

In contrast, in [2], the FM-representation of the quantization noise is applied for band-limited Gaussian noise input signals and clear quantitative limits for the quantization noise being uniformly distributed and spectrally white are given. It is also disclosed that in the discrete-time case with a fixed sampling rate  $f_s$ , the condition for the quantization noise being uniformly distributed is not sufficient for the quantization noise to be spectrally white. If the bandwidth  $B$  of the input signal is considerably smaller than  $f_s/2$ , i.e., smaller than  $B/(f_s/2) \ll 0.5$ , an additional increase of the input power is necessary to obtain a white spectrum, although the uniform distribution condition is met.

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## A Comment on "A Blind OFDM Synchronization Algorithm Based on Cyclic Correlation"

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**Abstract**—This comment points out several errors in the above letter. The correct frequency offset estimator is then proposed. Monte Carlo simulation results validate our analytical observations.

**Index Terms**—Blind synchronization, frequency offset estimator, orthogonal frequency division multiplexing.

### I. COMMENT ON [1]

In Section III of [1], (6) based on Parseval's Theorem is incorrect. The incorrect equation shown in [1, eq. (6)] is

$$\sum_{n=-\infty}^{\infty} g'[n]g'^*[n+\tau]e^{-j\frac{2\pi}{P}kn} = e^{-j2\pi\frac{k}{P}n\epsilon} \int_{-1/2}^{1/2} G^*\left(\beta - \frac{k}{P}\right)G(\beta)e^{-j2\pi\beta\tau}d\beta. \quad (1)$$

Our correct version is

$$\sum_{n=-\infty}^{\infty} g'[n]g'^*[n+\tau]e^{-j\frac{2\pi}{P}kn} = e^{-j2\pi\frac{k}{P}(n\epsilon-\tau)} \times \int_{-1/2}^{1/2} G^*\left(\beta - \frac{k}{P}\right)G(\beta)e^{-j2\pi\beta\tau}d\beta. \quad (2)$$

It is clear that the term  $e^{j(2\pi/P)k\tau}$  was neglected in (1). The proof of (2)<sup>1</sup> will be given in the Appendix.

Correspondingly, (7) and (8) of [1] should be modified as

$$R_y[k; \tau] = \frac{1}{P} \sum_{m=0}^{L-1} \sigma_{h[m]}^2 e^{j2\pi\theta\epsilon\tau} \sigma_c^2 \cdot \Gamma[\tau] G_2(k; \tau) e^{-j\frac{2\pi}{P}k(n\epsilon-\tau)} + r_w[\tau]\delta[k] \quad (3)$$

$$M[k; \tau] = \frac{1}{P} \sum_{m=0}^{L-1} \sigma_{h[m]}^2 e^{j2\pi\theta\epsilon\tau} \sigma_c^2 \cdot \Gamma[\tau] e^{-j\frac{2\pi}{P}k(n\epsilon-\tau)} + G_2^{-1}(k; \tau)r_w[\tau]\delta[k] \quad (4)$$

where  $G(\beta)$ ,  $R[k; \tau]$ ,  $M[k; \tau]$  are defined as the same as [1].

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<sup>1</sup>It should be mentioned the similar equation can be found in [2, eq. (4)].

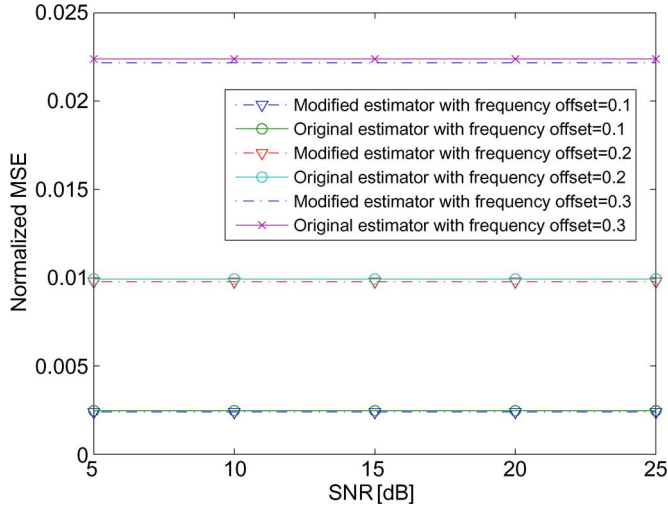


Fig. 1. MSE for frequency offset estimator in multipath fading channel.

Based on (3) and (4), the modified timing and frequency offset estimators are given as follows:

$$\hat{n}_\varepsilon = -\frac{1}{2\pi\frac{k}{P}} \arg [M[k; 0]] \quad k > 0 \quad (5)$$

$$\hat{\theta}_\varepsilon = \frac{1}{2\pi N} \arg [M[k; N] \cdot e^{j2\pi\frac{k}{P}(\hat{n}_\varepsilon - N)}] \quad k > 0 \quad (6)$$

while the estimators in [1] are

$$\hat{n}_\varepsilon = -\frac{1}{2\pi\frac{k}{P}} \arg [M[k; 0]] \quad k > 0 \quad (7)$$

$$\hat{\theta}_\varepsilon = \frac{1}{2\pi N} \arg [M[k; N] \cdot e^{j2\pi\frac{k}{P}\hat{n}_\varepsilon}] \quad k > 0. \quad (8)$$

From our corrected estimators (5) and (6), it can be found that the timing offset estimator is the same as [1], while the frequency offset estimator should be modified by the term of  $(-N)$ .

## II. SIMULATION RESULTS

In this section, we present the Monte Carlo simulation to evaluate the performance of the modified estimator. All of the simulation assumptions are the same as [1]. Since the two timing offset estimators are identical, we only propose the MSE property of the modified frequency offset estimator of (6).

As seen in Fig. 1, the MSE property of the modified frequency offset estimator is better than the original estimator, especially when the frequency offset is large. Moreover, the MSE of the modified estimator

does not vary with SNR, which means our new estimator is also reliable [1].

## III. CONCLUSION

In this comment, an incorrect equation in [1] is pointed out. The correct version of the equation and corresponding estimator is then proposed. Simulation results illustrate the performance of our modified estimator is better than the original one in [1].

## APPENDIX

In this section, we present the detailed derivation of (2).

*Proof:*

$$\begin{aligned} & \sum_{n=-\infty}^{\infty} g'[n]g'^*[n+\tau]e^{-j\frac{2\pi}{P}kn} \\ &= \left\langle g'[n], g'^*[n+\tau]e^{j\frac{2\pi}{P}kn} \right\rangle_n \\ &= \left\langle g[n-n_\varepsilon], g[n-n_\varepsilon+\tau]e^{j\frac{2\pi}{P}kn} \right\rangle_n \\ &= \left\langle e^{-j2\pi\beta n_\varepsilon} G(\beta), G\left(\beta-\frac{k}{P}\right)e^{j2\pi\left(\beta-\frac{k}{P}\right)(\tau-n_\varepsilon)} \right\rangle_\beta \\ &= \int_{-1/2}^{1/2} e^{-j2\pi\beta n_\varepsilon} G(\beta) \cdot G^*\left(\beta-\frac{k}{P}\right)e^{-j2\pi\left(\beta-\frac{k}{P}\right)(\tau-n_\varepsilon)} d\beta \\ &= e^{-j2\pi\frac{k}{P}(n_\varepsilon-\tau)} \int_{-1/2}^{1/2} G^*\left(\beta-\frac{k}{P}\right)G(\beta)e^{-j2\pi\beta\tau} d\beta \end{aligned}$$

where  $g'[n] = g[n-n_\varepsilon]$ ,  $G(\beta) = \sum_{n=-\infty}^{\infty} g(n)e^{-j2\pi n\beta}$ , and  $\langle \cdot, \cdot \rangle$  denotes inner product. ■

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