



Performance of transmit diversity assisted amplify-and-forward relay system with partial relay selection in mixed Rayleigh and Rician fading channels

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Abstract

The performance of transmit diversity (TD) assisted amplify-and-forward (AF) relay system with partial relay selection, which experiences mixed Rayleigh and Rician fading channels, is investigated. We first investigate the closed-form expression of the cumulative distribution function for the end-to-end equivalent signal-to-noise ratio (SNR), and then the exact expressions of outage probability and average symbol error probability (SEP) are derived. The theoretical observations are verified by the Monte Carlo simulation results.

Keywords transmit diversity, amplify-and-forward relay, Rician fading

1 Introduction

Relay technique has become a major topic in both the wireless research community [1] and standardization groups [2] because a relay station can be deployed to extend the network coverage or improve the system throughput. In many practical scenarios, a lot of relay channel measurement results [3] show different links in the relay system experience different types of fading conditions [4]. For instance, in a practical indoor relay system [5], the base station-to-relay link should be modeled as Rayleigh fading because there is no line-of-sight (NLoS) component between the base station and relay node, whereas there is always a strong line-of-sight (LoS) component between relay node and user equipment, thus the corresponding link should be modeled as Rician fading. Such asymmetric nature of the relay channel is termed as mixed type fading [4]. For the practical relay applications, since only the channel state information (CSI) of base station-relay link, also known as the local CSI, is available at the base station, the partial relay selection scheme is proposed in Ref. [6]. In Refs. [7–8], the performance of

partial relay selection in fixed gain AF relay, which experience Rayleigh fading channels, has been investigated. In Ref. [6], Krikidis et al. studied the closed-form probability density function (PDF) of the received signal to noise ratio (SNR) for an AF relay system in Rayleigh channels. In Ref. [9], both the exact and asymptotic expressions of outage probability and SEP of the partial relay selection scheme in Rayleigh fading channels are proposed.

Although a lot of papers have investigated the performance of partial relay selection scheme in symmetric fading channels [6–9], to the best of our knowledge, the performance of partial relay selection for AF relay systems in practical mixed type fading channels has not been studied. Moreover, since the base station is always equipped with multiple antennas, transmit diversity (TD) can be exploited in the practical relay system [10–11]. Based on the above observations, we proposed the practical TD assisted two-hop AF relay system in practical mixed Rayleigh and Rician fading channels, in which the base station is equipped with two antennas while relay node and user are both equipped with single antenna. For each transmission, with the help of local CSI, the base station selects the best relay node, which has the maximal received SNR, and then base station transmits to the best relay node with TD. To the best of our

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knowledge, such TD aided partial relay selection system in mixed type fading channels has not been reported in the literature. In particular, we derive the exact expressions of outage probability and SEP of the above relay system. Simulation results are also presented to validate the theoretical analysis.

The remainder of this paper is organized as follows. In Sect. 2, the system and channel model is described. In Sect. 3, system performance of the above relay system is analyzed. We derive the closed-form expressions for the outage probability and the symbol error probability, and the convergence properties of the infinite series are also proposed. In Sect. 4, the simulation results are presented to verify the correctness of the theoretic derivation. The conclusion is given in Sect. 5, and the detailed derivation is proposed as an Appendix

2. System model

We consider a two-hop AF relay system in Fig. 1. In the first time slot, the base station S selects the best relay based on the above SNR criterion and then transmits the symbol to relay node R with TD technique¹. In the second time slot, R amplifies the received signal by a gain factor G and forwards the amplified signal to the user D .

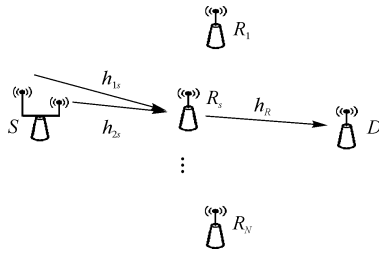


Fig. 1 TD assisted relay system with partial relay selection

Suppose that the s th R is selected by the S with partial CSI among the total N relays, so the signal vector at S is $(1/\sqrt{|h_{1s}|^2 + |h_{2s}|^2})(h_{1s}^*x, h_{2s}^*x)$. The received signal at the s th relay is

$$y_s = \sqrt{|h_{1s}|^2 + |h_{2s}|^2}x + w \quad (1)$$

where h_{ki} is the complex channel fading between the k th antenna of S and the antenna of i th R , x is the transmitted symbol, and w is the complex additive white Gaussian noise (AWGN) with the variance of $N_0/2$ per dimension.

¹ It should be noted that in this paper we applied the closed-loop TD in S-R link, however for the practical consideration, open-loop Alamouti code can also be implemented in the S-R link. Furthermore, the corresponding equivalent end-to-end SNR is the same as Eq. (2), and hence the performance analysis in this paper is applicable in both closed-loop TD and open-loop Alamouti code cases.

Since $|h_{ki}|$ is a Rayleigh random variable, $|h_{ki}|^2$ is an exponential random variable with the mean of $\bar{\gamma}$. We denote

$$\gamma_{R_i} = \sum_{k=1}^2 |h_{ki}|^2 \frac{1}{N_0} \quad (2)$$

The cumulative distribution function (CDF) of γ_{R_i} is [12, Eq. (2.1.114)]

$$F(\gamma_{R_i}) = \left(1 - \frac{1 + \frac{1}{\bar{\gamma}}\gamma_{R_i}}{\exp\left(\frac{1}{\bar{\gamma}}\gamma_{R_i}\right)} \right) U(\gamma_{R_i}) \quad (3)$$

where $U(\gamma)$ is the unit-step function.

Specifically, S choose the s th relay as $S = \arg \max_{i=1,2,\dots,N} \{\gamma_{R_i}\}$ (4)

At R , it amplifies the received signal with the gain factor $G = \left(\sum_{k=1}^2 |h_{ks}|^2 \right)^{-\frac{1}{2}}$ [13]. Therefore, the received signal at D

can be expressed as $y_d = h_R G y_s + n$ (5)

where h_R is the complex channel fading between R and D , and n is the complex additive white Gaussian noise (AWGN) with the variance of $N_0/2$ per dimension. Then the instantaneous equivalent end-to-end SNR at the UE is given by²

$$\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2} \quad (6)$$

where $\gamma_1 = \max_{i=1,2,\dots,N} \{\gamma_{R_i}\}$, $\gamma_2 = |h_R|^2 \frac{1}{N_0}$.

Since γ_1 is an order statistics, the CDF of γ_1 can be expressed as follows

$$F_{\gamma_1}(\gamma) = [F_{R_i}(\gamma)]^N = \left[1 - \frac{1 + \frac{1}{\bar{\gamma}}\gamma}{\exp\left(\frac{1}{\bar{\gamma}}\gamma\right)} \right]^N U(\gamma) \quad (7)$$

As we have discussed above, $|h_R|$ is a Rician random variable, and the PDF of γ_2 is [12, Eq. (2.1.140)]

$$f_{\gamma_2}(\gamma_2) = \frac{1}{2\sigma^2} \exp\left(-\frac{\gamma_2 + A^2}{2\sigma^2}\right) I_0\left(\frac{A\sqrt{\gamma_2}}{\sigma^2}\right) U(\gamma_2) \quad (8)$$

where the Rician factor $K = A^2/(2\sigma^2)$, $E(\gamma_2) = \bar{\gamma}_2 = A^2 + 2\sigma^2$

² It should be noted that there is another equivalent end-to-end SNR form as

$$\gamma = \gamma_1 \gamma_2 / (\gamma_1 + \gamma_2 + 1) \quad \text{with the gain factor } G = \left(\sum_{k=1}^2 |h_{ks}|^2 + N_0 \right)^{-\frac{1}{2}}.$$

However, the equivalent end-to-end SNR form in Eq. (1) has the advantage of mathematical tractability and its corresponding system performance can serve as a benchmark of all practical relays [13]. So we only focus on the equivalent end-to-end SNR form in Eq. (5).

and $I_0(x)$ is the zeroth modified Bessel function of the first kind [12, Eq. (8.406.1)]. $E[\cdot]$ is the mathematical expectation operator.

3 Performance analysis

In this section, we present the outage probability and SEP analysis of the TD assisted relay system with partial relay selection. The following theorem provides the CDF of the equivalent end-to-end SNR.

Theorem 1 The CDF of equivalent end-to-end SNR is given as follows.

$$F_\gamma(r) = 1 + \sum_{k=1}^N \sum_{m=0}^k \sum_{n=0}^m \sum_{i=0}^{+\infty} \sum_{j=0}^i \frac{I(k, m, n, i, j)}{\exp\left(kb + \frac{1}{2\sigma^2}\right) r} r^{m+i+1} K_{j-n+1}\left(r\sqrt{\frac{2kb}{\sigma^2}}\right) \quad (9)$$

where $I(k, m, n, i, j) = (1/\sigma^2) C_N^k C_k^m C_m^n C_i^j (-1)^k b^m (A/(2\sigma^2))^{2i} 1/(i!)^2 (2kb\sigma^2)^{(j-n+1)/2} \exp(-A^2/2\sigma^2)$, $b = 1/\bar{\gamma}$, $C_i^j = i!/[j!(i-j)!]$. $K_n(x)$ is the n th order modified Bessel function of the second kind [14, Eq. (8.407)].

Proof Please refer to Appendix A.

3.1 Outage probability analysis

Outage probability is defined as the probability that the instantaneous SNR falls below the threshold SNR γ_{th} . Thus the outage probability P_{out} is given as

$$P_{out} = P(\gamma < \gamma_{th}) = F_\gamma(\gamma_{th}) \quad (10)$$

Note that for the case of $N=1$, $m=1$, that is, mixed Rayleigh and Rician fading channels, Eq. (10) is exactly the same as Eq. (8) in Ref. [4].

3.2 SEP analysis

SEP is an important performance measure for communication systems in fading channels. Generally, SEP has a uniform expression [15, Eq. (5.1)]

$$P_s = E[Q(\sqrt{a\gamma})] = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{-\frac{x^2}{2}} F_\gamma\left(\frac{x^2}{a}\right) dx \quad (11)$$

where $F_\gamma(x)$ is the CDF of random variable, $Q(x)$ is the Gaussian Q -function, and a is a constant related to the modulation scheme. For instance, $a=4$ for BPSK, and $a=1$ for QPSK.

The SEP for the aforementioned relay system is given as follows

$$P_s = \frac{1}{2} + \sum_{k=1}^N \sum_{m=0}^k \sum_{n=0}^m \sum_{i=0}^{+\infty} \sum_{j=0}^i \frac{I(k, m, n, i, j)}{2\sqrt{2}a^{m+i+1}} \left[\frac{\left(\frac{2}{a\sigma}\sqrt{2kb}\right)^{j-n+1}}{\left(\frac{1}{2} + \frac{kb}{a} + \frac{1}{2a\sigma^2} + \frac{1}{a\sigma}\sqrt{2kb}\right)^{m+i+j-n+\frac{5}{2}}} \frac{\Gamma\left(m+i+j-n+\frac{5}{2}\right)\Gamma\left(m+i+n-j+\frac{1}{2}\right)}{\Gamma(m+i+2)} \right] F\left(m+i+j-n+\frac{5}{2}, j-n+\frac{3}{2}; m+i+2; \frac{1}{2} + \frac{kb}{a} + \frac{1}{2a\sigma^2} - \frac{1}{a\sigma}\sqrt{2kb}, \frac{1}{2} + \frac{kb}{a} + \frac{1}{2a\sigma^2} + \frac{1}{a\sigma}\sqrt{2kb}\right) \quad (12)$$

where $F(x, y, z; t)$ is the Gaussian hyper-geometric function defined in [14, Eq. (9.100)]. Note that for the mixed Rayleigh and Rician fading channels, we have $N=1$, $m=1$ and Eq. (12) reduces to the previously published result presented in Ref. [4, Eq. (14)].

Proof Please refer to Appendix B.

3.3 Analysis of convergence

Concerning the convergence of the infinite series expressions of Eqs. (10) and (12), it can be proved theoretically that the infinite series of Eq. (10) converges quickly, seen in Appendix C. As for the series of Eq. (12), it involves several complicated special functions, which make the theoretical evaluation intractable, however as we see later in the numerical simulation part, Eq. (12) will also converge with a finite summation terms over i . Thus, we can calculate Eqs. (10) and (12) using only a finite summation terms with neglectable loss of accuracy.

4 Simulation results

In this section, we present some simulation results to verify the theoretical analysis. We adopt the channel model in Sect. 2. It is assumed that number of relay node is $N=4$, and the threshold SNR for outage probability $\gamma_{th} = 5$ dB. The theoretical and simulation results for outage probability and SEP are proposed with two different Rician K factors under both balanced link ($E[\gamma_1] = E[\gamma_2]$) and unbalanced link

($E[\gamma_1]=1.5E[\gamma_2]$) scenarios³. Fig. 2 shows the outage probability with two different K factors. The analytical curves are calculated from Eq. (10) using 30 summation terms over i , and it is clear that the analytical results match the simulation results very well. Fig. 3 shows the SEP performance of QPSK modulation with two different K factors. The analytical curves are calculated from Eq. (12) using 30 summation terms over i , and it can be seen that the simulation results match the analytical results exactly.

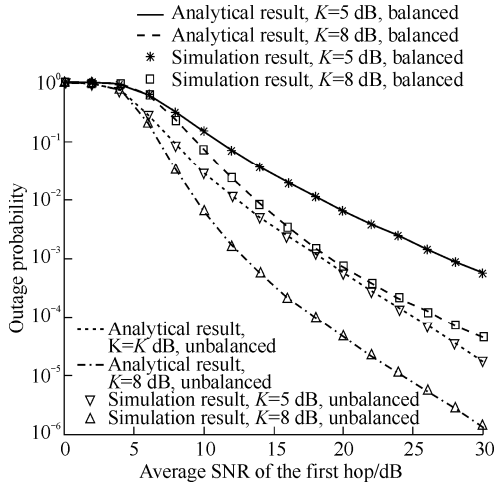


Fig. 2 Outage Probability for different K factors under both balanced and unbalanced link scenarios

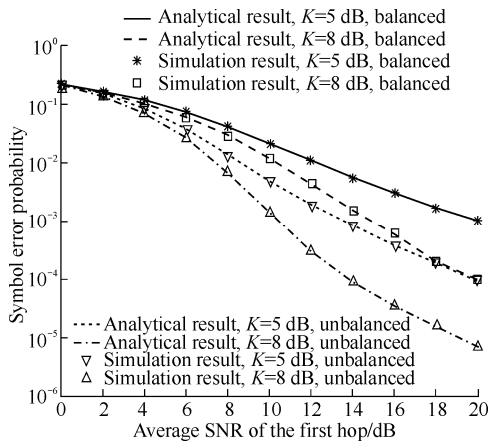


Fig. 3 SEP for different K factors with QPSK modulation under both balanced and unbalanced link scenarios

5 Conclusions

In this paper, we derived the closed-form CDF of the equivalent end-to-end SNR for the TD assisted relay systems

³ It should be mentioned that in practical relay applications, the transmit power of the first link (base station to relay station) is typically larger than the transmit power of the second link (relay station to mobile user), therefore we choose $E[\gamma_1]=1.5E[\gamma_2]$ as the unbalanced link configuration.

with partial relay selection, based on which we have derived the exact expressions for the outage probability and SEP in practical mixed Rayleigh and Rician fading channels. Monte Carlo simulation results validate the theoretical analysis. To the best of our knowledge, such exact results have not been published in the literature.

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Appendix A Proof of Theorem 1

According to definition of CDF, with the series representation of $I_0(x)$ [14, Eq. (8.445.1)] and binomial theorem [14, Eq. (1.111)], the CDF of the end-to-end SNR can be calculated as

$$\begin{aligned}
 F_\gamma(r) &= P\left(\frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} < r\right) = \int_0^r P\left(\gamma_1 > \frac{\gamma_2 r}{\gamma_2 - r}\right) \\
 & f_{\gamma_2}(\gamma_2) d\gamma_2 + \int_r^{+\infty} F_{\gamma_1}\left(\frac{\gamma_2 r}{\gamma_2 - r}\right) f_{\gamma_2}(\gamma_2) d\gamma_2 = \\
 & 1 - \int_r^{+\infty} \left[1 - F_{\gamma_1}\left(\frac{\gamma_2 r}{\gamma_2 - r}\right)\right] f_{\gamma_2}(\gamma_2) d\gamma_2 = \\
 & 1 + \sum_{k=1}^N \sum_{m=0}^k (-1)^k C_N^k C_k^m b^m \gamma^m \cdot \\
 & \int_r^{+\infty} \left[\frac{\gamma_2^m}{(\gamma_2 - r)^m} \exp\left(-\frac{bk\gamma_2 r}{\gamma_2 - r}\right)\right] f_{\gamma_2}(\gamma_2) d\gamma_2 = \\
 & 1 + \sum_{k=1}^N \sum_{m=0}^k (-1)^k C_N^k C_k^m b^m \gamma^m \cdot \\
 & \int_0^{+\infty} \left[\frac{(\gamma_2 + r)^m}{(\gamma_2)^m} \exp\left(-\frac{bk(\gamma_2 + r)r}{\gamma_2}\right)\right] f_{\gamma_2}(\gamma_2 + r) d\gamma_2 = \\
 & 1 + \sum_{k=1}^N \sum_{m=0}^k (-1)^k C_N^k C_k^m b^m \gamma^m I_1
 \end{aligned} \tag{A.1}$$

where we denote

$$\begin{aligned}
 I_1 &= \int_0^{+\infty} \left[\frac{(\gamma_2 + r)^m}{(\gamma_2)^m} \exp\left(-\frac{bk(\gamma_2 + r)r}{\gamma_2}\right)\right] f_{\gamma_2}(\gamma_2 + r) d\gamma_2 = \\
 & \int_0^{+\infty} \left[\left(1 + \frac{r}{\gamma_2}\right)^m \exp\left(-\frac{bk(\gamma_2 + r)r}{\gamma_2}\right)\right] \frac{1}{2\sigma^2} \cdot \\
 & \exp\left(-\frac{\gamma_2 + r + A^2}{2\sigma^2}\right) I_0\left(\frac{A\sqrt{\gamma_2 + r}}{\sigma^2}\right) d\gamma_2 =
 \end{aligned}$$

$$\begin{aligned} & \sum_{n=0}^m \sum_{i=0}^{+\infty} C_m^n \frac{(r)^n}{(i!)^2} \left(\frac{A}{2\sigma^2} \right)^{2i} \frac{1}{2\sigma^2} \exp\left(-\frac{r+A^2}{2\sigma^2} + kbr\right) \cdot \\ & \int_0^{+\infty} \left[\exp\left(-\frac{bkr^2}{\gamma_2} + \frac{\gamma_2}{2\sigma^2}\right) \right] (\gamma_2 + r)^i \gamma_2^{-n} d\gamma_2 = \\ & \sum_{n=0}^m \sum_{i=0}^{+\infty} \sum_{j=0}^i C_m^n C_i^j \frac{1}{(i!)^2} \left(\frac{A}{2\sigma^2} \right)^{2i} \frac{1}{2\sigma^2} \frac{r^{n+i-j}}{\exp\left(\frac{A^2}{2\sigma^2} + \left(kb + \frac{1}{2\sigma^2}\right)r\right)} \cdot \\ & \int_0^{+\infty} \gamma_2^{j-n} \left[\exp\left(-\frac{bkr^2}{\gamma_2} + \frac{\gamma_2}{2\sigma^2}\right) \right] d\gamma_2 \end{aligned} \quad (\text{A.2})$$

With the help of [14, Eq. (3.471.9)], it is easy to obtain

$$\int_0^{+\infty} \frac{\gamma_2^{j-n}}{\exp\left(\frac{bkr^2}{\gamma_2} + \frac{\gamma_2}{2\sigma^2}\right)} d\gamma_2 = 2(2kb\sigma^2 r^2)^{\frac{j-n+1}{2}} K_{j-n+1}\left(r\sqrt{\frac{2kb}{\sigma^2}}\right) \quad (\text{A.3})$$

so we can get

$$\begin{aligned} I_1 &= \sum_{n=0}^m \sum_{i=0}^{+\infty} \sum_{j=0}^i C_m^n C_i^j \frac{1}{(i!)^2} \left(\frac{A}{2\sigma^2} \right)^{2i} \frac{1}{\sigma^2} \cdot \\ & \frac{(2kb\sigma^2)^{\frac{j-n+1}{2}} r^{i+1}}{\exp\left(\frac{A^2}{2\sigma^2} + \left(kb + \frac{1}{2\sigma^2}\right)r\right)} K_{j-n+1}\left(r\sqrt{\frac{2kb}{\sigma^2}}\right) \end{aligned} \quad (\text{A.4})$$

Substitute Eq. (A.4) into Eq. (A.1), we can get the CDF of the end-to-end SNR as Eq. (9).

Appendix B Proof of Eq. (12)

Substituting Eq. (9) into Eq. (11), we can get [4]

$$\begin{aligned} P_s &= E\left[\mathcal{Q}(\sqrt{a\gamma})\right] = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{-\frac{x^2}{2}} F_\gamma\left(\frac{x^2}{a}\right) dx = \\ & \frac{1}{2} + \sum_{k,m,n,i,j} \frac{I(k,m,n,i,j)}{\sqrt{2\pi} a^{m+i+1}} \int_0^{+\infty} e^{-\frac{x^2}{2}} \frac{x^{2(m+i+1)}}{\exp\left[\left(kb + \frac{1}{2\sigma^2}\right)\frac{x^2}{a}\right]} \cdot \\ & K_{j-n+1}\left(\frac{x^2}{a}\sqrt{\frac{2kb}{\sigma^2}}\right) dx \end{aligned} \quad (\text{B.1})$$

Using the change of variables $t = x^2$ and with the help of [14, Eq. (6.621.3)] and [15, Eq. (5.13)], after some mathematical manipulations, we can get P_s as Eq. (12).

Appendix C Convergence Proof of Eq. (10)

Considering the convergence of Eq. (10), since the number of summation terms over k, m, n is finite, so we only focus on the infinite series over i . After some manipulation, we can formulize the infinite series as

$$T = \sum_{i=0}^{+\infty} \sum_{j=0}^i \frac{1}{(i!)^2} a_1^i a_2^j C_i^j K_{j-n+1}(a_3) \quad (\text{C.1})$$

where $a_1 = (A/2\sigma^2)^2 r$, $a_2 = \sqrt{2kb\sigma^2}$ and $a_3 = r\sqrt{2kb/\sigma^2}$.

So the truncation error is

$$R = \sum_{i=N}^{+\infty} \sum_{j=0}^i \frac{1}{(i!)^2} a_1^i a_2^j C_i^j K_{j-n+1}(a_3) \quad (\text{C.2})$$

without any loss of generality, we assume $N > n$. According to [16, Eq. (15)], for $v > 0$

$$K_v(x) \propto \frac{(v-1)!}{2} \left(\frac{x}{2}\right)^{-v} \quad (\text{C.3})$$

With the help of the property of modified Bessel function of the second kind [14, Eq. 8.486.16], substituting Eq. (C.3) into Eq. (C.2), we can get

$$\begin{aligned} R &= \sum_{i=N}^{+\infty} \sum_{j=0}^i \frac{1}{(i!)^2} a_1^i a_2^j C_i^j K_{|j-n+1|}(a_3) = \\ & \sum_{i=N}^{+\infty} \frac{a_1^i}{i!} \sum_{j=0}^i \frac{1}{j!(i-j)!} a_2^j \frac{(j-n+1)!}{2} \left(\frac{a_3}{2}\right)^{-|j-n+1|} = \\ & \sum_{i=N}^{+\infty} \frac{a_1^i}{i!} \left[\sum_{j=0}^{n-1} \frac{1}{j!(i-j)!} a_2^j \frac{(n-j-2)!}{2} \left(\frac{a_3}{2}\right)^{-(n-j-1)} + \right. \\ & \left. \sum_{j=n}^i \frac{1}{j!(i-j)!} a_2^j \frac{(j-n)!}{2} \left(\frac{a_3}{2}\right)^{-(j-n-1)} \right] \leq \\ & \sum_{i=N}^{+\infty} \frac{a_1^i}{i!} \left[R_1 + \frac{a_2^n}{a_3} \sum_{j=0}^{i-n} \left(\frac{2a_2}{a_3}\right)^j \right] = \\ & \sum_{i=N}^{+\infty} \frac{a_1^i}{i!} \left[R_1 + \frac{a_2^n \left[1 - \left(\frac{2a_2}{a_3}\right)^{i-n+1} \right]}{a_3 - 2a_2} \right] \end{aligned} \quad (\text{C.4})$$

where $R_1 = \sum_{j=0}^{n-1} \left\{ a_2^j / [j!(i-j)!] \right\} [(n-j-2)!/2] (a_3/2)^{-(n-j-1)}$.

It is clear that the convergence problem of Eq. (C.2) is a just special case of the typical series problem $\sum_{i=N}^{+\infty} s^i/i!$, where $s > 0$. According to the Stirling's formula [14, Eq. 8.327.2], the truncation error of Eq. (C.2) converges to zero quickly.

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