalign all the others. A common solution to this problem is to feedback the time and frequency information to the relays via a control channel and each relay properly adjust its transmitted signal. However, it will increase the burden on the system and introduce feedback delay [8]. A linear equalizer have been proposed in [9] to achieved considerable diversity gain in the presence of both time and frequency offsets, which means the assumption of perfect synchronization is not always necessary. Thus the multiple different time and frequency offsets at destination node may prohibit the direct application of the existing channel estimation methods such as [4]-[7], which assuming perfect synchronization.

In this paper, we study the training-based channel estimation methods for asynchronous AF relay networks. A piecewise channel estimator with respect to SNR is introduced to approximate the MMSE method and combat the effects of time and frequency offsets. By utilizing the training sequence, we avoid the matrix inverse operation, and the complexity at the destination is reduced significantly. Simulation results show that our method approaches the MMSE estimator and outperforms the linear least-square (LS) estimator.

II. System Model

A. Training Signal Model at the Relays

A half-duplex space-division multiple-access (SDMA) cooperative network with one source node $\mathbb{S}$, M relay nodes $\mathbb{R}_k$ ($k = 1, 2, \cdots, M$), and a single destination node $\mathbb{D}$ is considered in Fig. 1, where every node in the network has only one antenna that cannot transmit and receive simultaneously. Quasi-static and frequency flat-fading channel is considered, which is constant within one frame but may vary from frame

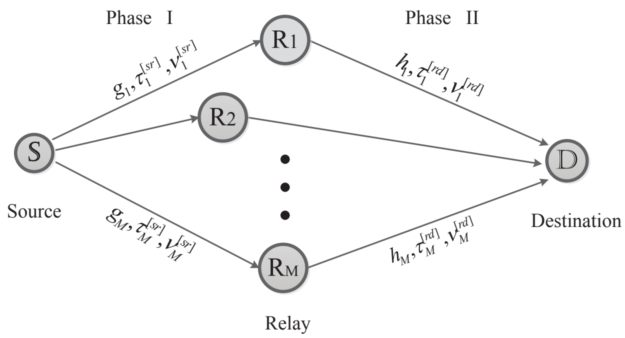

Fig. 1. The system model for the cooperative network.

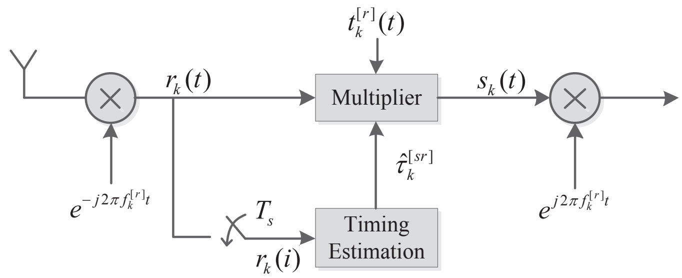

Fig. 2. Block diagram for AF kth relay transceiver.

to frame. We also assume that there is no direct link between the source and the destination. Denote the channel from $\mathbb{S}$ to $\mathbb{R}_k$, $\mathbb{R}_k$ to $\mathbb{D}$ and $\mathbb{S}-\mathbb{R}_k-\mathbb{D}$ as g_k, h_k, w_k,respectively; where $g_k \in \mathcal{CN}(0,\sigma_{gk}^2)$ and $h_k \in \mathcal{CN}(0,\sigma_{hk}^2)$. In Fig. 1, τ_k and ν_k are used to denote timing offsets and frequency offsets. Superscripts $(\cdot)^{[sr]}$, $(\cdot)^{[rd]}$ and $(\cdot)^{[sum]}$ denote offsets from $\mathbb{S}$ to $\mathbb{R}_k$, $\mathbb{R}_k$ to $\mathbb{D}$ and $\mathbb{S}-\mathbb{R}_k-\mathbb{D}$ respectively, and we assume that all the offsets have already been known at the destination. In the first phase, the source broadcasts its training sequence (TS) to the relays and the second phase the relays transmit M distinct TSs to the destination.

The block diagram for the AF transceiver at $\mathbb{R}_k$ is depicted in Fig. 2, which is used in timing and frequency estimation literature [10], [11]. The received signal at $\mathbb{R}_k$ is down converted to baseband by oscillator frequency $f_k^{[r]}$ to obtain the baseband received training signal $r_k(n)$ at time n, for $n = 1,\cdots,N$, is given by

$$r_k(n) = g_k e^{\frac{j2\pi n\nu_k^{[sr]}}{N}} t^{[s]}(n-\tau_k^{[sr]}) + u_k(n), \tag{1}$$

where

- N denotes the length of the TS;
- $\mathbf{t}^{[s]} \triangleq [t^{[s]}(1),\cdots,t^{[s]}(N)]^T$ is the known TS broadcast from the source to the relay nodes;
- $u_k(n)$ is the zero-mean additive white Gaussian noise (AWGN) at the kth relay. For convenience, all noise variances are assumed as N_0, namely $\mathcal{CN}(0, N_0)$.

As depicted in Fig. 2, we utilize a timing detector and a multiplier at each relay node to ensure successful cooperation for AF networks. After timing synchronization, the received signal $r_k(n)$ is multiplied by the unique unit amplitude training signal $t_k^{[r]}(n)$ at the appropriate time. This processing structure does not increase hardware complexity at the relays and can be used for both synchronization and channel estimation. Unlike conducting precoding at relay nodes, the phase shifting of this transceiver caused by the frequency offset is linear, thus the frequency offset $\nu_k^{[sr]}$ and $\nu_k^{[rd]}$ can be merged into $\nu_k^{[sum]}$, which lead to a simpler channel estimator expression. The design of training sequences will be discussed later. The output of the transceiver at the kth relay $s_k(n)$ is given by

$$\begin{aligned} s_k(n) =&\alpha_k g_k e^{\frac{j2\pi n\nu_k^{[sr]}}{N}} t^{[s]}(n-\tau_k^{[sr]}) t_k^{[r]}(n-\tau_k^{[sr]}) \\ &+ \alpha_k t_k^{[r]}(n-\tau_k^{[sr]}) v_k(n). \end{aligned} \tag{2}$$

α_k is the relay amplification factor to meet the power constraint for each relay terminal and is given by

$$\alpha_k = \sqrt{\frac{P_{rk}}{\sigma_{gk}^2 P_s + N_0}}, \tag{3}$$

where P_{rk} is the transmission power at the kth relay. The factor α_k considered in this paper does not depend on the instantaneous channel realization, thus no channel estimation is required at the relay terminals.

B. Training Signal Model at the Destination

The received signal at the destination is given by

$$\begin{aligned} y(n) &= \sum_{k=1}^{M} h_k e^{\frac{j2\pi n\nu_k^{[rd]}}{N}} s_k(n-\tau_k^{[rd]}) + v(n) \\ &= \sum_{k=1}^{M} \alpha_k w_k e^{\frac{j2\pi n\nu_k^{[sum]}}{N}} \bar{t}_k(n-\tau_k^{[sum]}) \\ &\quad \underbrace{+ \sum_{k=1}^{M} \alpha_k h_k e^{\frac{j2\pi n\nu_k^{[rd]}}{N}} t_k^{[r]}(n-\tau_k^{[sum]}) u_k(n-\tau_k^{[rd]}) + v(n),}_{\text{overall noise } \mathbf{n}_d(\text{n})} \end{aligned} \tag{4}$$

where

- $w_k = g_k h_k$ denotes the cascaded channel consisting of source-to relay and relay-to-destination links;
- $\bar{t}_k(n) = t^{[s]}(n) \cdot t_k^{[r]}(n)$ denotes the equivalent TS from $\mathbb{S}-\mathbb{R}_k-\mathbb{D}$ through the kth relay;
- $\tau_k^{[sum]} = \tau_k^{[sr]} + \tau_k^{[rd]}$ and $\nu_k^{[sum]} = \nu_k^{[sr]} + \nu_k^{[rd]}$ is the sum of timing and frequency offsets from $\mathbb{S}-\mathbb{R}_k-\mathbb{D}$, respectively;
- $v(n) \in \mathcal{CN}(0, N_0)$ represents the complex white Gaussian noise at $\mathbb{D}$.

Eq. (4) can be written in vector form as

$$\mathbf{y} = \sum_{k=1}^{M} \alpha_k w_k \mathbf{\Gamma}_k \bar{\mathbf{t}}_k(\tau_k^{[sum]}) + \mathbf{n}_d$$

$$
\begin{aligned}
&= \underbrace{[\boldsymbol{\Gamma}_1 \bar{\mathbf{t}}_1(\tau_1^{[sum]}), \boldsymbol{\Gamma}_2 \bar{\mathbf{t}}_2(\tau_2^{[sum]}), \cdots, \boldsymbol{\Gamma}_M \bar{\mathbf{t}}_M(\tau_M^{[sum]})]}_{\boldsymbol{\Xi}} \\
&\quad \times [\alpha_1 w_1, \cdots, \alpha_M w_M]^T + \mathbf{n}_d \\
&= \boldsymbol{\Xi}\boldsymbol{\Lambda}_\alpha \mathbf{w} + \mathbf{n}_d, \qquad (5)
\end{aligned}
$$

where $\boldsymbol{\Gamma}_{\rm k} \triangleq diag\{e^{j2\pi v_k^{[sum]}(0)/N}, \cdots, e^{j2\pi v_k^{[sum]}(N-1)/N}\}$, $\bar{\mathbf{t}}_k(\tau_k^{[sum]}) \triangleq [\bar{t}_k(-\tau_k^{[sum]}), \bar{t}_k(1-\tau_k^{[sum]}) \cdots, \bar{t}_k(N-1-\tau_k^{[sum]})]$, $\Lambda_\alpha = diag\{\alpha_1, \cdots, \alpha_M\}$, and $\mathbf{w} = [w_1, \cdots, w_M]^T$.

Furthermore, it can be easily obtained that the covariance of $\mathbf{n}_d$ conditioned on a specific h_k is

$$
Cov(\mathbf{n}_d | h_k, k = 1, \cdots, M) = (\sum_{k=1}^{M} |h_k|^2 \alpha_k^2 + 1) N_o \mathbf{I}, \qquad (6)
$$

when $\bar{t}_k(n)$ is unit amplitude signal. Therefore, the overall noise vector $\mathbf{n}_d$ is still white Gaussian but with a scaled covariance of $N_o\mathbf{I}$.

III. Training Based Channel Estimation

In this section, the overall channel w_k will be estimated at the destination in the presence of both time and frequency offsets. The statistics of g_k and h_k are assumed already known at $\mathbb{D}$ and the covariance of $\mathbf{h}$ and $\mathbf{g}$ are denoted as $\mathbf{R}_h$ and $\mathbf{R}_g$, respectively. Then, the covariance matrix of $\mathbf{w}$ is $\mathbf{R}_w = E\{\mathbf{w}\mathbf{w}^H\} = \mathbf{R}_g \odot \mathbf{R}_h$ where $\odot$ denotes the Hadamard product. We assume the channels from $\mathbb{S}$ to $\mathbb{R}_k$, $\mathbb{R}_k$ to $\mathbb{D}$ are independent, and the overall channels of $\mathbb{S}-\mathbb{R}_k-\mathbb{D}$ w_k through different relay are also independent. This assumption is valid, since the relay nodes in the distributed cooperative networks are usually far from each other, thus the channel correlation between different links can be ignored. Therefore the covariance matrix $\mathbf{R}_w$ turns to be a diagonal matrix.

A. LS Estimation

From (5), the LS estimation of $\mathbf{w}$ should be obtained from

$$
\hat{\mathbf{w}}_{LS} = \boldsymbol{\Lambda}_\alpha^{-1} (\boldsymbol{\Xi}^H \boldsymbol{\Xi})^{-1} \boldsymbol{\Xi}\mathbf{y}. \qquad (7)
$$

The MSE of the LS channel estimation can be derived as

$$
MSE(\hat{\mathbf{w}}_{LS}) = N_0 (\sum_{k=1}^{M} |h_k|^2 \alpha_k^2 + 1) \boldsymbol{\Lambda}_\alpha^{-1} (\boldsymbol{\Xi}^H \boldsymbol{\Xi})^{-1} \boldsymbol{\Lambda}_\alpha^{-1}. \qquad (8)
$$

B. MMSE Estimation

The linear MMSE estimator of $\mathbf{w}$ is expressed as

$$
\hat{\mathbf{w}}_{MMSE} = E\{\mathbf{w}\mathbf{y}^H\} (E\{\mathbf{y}\mathbf{y}^H\})^{-1} \mathbf{y}. \qquad (9)
$$

With straightforward calculations,

$$
\begin{aligned}
&E\{\mathbf{w}\mathbf{y}^H\} = \mathbf{R}_w \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H, \\
&\mathbf{R}_y = E\{\mathbf{y}\mathbf{y}^H\} = \boldsymbol{\Xi}\boldsymbol{\Lambda}_\alpha \mathbf{R}_w \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H + \underbrace{N_0 (\sum_{k=1}^{M} \sigma_{hk}^2 \alpha_k^2 + 1)}_{\sigma_\eta^2} \mathbf{I} \\
&\quad = \boldsymbol{\Xi}\boldsymbol{\Lambda}_\alpha \mathbf{R}_w \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H + \sigma_\eta^2 \mathbf{I}.
\end{aligned}
$$

Then

$$
\hat{\mathbf{w}}_{MMSE} = \mathbf{R}_w \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H (\boldsymbol{\Xi}\boldsymbol{\Lambda}_\alpha \mathbf{R}_w \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H + \sigma_\eta^2 \mathbf{I})^{-1} \mathbf{y}. \qquad (10)
$$

The MSE of the MMSE channel estimation can be derived as

$$
MSE(\hat{\mathbf{w}}_{MMSE}) = (\mathbf{R}_w^{-1} + \sigma_\eta^{-2} \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H \boldsymbol{\Xi} \boldsymbol{\Lambda}_\alpha)^{-1}. \qquad (11)
$$

To avoid the large dimension matrix inverse operation in (10), we use the matrix inversion lemma [12] to rewrite the inverse of $\mathbf{R}_y$ as

$$
\begin{aligned}
\mathbf{R}_y^{-1} &= \sigma_\eta^{-2}\mathbf{I} - \sigma_\eta^{-4} \boldsymbol{\Xi}\boldsymbol{\Lambda}_\alpha (\mathbf{I} + \sigma_\eta^{-2} \mathbf{R}_w \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H \boldsymbol{\Xi} \boldsymbol{\Lambda}_\alpha)^{-1} \mathbf{R}_w \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H \\
&= \sigma_\eta^{-2}\mathbf{I} - \sigma_\eta^{-4} \boldsymbol{\Xi}\boldsymbol{\Lambda}_\alpha \mathbf{R}_w^{1/2} (\mathbf{I} + \sigma_\eta^{-2} \mathbf{R}_w^{1/2} \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H \boldsymbol{\Xi} \boldsymbol{\Lambda}_\alpha \mathbf{R}_w^{1/2})^{-1} \\
&\quad \times \mathbf{R}_\omega^{1/2} \boldsymbol{\Lambda}_\alpha \boldsymbol{\Xi}^H \\
&= \sigma_\eta^{-2}\mathbf{I} - \sigma_\eta^{-2} \bar{\mathbf{U}} (\sigma_\eta^2 \mathbf{I} + \bar{\mathbf{U}}^H \bar{\mathbf{U}})^{-1} \bar{\mathbf{U}}^H \\
&= \sigma_\eta^{-2}\mathbf{I} - \sigma_\eta^{-2} \bar{\mathbf{U}} (\mathbf{R}_s)^{-1} \bar{\mathbf{U}}^H, \qquad (12)
\end{aligned}
$$

where $\bar{\mathbf{U}} = \boldsymbol{\Xi}\boldsymbol{\Lambda}_\alpha \mathbf{R}_w^{1/2}$, $\mathbf{R}_s = (\sigma_\eta^2 \mathbf{I} + \bar{\mathbf{U}}^H \bar{\mathbf{U}})$, and $\mathbf{R}_w$ is a diagonal matrix. With (12), we only need the inverse of an M-by-M matrix $\mathbf{R}_s$ rather than a N-by-N matrix $\mathbf{R}_y$. Thus the computational complexity is reduced, especially when the number of relays M is small.

C. Low Complexity Channel Estimator

In order to further reduce the complexity of Eq. (12), the Chu sequence is employed as the training sequence to get a low-complexity channel estimator.

1) Training Sequence Design: In [7], the authors studied the optimal training design for synchronous AF relays network. Through similar derivation, the optimal training in our systems should satisfy $\boldsymbol{\Xi}^H \boldsymbol{\Xi} = N\mathbf{I}$, where N is the length of TS (see [7] for more detail). In other words, the column vectors $\boldsymbol{\Gamma}_k \bar{\mathbf{t}}_k(\tau_k^{[sum]})$, $k = 1, \cdots, M$ of $\boldsymbol{\Xi}$ are orthogonal, which means the equivalent TSs $\bar{\mathbf{t}}_k$, $k = 1, \cdots, M$ must be remained orthogonal in the presence of time and frequency offsets. This condition is difficult to be satisfied, since the offsets are usually unknown at the transmitter. Here, we employ the Chu sequence as a suboptimal TS, thus $\boldsymbol{\Xi}^H \boldsymbol{\Xi}$ will approximate to a diagonal matrix if the frequency offsets are small (see appendix for proof). The Chu sequence is an N-phase sequence, which have perfect autocorrelation property and constant magnitude in both the time domain and the frequency domain. A Chu sequence with length N is given as:

$$
c(n) = \begin{cases} e^{j\pi q n^2/N}, & \text{for even N,} \\ e^{j\pi q n(n+1)/N}, & \text{for odd N,} \end{cases} \qquad (13)
$$

where $n \in (0, \cdots, N-1)$ and q are relatively prime to N. If $c(n)$ is assigned to the 1th relay as the training sequence $\bar{\mathbf{t}}_1$, then a cyclic shift version of $c(n)$ is assigned to the next relay, where the cyclic shift length is K_0. Thus the training sequence $\bar{\mathbf{t}}_k$ for the kth relay can be obtained by $c(\lfloor n + (k-1) \times K_0 \rfloor_N)$. Notice that K_0 must be larger than $\max\{\tau_1^{[sum]}, \cdots, \tau_M^{[sum]}\}$, in order to maintain the orthogonality between $\bar{\mathbf{t}}_k(\tau_k^{[sum]})$, $k = 1, \cdots, M$. Furthermore, the maximum number of active relays in the network is $\lfloor N/K_0 \rfloor$.

2) Channel Estimation with Time Offsets Only: Assume that only time offsets exist in the network, then the training sequence $\bar{\mathbf{t}}_k(\tau_k^{[sum]}), k = 1, \cdots, M$ are orthogonal when $K_0 > \max\{\tau_1^{[sum]}, \cdots, \tau_M^{[sum]}\}$, which means $\mathbf{\Xi}^H\mathbf{\Xi}$ a diagonal matrix. Thus the inverse of $\mathbf{R}_s$ is a diagonal matrix, written as

$$R_s^{-1} = (\sigma_\eta^2\mathbf{I} + \bar{\mathbf{U}}^H\bar{\mathbf{U}})^{-1} = (\sigma_\eta^2\mathbf{I} + N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1} = \Lambda_s. \quad (14)$$

Using (12) and (14), (9) can be rewritten as

$$\hat{\mathbf{w}}_{L-complex} = \sigma_\eta^{-2}\mathbf{R}_\omega^{1/2}\bar{\mathbf{U}}^H[\mathbf{I} + \bar{\mathbf{U}}\Lambda_s\bar{\mathbf{U}}^H]\mathbf{y}, \quad (15)$$

where the kth term on the diagonal of Λ_s is $(\Lambda_s)_{k,k} = \frac{1}{\sigma_\eta^2 + N\alpha_k^2\sigma_{hk}^2\sigma_{gk}^2}$.

3) Channel Estimation with Both Time and Frequency Offsets: In the presence of multiple frequency offsets, $\mathbf{R}_s$ is no longer a diagonal matrix, then the utilizing of (15) will lead to approximate error since the entries not on the diagonal are ignored. From (12), we find that the approximate error is multiplied by σ_η^{-2}, which means this error is negligible when SNR is low and increases with the rise of SNR. In Fig. 3, we display the MSEs versus SNR of the LS estimator (7), the MMSE estimator (10) and the low complexity estimator (15). There are two relays in the network, where the time offsets are uniform distribution in the interval $[0, K_0-1]$ and the frequency offsets $\nu^{[sum]} = \{0.2, 0.21\}$. It can be seen that Eq. (15) have the same MSE with MMSE estimation at low SNR, however, its performance is very poor under high SNR region due to the approximate error. Moreover, the LS and MMSE method have the same MSE at high SNR. Thus, in order to improve the performance of (15), a piecewise estimator is proposed as

$$\hat{\mathbf{w}}_{proposed} = \begin{cases} \hat{\mathbf{w}}_{L-complex} & \text{in(15)}, \quad \sigma_\eta^2 \geq \lambda_{Th} \\ \hat{\mathbf{w}}_{LS} & \text{in(6)}, \quad \sigma_\eta^2 < \lambda_{Th} \end{cases} \quad (16)$$

where λ_{Th} is a threshold chosen as the σ_η^2 which satisfies $MSE(\hat{\mathbf{w}}_{L-complex}) = MSE(\hat{\mathbf{w}}_{LS})$.

To complete this scheme, the closed-form solution of λ_{Th} is derived. First we rewrite $\mathbf{R}_s^{-1}$ as

$$\begin{aligned} R_s^{-1} &= (\sigma_\eta^2\mathbf{I} + \bar{\mathbf{U}}^H\bar{\mathbf{U}})^{-1} = (\sigma_\eta^2\mathbf{I} + N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w + \bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1} \\ &= (\sigma_\eta^2\mathbf{I} + N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1} - (\sigma_\eta^2\mathbf{I} + \bar{\mathbf{U}}^H\bar{\mathbf{U}})^{-1}(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w) \\ &\quad \times(\sigma_\eta^2\mathbf{I} + N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1}. \end{aligned} \quad (17)$$

The second term is the approximate error of (14), which leads to the estimate error of $\mathbf{w}$,

$$\begin{aligned} \Delta\mathbf{w} =& \sigma_\eta^{-2}\mathbf{R}_w^{1/2}\bar{\mathbf{U}}^H\bar{\mathbf{U}}(\sigma_\eta^2\mathbf{I} + \bar{\mathbf{U}}^H\bar{\mathbf{U}})^{-1}(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w) \\ &\times(\sigma_\eta^2\mathbf{I} + N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1}\bar{\mathbf{U}}^H\mathbf{y}. \end{aligned} \quad (18)$$

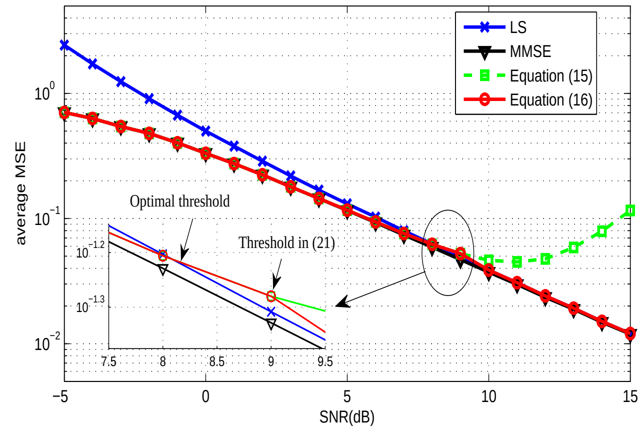

Fig. 3. Channel estimation MSEs versus SNR with both time and frequency offsets. The number of relays M is 2, N = 8, $K_0 = 4$ and $\nu^{[sum]} = \{0.2, 0.21\}$.

Notice that $\sigma_\eta^2\mathbf{I} \ll \bar{\mathbf{U}}^H\bar{\mathbf{U}}$ or $N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w$ at middle and high SNR, thus the entries $\sigma_\eta^2\mathbf{I}$ in (18) are ignored. After that

$$\begin{aligned} \Delta\mathbf{w}\Delta\mathbf{w}^H =& \sigma_\eta^{-4}\mathbf{R}_w^{1/2}(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)(N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1}\bar{\mathbf{U}}^H\mathbf{R}_y\bar{\mathbf{U}} \\ &\times(N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1}(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)\mathbf{R}_w^{1/2} \\ \approx& \sigma_\eta^{-4}\mathbf{R}_w^{1/2}(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)(N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1}(\bar{\mathbf{U}}^H\bar{\mathbf{U}})^2(N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1} \\ &\times(N(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)\mathbf{R}_w^{1/2}. \end{aligned} \quad (19)$$

The MSE of $\hat{\mathbf{w}}_{L-complex}$ is defined as

$$\begin{aligned} MSE(\hat{\mathbf{w}}_{L-complex}) &= \frac{1}{M}\mathrm{E}\{\left\|\hat{\mathbf{w}}_{L-complex} - \mathbf{w}\right\|^2\} \\ &= \frac{1}{M}\mathrm{E}\{\|\Delta\mathbf{w} + \Delta\mathbf{w}_n\|^2\} \\ &= \frac{1}{M}trace(\mathrm{E}\{\Delta\mathbf{w}\Delta\mathbf{w}^H + \Delta\mathbf{w}_n\Delta\mathbf{w}_n^H + \Delta\mathbf{w}\Delta\mathbf{w}_n^H + \Delta\mathbf{w}_n\Delta\mathbf{w}^H\}). \end{aligned} \quad (20)$$

Where $\Delta\mathbf{w}_n$ is the estimation error caused by Gaussian noise, while $\Delta\mathbf{w}$ derived in (18) is caused by the approximate error, we assume they have similar values around the threshold. Thus, the MSE turn to be $\frac{4}{M}trace(\mathrm{E}\{\Delta\mathbf{w}\Delta\mathbf{w}^H\})$. Moreover, the relay amplification factor α_k is set to be $\sqrt{P_{rk}/\sigma_{gk}^2 P_s}$ to get a direct solution.

Finally, λ_{Th} is shown by (21) at the bottom of the page. In order to prove the effectiveness of (21), Table I provides the threshold of SNR calculated from λ_{Th} and the optimal threshold obtained through simulation. The system parameters are set to the same values as Fig. 3 except frequency offsets. It can be found that the frequency difference $\Delta\nu^{[sum]}$ between relays influences the value of threshold, since $\Delta\nu^{[sum]}$ will destroy the orthogonality of TSs. Notice that the estimation error

$$\lambda_{Th} = \left(\frac{4trace\{\mathbf{R}_w^{1/2}(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)(N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1}(\bar{\mathbf{U}}^H\bar{\mathbf{U}})^2(N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)^{-1}(\bar{\mathbf{U}}^H\bar{\mathbf{U}} - N\mathbf{\Lambda}_\alpha^2\mathbf{R}_w)\mathbf{R}_w^{1/2}\}}{trace\{(\mathbf{\Lambda}_\alpha\mathbf{\Xi}^H\mathbf{\Xi}\mathbf{\Lambda}_\alpha)^{-1}\}}\right)^{1/3}. \quad (21)$$

of the threshold is usually within 1 dB, and the performance degradation is acceptable in this case.

TABLE I
COMPARISON OF SNR THRESHOLD

SNR threshold (dB)	$\Delta\nu^{[sum]} = \left|\nu_1^{[sum]} - \nu_2^{[sum]}\right|$				
	0.01	0.05	0.1	0.2	0.5
Optimal	8.1	5.0	3.7	2.6	1.6
Calculated	8.77	5.12	3.15	1.78	0.63

D. Computational Complexity

We briefly compare the complexities of the different algorithms, where the number of complex multiplications is considered as a complexity metric. The product of an $(a \times b)$ matrix with an $(b \times c)$ matrix requires $O(abc)$, the inversion of an $(a \times a)$ matrix requires $O(a^3)$ operations. Notice the value of threshold λ_{Th} is only affected by the frequency offsets and the length of TS N, thus λ_{Th} can be calculated and stored beforehand at the destination. Therefore the complexity of calculating λ_{Th} can be neglected. The complexity of each estimation algorithm is shown in Tables II. As expected, the proposed algorithm reduces the complexity of the MMSE method.

TABLE II
COMPLEXITIES OF FREQUENCY OFFSETS ESTIMATION ALGORITHMS

Estimator	Computational Complexity	Example: M=3,N=128
LS	$O(M^3 + 2M^2N + MN)$	2.7×10^3
MMSE using (10)	$O(N^3 + 2N^2M + N^2)$	2.2×10^6
Eq. (16)	$O(2N^2M + NM)$ or $O(M^3 + 2M^2N + MN)$	9.8×10^4 or 2.7×10^3

IV. SIMULATION RESULTS

In this section, the performance of the proposed channel estimation algorithms for cooperative relay systems with time and frequency offsets is evaluated by simulations. The channels g_k, h_k are assumed as independent circularly symmetric complex Gaussian random variables with unit variances. The additive noise at the relays and destination is considered white Gaussian with zero mean and the variance according to the received SNR. The overall time offset for each relay is randomly generated according to the cyclic shifting length K_0, which uniform distributed over $[0, K_0 - 1]$. Furthermore, the performance of channel estimation will be measured by the MSE which is defined as $E\{\|\hat{\omega} - \omega\|\}/M$.

Fig. 4 shows the MSE versus SNR of LS estimator (7), MMSE estimator (10) and the low-complexity estimator (15) for M= 3 relay nodes only in the presence of time offsets only. The MMSE estimator and the low-complexity estimator have the same performance which outperform the LS estimator at lower SNR region, whereas these estimators have nearly the same performance at higher SNR regimes. This is a consistent phenomenon as in the synchronous relay networks channel estimation [7]. The result illustrates that our scheme completely eliminates the effects of time offsets and approaches the MMSE estimator with lower complexity. It is also seen that the channel estimation MSE decreases when N increases.

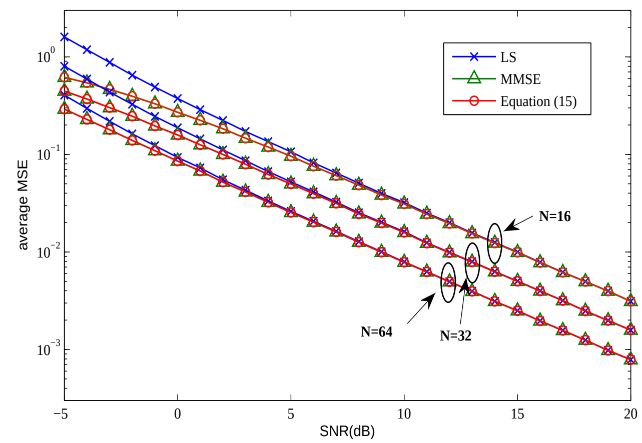

Fig. 4. Channel estimation MSEs versus SNR with time offsets only. The number of relays M is 3, K_0 = 4 and the length of cyclic prefix is 4.

The MSE performance of the channel estimators for M = 3 relay nodes with both time and frequency offsets is shown in Fig. 5. It is seen that the channel estimation MSE decreases when N increases and the values of frequency offsets influence the threshold of SNR, where large offsets will lead to a low threshold value. Results illustrate that the MSE performance of the proposed method (16) can approach very closely with that of the MMSE method, and outperform the LS method.

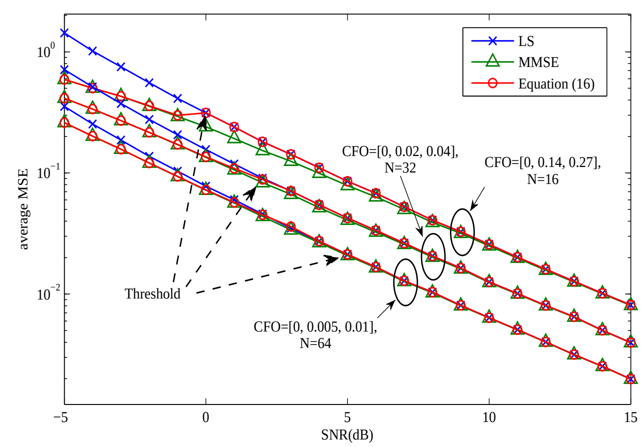

Fig. 5. Channel estimation MSEs versus SNR with both time and frequency offsets. The number of relays M is 3, K_0 = 4, and the length of cyclic prefix is 4.

V. CONCLUSIONS

In this paper, we have studied the training-based channel estimation for AF relay networks with both time and frequency offsets. The LS and MMSE methods have both been provided, and then a piecewise method was proposed to reduce the complexity and to combat the effects of the offsets in the network. A direct solution of the threshold of the proposed piecewise method was derived. Utilizing the threshold, the MSE performance of the proposed estimator can approach very closely with that of the MMSE estimator with lower complexity. Numerical results have been provided to validate the proposed studies.

Appendix
Analysis of $\boldsymbol{\Xi}^H\boldsymbol{\Xi}$ with Frequency Offsets

To simplify the derivation, we assume time offsets are zero since they do not destroy the orthogonality between training sequences when $\tau^{[sum]} < K_0$ and the length of training sequence are even. From (5), it can be obtained that

$$[\boldsymbol{\Xi}^H\boldsymbol{\Xi}]_{ij} = \begin{cases} N, & i = j \\ \bar{\mathbf{t}}_i^H \boldsymbol{\Gamma}_i^H \boldsymbol{\Gamma}_j \bar{\mathbf{t}}_j, & i \neq j \end{cases} . \quad (22)$$

We define

$$\begin{cases} b_1(n) = c(n) \cdot e^{j2\pi\nu_i n/N} \\ b_2(n) = c(\lfloor n + (j-i)K_0 \rfloor_N) \cdot e^{j2\pi\nu_j n/N} = c(\lfloor n+m \rfloor_N) e^{j2\pi\nu_j n/N} \end{cases}$$

where $m = (j-i)K_0$. Then

$$\begin{aligned} [\boldsymbol{\Xi}^H\boldsymbol{\Xi}]_{ij,i\neq j} &= \sum_{n=0}^{N-1} b_1(n) b_2^*(n) \\ &= \sum_{n=0}^{N-m-1} c(n) \cdot c^*(n+m) \cdot e^{j2\pi(\nu_i-\nu_j)n/N} \\ &\quad + \sum_{n=N-m}^{N-1} c(n) \cdot c^*(n+m-N) \cdot e^{j2\pi(\nu_i-\nu_j)n/N} \\ &= \sum_{n=0}^{N-1} e^{\frac{j\pi q[n^2-(n+m)^2]}{N}} \cdot e^{j\frac{2\pi(\nu_i-\nu_j)n}{N}} \\ &= \frac{\sin(\pi\Delta\nu)}{\sin(\frac{\pi}{N}(qm - \Delta\nu))} \cdot e^{j\pi[\Delta\nu(N-1)+qm]/N} . \end{aligned} \quad (23)$$

Notice that q and N are relatively prime, thus the entry $[\boldsymbol{\Xi}^H\boldsymbol{\Xi}]_{ij,i\neq j}$ is much smaller than $[\boldsymbol{\Xi}^H\boldsymbol{\Xi}]_{ii} = N$, when $\Delta\nu$ is small.

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