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# Comments on "Performance Analysis of MRC Diversity for Cognitive Radio Systems"

Wei Xu, Jianhua Zhang, and Ping Zhang

***Abstract*—The exact asymptotic symbol error rate (SER) for cognitive ratio systems with maximum ratio combining (MRC) considered by Li is derived, which circumvents the precondition requirement in the derivation made by Li. Monte Carlo simulations validate the theoretical analysis.**

***Index Terms*—Cognitive radio, diversity order, maximum ratio combining (MRC), symbol error rate (SER).**

## I. Introduction

In [1], the ergodic capacity and average symbol error rate (SER) are asymptotically investigated to show the capacity scaling law and the achievable diversity order for the cognitive ratio system with maximum ratio combining (MRC). However, the asymptotic average SER analysis in [1, Sec. IV] is only evaluated under the condition $P \ll Q$, where $P$ is the maximum transmit power of the cognitive user and $Q$ is the interference power constraint at the primary user. However, if this condition does not hold,[1] the accuracy of the asymptotic SER in [1] will decrease. In this commentary, it is pointed out that the requirement $P \ll Q$ is not necessary for the derivation of exact asymptotic SER. Moreover, it is shown that the asymptotic SER in [1] is just a special case of the general results, which was derived in this commentary. The numerical and Monte Carlo simulations are also presented to verify the theoretical observations.

## II. Exact Asymptotic Symbol Error Rate Analysis

According to [1, eq. (3)], the received signal-to-noise ratio (SNR) at the cognitive receiver can be formulated as

$$\gamma_{\mathrm{MRC}} = p\overline{\gamma} \sum_{i=1}^{K} g_{c_i} \tag{1}$$

where $p = \min\{P, Q/g_p\}$ is the transmit power of the cognitive transmitter, $\overline{\gamma}$ is the inverse of the power of additive white Gaussian noise, $K$ is the number of the cognitive receive antennas, and $g_{c_i}$, $i = 1, 2, \ldots K$ is the channel power from the cognitive transmitter to the $i$th cognitive receive antenna [1].

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[1]It should be noted that $P \ll Q$ may not be always satisfied for the practical cognitive radio system, in which the interference tolerance level, i.e., $Q$, is always finite [2].

For arbitrary $p$, $\gamma_{\mathrm{MRC}}$ is the summation of independently and identically distributed exponential random variables $\gamma_i = p\overline{\gamma}g_{c_i}, i = 1, 2, \ldots, K$, with the mean of $p\overline{\gamma}$. Using the Taylor series of $e^x$, we have $e^x = 1 + x + o(x), x \to 0$.[2] Therefore, the conditional cumulative distribution functions of $\gamma_i, i = 1, 2, \ldots, K$ can be written as

$$F_{\gamma_i|p}(x) = \frac{x}{p\overline{\gamma}} + o(x). \quad (2)$$

With the help of [3, Lemma 1], the asymptotic conditional probability distribution function (pdf) of $\gamma_{\mathrm{MRC}}$ can be derived as

$$f_{\gamma_{\mathrm{MRC}}|p}(x) = \left(\frac{1}{p\overline{\gamma}}\right)^K \frac{1}{\Gamma(K)} x^{K-1} + o(x^{K-1}) \quad (3)$$

where $\Gamma(\cdot)$ is the gamma function [4, eq. (8.310.1)].

Now, averaging the preceding conditional pdf over $p$, the asymptotic pdf of $\gamma_{\mathrm{MRC}}$ can be expressed as

$$f_{\gamma_{\mathrm{MRC}}}(x) = E\left[(1/p)^K\right] (\overline{\gamma})^{-K} \frac{1}{\Gamma(K)} x^{K-1} + o(x^{K-1}) \quad (4)$$

where $E[\cdot]$ is the statistical expectation. According to the definition of $p$, $E[(1/p)^K]$ can be calculated as

$$\begin{aligned} E\left[(1/p)^K\right] &= \int_0^{Q/P} P^{-K} \exp(-x)\, dx + \int_{Q/P}^{\infty} (x/Q)^K \exp(-x)\, dx \\ &= P^{-K}\left[1 - \exp(Q/P)\right] + Q^{-K} \int_{Q/P}^{\infty} x^K \exp(-x)\, dx. \end{aligned} \quad (5)$$

With the help of [4, eq. (3.351.2)], we have

$$E\left[(1/p)^K\right] = P^{-K}\left[1 - \exp(-Q/P)\right] + Q^{-K}\Gamma(K+1, Q/P) \quad (6)$$

where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function [4, eq. (8.350.2)].

Substituting (6) into (4) and simplifying, the exact asymptotic pdf can be finally obtained as

$$\begin{aligned} f_{\gamma_{\mathrm{MRC}}}(x) = &\left[P^{-K}\left(1 - \exp(-Q/P)\right) + Q^{-K}\Gamma(K+1, Q/P)\right] \\ &\times \left(\overline{\gamma}^K \Gamma(K)\right)^{-1} x^{K-1} + o(x^{K-1}). \end{aligned} \quad (7)$$

According to [5, Prop. 1], the exact asymptotic SER can be derived as

$$P_E \to \frac{aL(P, Q, K)\Gamma(K + 1/2)}{2\sqrt{\pi}Kb^K} \overline{\gamma}^{-K} \quad (8)$$

where $L(P, Q, K) \triangleq 1/\Gamma(K)[(1 - \exp(-(Q/P))/P^K + \Gamma(K + 1, Q/P)/Q^K]$, and $a$ and $b$ are the modulation-dependent parameters defined in [1].

It can also be obtained that the diversity and coding gain for the preceding cognitive radio system with MRC is $G_d = K$ and $G_c = [aL(P, Q, K)\Gamma(K + 1/2)/2\sqrt{\pi}Kb^K]^{-1/K}$, respectively; hence, full diversity order can be achieved for all $P$ and $Q$ values.

*Remark (Special Case of $P \ll Q$ in [1]):* According to the property of exponential function and the asymptotic formula in [4, eq. (8.350.4)], we find out that $\exp(-Q/P) \approx 0$ and $\Gamma(K + 1, Q/P) \approx 0$, for $P \ll Q$. Therefore, in this case, the asymptotic pdf in (7) can be formulated as

$$f_{\gamma_{\mathrm{MRC}}}(x) = (P\overline{\gamma})^{-K} \left(\Gamma(K)\right)^{-1} x^{K-1} + o(x^{K-1}). \quad (9)$$

[2] We use $o(\cdot)$ to represent higher order terms, where $f(x) = o(g(x)), x \to x_0$, if $\lim_{x \to x_0} f(x)/g(x) = 0$.

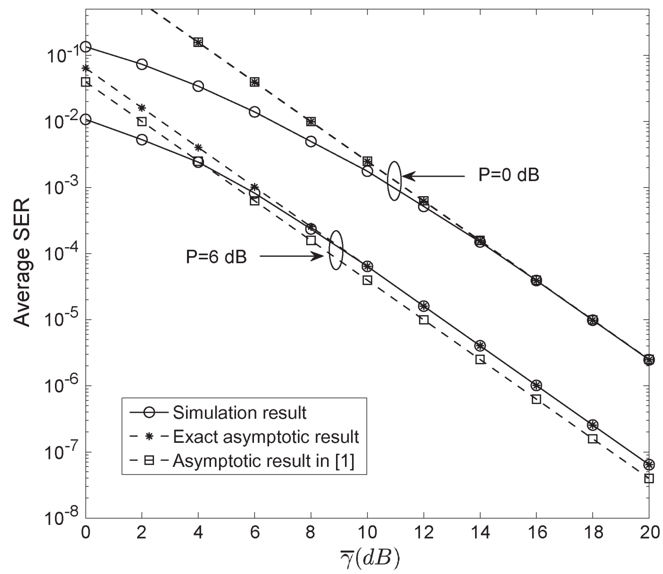


Fig. 1. Average SER versus $\overline{\gamma}$ with different $P$ values.

Taking the $(K - 1)$th derivation of $f_{\gamma_{\mathrm{MRC}}}(x)$ and using the formula in [4, eq. (8.339.1)], we get

$$\frac{\partial^{K-1}}{\partial x^{K-1}} f_{\gamma_{\mathrm{MRC}}}(x) = (P\overline{\gamma})^{-K} \quad (10)$$

which reduces to [1, eq. (15)] and is shown as

$$\frac{\partial^{K-1}}{\partial x^{K-1}} f_{\gamma_{\mathrm{MRC}}}(0) = \prod_{i=1}^{K} f_{\gamma_i}(0) \quad (11)$$

where $f_{\gamma_i}(0) \approx 1/P\bar{\gamma}$, for $P \ll Q$ [1].

Hence, the asymptotic SER pertaining to $P \ll Q$ in [1, eq. (16)]

$$P_s = \frac{a(2K-1)!!}{2^{K+1}K!b^K} \prod_{i=1}^{K} f_{\gamma_i}(0) \approx \frac{a(2K-1)!!}{2^{K+1}K!(bP)^K} \overline{\gamma}^{-K} \quad (12)$$

is a special case of (8).

## III. Simulations

As in [1], assume that $K = 3$, $Q = 6$ dB, and quadrature phase-shift keying modulation with $a = 2$ and $b = 0.5$. Fig. 1 shows the average SER performance with respect to the average SNR $\overline{\gamma}$ for different values of $P$. The exact asymptotic results are calculated by (8), and the asymptotic results in [1] are obtained by [1, eq. (17)]. Unsurprisingly, for $P = 6$ dB, since the condition $P \ll Q$ does not hold, the asymptotic results in [1] become very loose. However, it can be observed that the exact asymptotic results are highly accurate for all different $P$ and $Q$ values in the high-SNR regime.

## IV. Conclusion

In this commentary, the exact asymptotic SER for cognitive radio systems with MRC in [1] has been reinvestigated, which holds for all the parameter values, and has been proved accurate in the high-SNR regime. It has also been revealed that the asymptotic SER in [1] was just a special case of our general results under certain preconditions.

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# Interference Alignment With Differential Feedback


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***Abstract*—Interference alignment (IA) has been recognized as a promising technique to obtain large multiplexing gain in multiple-input–multiple-output (MIMO) interference channels. Most existing IA schemes require global channel state information (CSI) at the transmitter to design precoding vectors and, thus, result in significant capacity overhead in the feedback link. To reduce the feedback overhead, in this paper, we investigate the IA scheme employing differential CSI feedback over time-correlated MIMO channels. Specifically, we analyze the achievable sum-rate performance using differential feedback and derive the minimum differential feedback rate to achieve the maximum multiplexing gain. We also provide the upper bound of the sum-rate performance loss due to the limited differential feedback rate. Finally, the analytical results are verified by simulations in a practical IA scheme with differential CSI feedback using Lloyd's quantization algorithm.**



***Index Terms*—Channel state information (CSI), differential feedback, interference alignment (IA), time correlation.**


## I. Introduction

Interference alignment (IA) has been developed as a promising transmission scheme to achieve a large capacity scaling, which is also known as multiplexing gain or degrees of freedom (DOFs) in the $K$-user interference channels [1]. Compared with traditional interference avoidance schemes such as orthogonal channel access, in the IA scheme, by forcing interference into a predefined reduced-dimensional subspace and maximizing the interference-free subspace, the achievable sum rate linearly scales with the number of users in the interference channels. It has been proven that, in a $K$-user single-input–single-output (SISO) system, the achievable DOF is $K/2$ with the IA scheme [2]. Moreover, for the MIMO case, the maximum Dof is also linearly scaled with the number of users and antennas with the IA scheme [2], [3].

For the application of IA to practical communication systems, various methods of designing the precoder vectors have been proposed. For instance, the closed-form solutions for the $K$-user interference systems were provided in [2], and the iterative solutions with different criteria (minimum interference leakage, maximum signal-to-interference-plus-noise ratio, etc.) have been developed in [4] and [5]. In these aforementioned works, global channel state information (CSI) is typically assumed at the transmitters. However, in certain wireless communication systems, the CSI can only be obtained through feedback since channel reciprocity may not exist, e.g., for a frequency-division duplex system. Due to the receiver's estimation error and the limit on feedback capacity, the perfect CSI cannot be obtained at the transmitters. Therefore, the impact of imperfect CSI on the IA systems and the practical feedback scheme for IA systems were studied in recent works [6]–[8]. In [6], Grassmannian codebooks were used to quantize the channel coefficients and indicated that, with limited feedback bits, the maximum multiplexing gain can still be achieved in a SISO system. In addition, then, this work was extended to single-input–multiple-output and MIMO systems in [7]. In [8], the analog feedback scheme was used to reduce the feedback overhead. However, the properties of wireless channels, such as time correlation, are not taken into account in those papers. To further reduce the feedback overhead, it would be useful to exploit the time correlation of wireless channels [9], [16]. For temporal-correlated channels, it would be sufficient to feed back the difference of consecutive CSIs, which is called the differential feedback scheme, such that the feedback overhead can be significantly reduced.

In this paper, not focusing on a specific differential feedback scheme as [9], we investigate the theoretical bounds of differential feedback over time-correlated channels for IA systems based on Shannon's Rate Distortion Theory. The main contributions are summarized here.

1) A closed-form expression of the minimum differential feedback rate is derived to preserve the maximum multiplexing gain for the time-correlated MIMO interference channels.
2) The relationship between the achievable sum rate and the number of differential feedback bits has been investigated. In addition, an upper bound of sum-rate performance loss with the differential feedback bits is provided.
3) A practical IA system using the alternating-minimization precoding scheme and differential feedback scheme is employed to verify the theoretical analysis. Furthermore, a differential feedback scheme with Lloyd's algorithm and fixed feedback bits is also used to verify the theoretical calculation.

*Notations:* Bold uppercase (lowercase) letters denote matrices (vectors), $(\cdot)^H$ is the Hermitian transpose, and $\|\cdot\|^2$ and $|\cdot|$ represent Euclidean norm and absolute value, respectively. $\mathbb{E}[\cdot]$ is used for expectation over random variables, and $\log(\cdot)$ stands for logarithm to the base 2.


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