Improved Channel Estimation Based on Parametric Channel Approximation Modeling for OFDM Systems

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***Abstract*—Orthogonal frequency division multiplexing (OFDM) together with high order modulation scheme requires accurate channel estimation to perform coherent demodulation. In this paper, improved channel estimation methods based on a parametric channel approximation model are proposed for the OFDM system using pilot subcarriers. This channel model is called fraction taps channel approximation (FTCA) model, which is defined as a finite impulse response (FIR) on some definitive delay taps that have a fraction tap delay spacing relative to the sampling interval. Then, based on the FTCA channel model, the minimum mean square error (MMSE) and least square (LS) estimators are derived. Simulations over non-sample-spaced channels prove that the use of the FTCA channel model can effectively eliminate the problem of multi-path delay estimation and reduce the signal subspace dimension of the channel correlation matrix, where the full-rank estimators using pilot subcarriers can be adopted, and consequently, improve the channel estimation performance.**

***Index Terms*—Channel estimation, fraction tap delay spacing, fraction taps channel approximation (FTCA), LS, MMSE, orthogonal frequency division multiplexing (OFDM).**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) has recently received considerable interest in wireless communication systems due to its advantages in lessening the severe effects of frequency-selective fading [1]. OFDM together with high order modulation scheme [2], such as multilevel quadrature amplitude modulation (M-QAM), can provide high data rate transmission with high bandwidth efficiency. However, it requires accurate channel estimation to perform coherent demodulation due to its sensitivity to noise.

Currently, there are two different types of channel estimations: i) parametric and ii) non-parametric. The parametric channel estimation [3]–[9] commonly employs a deterministic channel model, which is based on a finite number of delay paths, and estimates the gain and delay of each path. It usually outperforms the non-parametric channel estimation and provides efficient channel estimation [9]–[11]. Recently, many parametric channel estimation techniques have been proposed for OFDM systems and studied in depth. In [6], a least square (LS) based parametric channel estimation is proposed for the sample-spaced channel. However, it suffers severe performance loss in non-sample-spaced channels. The minimum mean-square error (MMSE) estimator [7] is introduced to estimate the complex tap gains of the observed channel. It performs well and achieves much performance gain over the LS estimator [7] except for its large computational complexity. Then, in order to reduce the computational complexity, the low-rank estimators [7] are also introduced. They have limited performances in non-sample-spaced channels and even suffer error floors for high signal-to-noise ratios (SNRs). Then, in [9], the channel estimation based on the parametric channel modeling is proposed. It achieves perfect performance in sparse multi-path fading channels. However, it suffers the problem of slow convergence in multi-path delay estimation in relatively slow-fading channels and even suffers errors in multi-path delay estimation when many propagation paths exist in the channel, which commonly exists in a practical radio channel.

The non-parametric channel estimation makes few assumptions of the channel and is widely used. It normally consists of algorithms to estimate the pilot signals and interpolate the channel. Many non-parametric channel estimation methods [12]–[18] have been proposed and studied in depth. Meanwhile, the method, LS estimate of the pilot signals together with linear interpolation, is of much simplicity and widely used.

In this paper, improved channel estimation methods based on the parametric channel approximation model using pilot tones are proposed for the OFDM system. These works are mainly inspired by the observations in [7], where the full-rank MMSE estimator has large computational complexity while the low-rank estimators have limited performances and even suffer error floors in non-sample-spaced channels for high SNRs due to the large dimension of the observed channel, and in [9], where the parametric channel estimator can only be adopted in sparse multi-path fading channels and suffers trouble in multi-path delay estimation since these parameters, such as the number of the delay paths and each path delay, in the L-path model are all unknown and need to be sought. In order to solve these problems lying in with the channel models, a parametric channel approximation model is proposed. This approximation model is called fraction taps channel approximation (FTCA) channel model, which is modeled as a finite impulse response on some definitive delay taps that have a fraction tap delay spacing relative to the sampling interval and uses these definitive delay taps to approximate the real channel. Then, based on the FTCA channel model, the MMSE and LS estimators are derived. Through simulations and analyses, it is demonstrated that

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the FTCA channel model can effectively approximate the real channel and provides two key advantages. Firstly, it eliminates the problem of multi-path delay estimation and can be adopted in a channel not restricted to a sparse multi-path fading since these parameters, such as the number of the delay taps and each tap delay, in the FTCA channel model are all definitive terms and need not to be sought. Secondly, as compared with the observed channel model [7], the dimension of its channel correlation matrix is reduced, where the full-rank estimators using pilot sub-carries can be adopted, and consequently, improves the channel estimation performance.

In this paper, only frequency-domain channel estimation over multi-path fading channels is considered, and it is also assumed that the time synchronization is well achieved [19]–[21] and the inter-carrier interference (ICI) is effectively suppressed [22]–[24] in the system by some proper methods. This paper is organized as follows. Section II introduces some preliminaries of the OFDM system. Then, the FTCA channel model is proposed and the FTCA estimators are derived in Section III. Section IV presents some simulation results to demonstrate the effectiveness of the use of the FTCA channel model. Finally, conclusions are made in Section V.

II. Preliminaries

A. Channel Model

In the wireless communication environments, the received signals are a superposition of waves coming from all directions due to reflection, diffraction, and scattering. This effect is known as multi-path propagation [25]. In a complex notation, the channel impulse response (CIR) of the multi-path fading channel [26] is expressed as follows

$$h(\tau)=\sum_{i=1}^{L}h_i\delta(\tau-\tau_i) \quad (1)$$

where L is the number of propagation paths, h_i denotes the complex gain of the ith propagation path, and τ_i is the delay of the ith propagation path and is assumed that τ_1 always has zero delay while the other $L-1$ delay paths are uniformly distributed over (0, τ_{max}], where τ_{max} is the maximum delay spread in the channel. At the same time, the multi-path power delay profile (PDP) is assumed to be of exponential distribution $\varphi(\tau)=e^{-\beta\tau}$ and its average channel energy is normalized to one, where $\beta=4/\tau_{max}$. This is the well-known L-path channel model and its frequency response is described by

$$H(f)=\int_{-\infty}^{+\infty}h(\tau)e^{-j2\pi f\tau}d\tau =\sum_{i=1}^{L}h_ie^{-j2\pi f\tau_i} \quad (2)$$

For an OFDM system with proper guard interval and timing, its channel frequency response can be expressed as

$$H(k)=\sum_{i=1}^{L}h_ie^{-j\frac{2\pi k\tau_i}{NT}},\qquad k=0,1,\ldots,N-1 \quad (3)$$

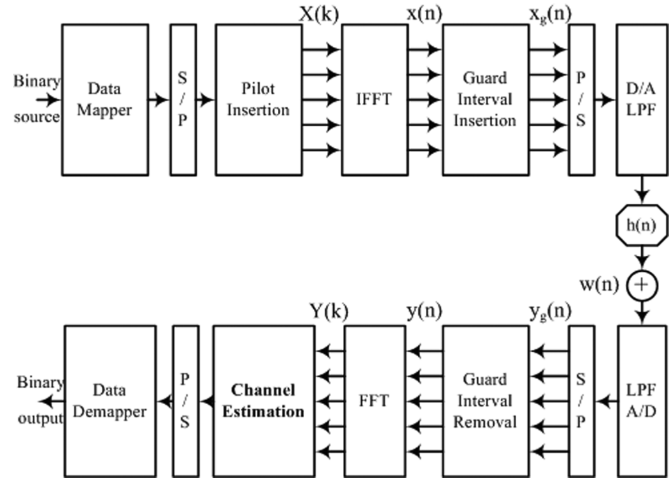

Fig. 1. Discrete-time baseband equivalent model of OFDM system.

where N is the number of the subcarriers of the OFDM system, k is the subcarrier index, $1/T$ refers to the Nyquist sampling rate, and T is the sampling interval.

B. OFDM System Descriptions

Fig. 1 shows the discrete-time baseband equivalent model of the considered OFDM system. The binary information data is first grouped and mapped according to the modulation in the data mapper block. In this paper, 16-QAM modulation is considered. After pilot data insertion, the modulated data $X(k)$ is sent to the inverse discrete Fourier transform (IDFT) block and transformed and multiplexed into the time-domain signal $x(n)$, which is expressed with

$$x(n)=\sqrt{N}\times IDFT\{X(k)\} =\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X(k)e^{j2\pi kn/N},\quad n=0,1,\ldots,N-1 \quad (4)$$

Then, following the IDFT block, the guard interval, which is chosen to be larger than the maximum delay spread τ_{max} in the channel and contains the cyclically extended part of the OFDM symbol, is inserted to prevent both inter-symbol interference (ISI) and inter-carrier interference (ICI). Thus, the time-domain transmitted signal $x_g(n)$ is given as follows

$$x_g(n)=\begin{cases}x(N+n), & n=-N_g,-N_g+1,\ldots,-1\\ x(n), & n=0,1,\ldots,N-1\end{cases} \quad (5)$$

where N_g is the number of samples in the guard interval which satisfies the constraint $N_g\times T\geq\tau_{max}$. Then, $x_g(n)$ passes through the frequency-selective fading channel $h(n)$, which is the observed channel impulse response after sampling the frequency response of the channel $h(\tau)$ described in (1), with the additive noise $w(n)$. Here, it is assumed that the D/A and A/D converters contain ideal low-pass filters with bandwidth $1/T$ and the guard interval is properly removed from the received signal $y_g(n)$ with perfect time synchronization being achieved. Then, the received baseband signal $y(n)$ is obtained as follows

$$y(n)=x(n)\otimes\frac{h(n)}{\sqrt{N}}+w(n),\qquad n=0,1,\ldots,N-1 \quad (6)$$

where $\otimes$ denotes circular convolution, and $w(n)$ is the complex additive white Gaussian noise (AWGN) with zero mean and variance σ^2. Then, the received data $Y(k)$ is obtained as follows

$$\begin{aligned} Y(k) &= \frac{1}{\sqrt{N}} \times DFT\{y(n)\} \\ &= X(k)H(k) + W(k), \quad k = 0, 1, \ldots, N-1 \end{aligned} \tag{7}$$

where $W(k)$ and $H(k)$ are the Fourier transforms of $w(n)$ and $h(n)$, respectively. Then, the observed channel $h(n)$, which is determined by the cyclic equivalent of sinc-functions, is achieved as [7]

$$\begin{aligned} h(n) &= \sqrt{N} \times IDFT\{H(k)\} \\ &= \sum_{i=1}^{L} h_i e^{-j\frac{\pi}{N}\left(n+(N-1)\frac{\tau_i}{T}\right)} \frac{\sin(\pi\tau_i/T)}{\sqrt{N}\sin\left(\pi(\tau_i/T - n)/N\right)}, \\ & \quad n = 0, 1, \ldots, N-1 \end{aligned} \tag{8}$$

Therefore, if the delay τ_i/T is an integer, then all the energy from h_i will be mapped to tap $h(\tau_i/T)$. However, if τ_i/T is not an integer, then its energy will leak to all the taps $\{h(n)\}$.

After $Y(k)$ has been achieved, the pilot signal $Y_P(k)$ is extracted from $Y(k)$ and the channel frequency response at the pilot subcarriers $H_P(K)$ is obtained with the information carried by $Y_P(k)$. Then, the channel transfer function $H(k)$ can be achieved by interpolating $H_P(k)$. With the knowledge of $H(k)$, the transmitted data $X(k)$ can be restored by simply dividing the received data $Y(k)$ by $H(k)$

$$\hat{X}(k) = \frac{Y(k)}{\hat{H}(k)}, \qquad k = 0, 1, \ldots, N-1 \tag{9}$$

where $\hat{H}(k)$ is the estimate of $H(k)$. After data de-mapping, the source binary information data is reconstructed at the receiver.

III. Fraction TAPS Channel Approximation Estimations

When the parametric channel estimations are applied, they are usually based on the two types of the channel models: i) the L-path channel model $h(\tau)$ [9] described in (1) and ii) the observed channel $h(n)$ [7] described in (8).

In the L-path channel model $h(\tau)$, these parameters, such as the number of the propagation paths L, each path delay τ_i and gain h_i, are all unknown terms. When a parametric channel estimator [9] based on $h(\tau)$ is used, all these parameters need to be estimated. In a sparse multi-path fading channel, the path delays are tracked and the estimator performs well. However, it brings slow convergence in multi-path delay estimation in relatively slow-fading channels since the number of OFDM symbols needed is large. Moreover, it even suffers errors in multi-path delay estimation when many propagation paths exist in the channel, which commonly exists in a practical radio channel.

In the observed channel model $h(n)$, where the real channel $h(\tau)$ is approximated by the finite impulse response (FIR) on the N delay taps $\{nT\}$ that have a tap delay spacing T, the parameters, such as the number of the observed taps N and the tap delays $\{nT\}$, are all definitive terms. Therefore, When the parametric channel estimator based on $h(n)$ is used, these definitive parameters need not to be sought and the estimator can well deal with the situations with many propagation paths except that its rank grows with N. This is a serious problem, especially when N is large. When the full-rank MMSE estimator [7] is applied, the rank of the estimator will be quite large and lead to huge computational complexity. However, if the low-rank estimators [7] are applied, they will have limited performances and even suffer error floors for high SNRs since the channel energy loss in the excluded taps is large and can not be neglected.

In order to solve these problems lying in with the channel models, a parametric approximation channel model is proposed in this paper, which is called the FTCA channel model.

A. The FTCA Channel Model

In order to describe the FTCA channel model, an assumption is first made that the channel frequency response $H(k)$ described in (3) can be well approximated by the frequency response $H_F(k)$ of the channel $h_F(\tau)$, which is modeled as a FIR on some definitive delay taps $\{lK_aT\}$ that have a fraction tap delay spacing K_aT relative to the sampling interval T, when K_a is properly chosen. $H(k)$, $H_F(k)$ and $h_F(\tau)$ are described as follows

$$H(k) = H_F(k) + H_e(k), \quad k = 0, 1, \ldots, N-1 \tag{10}$$

$$H_F(k) = \sum_{l=1}^{M} g_l e^{-j\frac{2\pi kl(K_aT)}{NT}}, \quad k = 0, 1, \ldots, N-1 \tag{11}$$

$$h_F(\tau) = \sum_{l=1}^{M} g_l \delta(\tau - lK_aT) \tag{12}$$

where $H_e(k)$ denotes the channel approximation error, which is assumed to be a complex Gaussian term with zero mean and variance B_e described in (29) and uncorrelated with $H_F(k)$, K_a is a fraction factor selected from (0, 1], g_l is the complex gain of the lth approximation tap, and M is the number of approximation taps and satisfies the following constraint

$$M = \left\lceil \frac{\tau_{max}}{K_aT} + 1 \right\rceil \tag{13}$$

where $\lceil x \rceil$ denotes the nearest integer that is larger than or equal to x. M is commonly less than N, where the dimension of the observed channel $h(n)$ is $N \times 1$. Here, τ_{max} is assumed to be known in advance and a new variable called the normalized maximum delay spread $N_D = \lceil \tau_{max}/T \rceil$ is defined, which denotes the maximum delay spread normalized by the sampling interval.

In this paper, $h_F(\tau)$ is called the FTCA channel model and its sampled channel frequency response is $H_F(k)$. When the FTCA channel model is used, a fraction value is chosen for K_a, so K_a can be viewed as a known parameter. Therefore, these parameters, such as the number of the approximation taps M and each approximation tap delay lK_aT, are all known terms.

Then, in a matrix notation, the channel frequency response vector $\mathbf{H} = [H(0), \ldots, H(N-1)]^T$ can be expressed as follows

$$\mathbf{H} = \mathbf{H}_F + \mathbf{H}_e = \mathbf{F}\mathbf{g} + \mathbf{H}_e = \mathbf{W}_N\mathbf{h} \tag{14}$$

where $(.)^T$ is the transpose operation, $\mathbf{g}$ and $\mathbf{H}_e$ are all Gaussian vectors and independent of each other, and

$$\begin{aligned} \mathbf{H}_F &= [H_F(0), H_F(1), \ldots, H_F(N-1)]^T, \\ \mathbf{H}_e &= [H_e(0), H_e(1), \ldots, H_e(N-1)]^T, \\ \mathbf{g} &= [g_1, g_2, \ldots, g_M]^T, \\ \mathbf{h} &= [h_1, h_2, \ldots, h_L]^T, \\ [\mathbf{F}]_{k,l} &= e^{-j2\pi lkK_a/N}, \\ & \quad k = 0, \ldots, N-1 \text{ and } l = 1, \ldots, M, \\ [\mathbf{W}_N]_{k,i} &= e^{-j2\pi k\tau_i/N}, \\ & \quad k = 0, \ldots, N-1 \text{ and } i = 1, \ldots, L \end{aligned} \tag{15}$$

Based on the LS criterion, $\mathbf{g}$ can be achieved simply by minimizing $(\mathbf{H}-\mathbf{Fg})^H(\mathbf{H}-\mathbf{Fg})$ and is described by

$$\mathbf{g} = (\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathbf{H} = (\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathbf{W}_N\mathbf{h} \tag{16}$$

where $(.)^H$ denotes the Hermitian transpose operation. $\mathbf{F}$ and $\mathbf{W}_N$ are both definitive matrixes. Thus, $\mathbf{g}$ is a linear transform of $\mathbf{h}$ and each propagation path opponent h_i in the real channel $h(\tau)$ is approximated by the finite impulse response on these definitive delay taps $\{lK_aT\}$ in the FTCA channel model $h_F(\tau)$. Therefore, $h_F(\tau)$ relates little to the types of the real channel, such as sparse or non-sparse , sample- or non-sample-spaced.

Then, the channel approximation error vector $\mathbf{H}_e$ is obtained as follows

$$\mathbf{H}_e = \mathbf{H} - \mathbf{Fg} = \mathbf{H} - \mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathbf{H} \tag{17}$$

$\mathbf{H}_e$ is commonly very small and can be negligible when K_a is properly selected. Thus, it is derived that $\mathbf{H} \approx \mathbf{H}_F = \mathbf{Fg}$ and the channel transform function $\mathbf{H}$ can be achieved by the estimate of the approximated FTCA channel $h_F(\tau)$, where only the complex tap gain vector $\mathbf{g}$ needs to be sought.

B. The Pilot Subcarriers Arrangement

In this paper, frequency domain channel estimation based on comb-type pilot arrangement is considered. Fig. 2 shows an example of the pilot arrangement pattern, where D_f denotes the interval in terms of the number of the subcarriers between two adjacent pilots in the frequency domain. In the OFDM system, the S pilot subcarriers are assumed to be evenly inserted into the N transmission subcarriers, then

$$S = \lceil N/D_f \rceil \tag{18}$$

At the same time, S should also satisfy the constraint that $S \geq M$. Let $\mathbf{P}$ denotes the set that contains the position indexes of the S pilot tones, then, $\mathbf{P}$ is described by

$$\mathbf{P} = \{p(m) | p(m) = mD_f, \qquad m = 0, 1, \ldots, S-1\} \tag{19}$$

Moreover, pilot signals $\{X(p(m))\}$ with the same amplitude are considered, where $A = |X(p(m))|^2$.

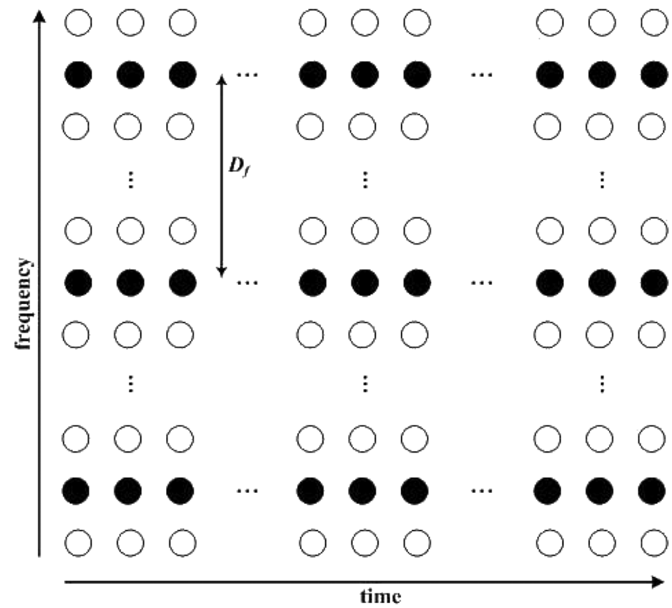

Fig. 2. Comb-type pilot arrangement for the OFDM system.

C. The FTCA Estimators

With the use of the FTCA channel model, the received pilot signal vector $\mathbf{Y_P} = [Y(p(0)), \ldots, Y(p(S-1))]^T$ is described by

$$\mathbf{Y_P} = \mathbf{X_P}\mathbf{F_P}\mathbf{g} + \mathbf{X_P}\mathbf{H}_{e,\mathbf{P}} + \mathbf{W_P} \tag{20}$$

where

$$\begin{aligned} \mathbf{X_P} &= \text{diag}\left\{[X(p(0)), X(p(1)), \ldots, X(p(S-1))]^T\right\}, \\ \mathbf{H}_{e,\mathbf{P}} &= [H_e(p(0)), H_e(p(1)), \ldots, H_e(p(S-1))]^T, \\ \mathbf{W_P} &= [W(p(0)), W(p(1)), \ldots, W(p(S-1))]^T, \\ [\mathbf{F_P}]_{m,l} &= e^{-j\frac{2\pi lK_a p(m)}{N}}, \\ & \quad m = 0, \ldots, S-1 \text{ and } l = 1, \ldots, M \end{aligned} \tag{21}$$

where $\text{diag}\{x\}$ denotes the diagonal matrix with the entries of the vector x along the main diagonal.

1) The FTCA-MMSE Estimator: In conventional comb-type pilot based channel estimation, the estimate of the channel frequency response at the pilot subcarriers, based on the LS criterion, is achieved as follows

$$\hat{\mathbf{H}}_{LS,\mathbf{P}} = \mathbf{X_P}^{-1}\mathbf{Y_P} = \mathbf{F_P}\mathbf{g} + \mathbf{H}_{e,\mathbf{p}} + \mathbf{X_P}^{-1}\mathbf{W_P} = \mathbf{F_P}\mathbf{g} + \mathbf{W}'' \tag{22}$$

Where

$$\mathbf{W}'' = \mathbf{H}_{e,\mathbf{p}} + \mathbf{X_P}^{-1}\mathbf{W_P} \tag{23}$$

Here, $\mathbf{H}_{e,\mathbf{P}}$ and $\mathbf{X_P}^{-1}\mathbf{W_P}$ are all Gaussian terms and assumed to be uncorrelated with each other. And, it is also assumed that $\mathbf{g}$

and $\mathbf{W}''$ are both Gaussian vectors with zero mean and independent of each other. Then, the MMSE estimate of $\mathbf{g}$ is achieved by [27]

$$\hat{\mathbf{g}}_{MMSE} = \mathbf{R}_{\mathbf{g},\hat{\mathbf{H}}_{LS,\mathbf{P}}} \mathbf{R}^{-1}_{\hat{\mathbf{H}}_{LS,\mathbf{P}},\hat{\mathbf{H}}_{LS,\mathbf{P}}} \hat{\mathbf{H}}_{LS,\mathbf{P}} \quad (24)$$

where

$$\begin{aligned} \mathbf{R}_{\mathbf{g},\hat{\mathbf{H}}_{LS,\mathbf{P}}} &= \mathrm{E}\left(\mathbf{g}\hat{\mathbf{H}}^H_{LS,\mathbf{P}}\right) = \mathbf{R}_{\mathbf{g},\mathbf{g}}\mathbf{F}^H_{\mathbf{P}} \\ \mathbf{R}_{\hat{\mathbf{H}}_{LS,\mathbf{P}},\hat{\mathbf{H}}_{LS,\mathbf{P}}} &= \mathrm{E}\left(\hat{\mathbf{H}}_{LS,\mathbf{P}}\hat{\mathbf{H}}^H_{LS,\mathbf{P}}\right) \\ &= \mathbf{F_P}\mathbf{R}_{\mathbf{g},\mathbf{g}}\mathbf{F}^H_{\mathbf{P}} + \left(B_e + \sigma^2/A\right)\mathbf{I}_S \quad (25) \end{aligned}$$

where $\mathbf{R}_{\mathbf{g},\mathbf{g}}$ is the auto-covariance matrix of $\mathbf{g}$, and $\mathbf{I}_S$ is an identity matrix of size $S \times S$. Here, B_e is neglected since it is very small compared to σ^2/A when K_a is properly chosen. Moreover, $\mathbf{R}_{\mathbf{g},\mathbf{g}}$ and σ^2 are assumed to be known in advance.

Therefore, the MMSE estimator $\hat{\mathbf{H}}_{FTCA-MMSE}$ for the channel frequency response is achieved by

$$\begin{aligned} \hat{\mathbf{H}}_{FTCA-MMSE} &= \mathbf{F}\mathbf{g}_{MMSE} \\ &= \mathbf{F}\mathbf{R}_{\mathbf{g},\hat{\mathbf{H}}_{LS,\mathbf{P}}} \mathbf{R}^{-1}_{\hat{\mathbf{H}}_{LS,\mathbf{P}},\hat{\mathbf{H}}_{LS,\mathbf{P}}} \hat{\mathbf{H}}_{LS,\mathbf{P}} \\ &= \mathbf{F}\left(\mathbf{F}^H_{\mathbf{P}}\mathbf{F_P} + \mathbf{R}^{-1}_{\mathbf{g},\mathbf{g}}\beta/D_{SNR}\right)^{-1} \\ &\quad \times \mathbf{F}^H_{\mathbf{P}}\hat{\mathbf{H}}_{LS,\mathbf{P}} \quad (26) \end{aligned}$$

where $\beta = E(|X(k)|^2)/A$ is the ratio of the average signal power to the pilot power, and $D_{SNR} = E(|X(k)|^2)/\sigma^2$ is the average SNR.

2) The FTCA-LS Estimator: In order to get the LS estimate of $\mathbf{g}$, it is simple to minimize $(\mathbf{Y_P} - \mathbf{X_P}\mathbf{F_P}\mathbf{g})^H(\mathbf{Y_P} - \mathbf{X_P}\mathbf{F_P}\mathbf{g})$. Then, the LS estimate of $\mathbf{g}$ is described as follows

$$\begin{aligned} \hat{\mathbf{g}}_{LS} &= \left(\mathbf{F}^H_{\mathbf{P}}\mathbf{X}^H_{\mathbf{P}}\mathbf{X_P}\mathbf{F_P}\right)^{-1}\mathbf{F}^H_{\mathbf{P}}\mathbf{X}^H_{\mathbf{P}}\mathbf{Y_P} \\ &= \left(\mathbf{F}^H_{\mathbf{P}}\mathbf{F_P}\right)^{-1}\mathbf{F}^H_{\mathbf{P}}\hat{\mathbf{H}}_{LS,\mathbf{P}} \quad (27) \end{aligned}$$

Therefore, the LS estimator $\hat{\mathbf{H}}_{FTCA-LS}$ for the channel frequency response is achieved by

$$\hat{\mathbf{H}}_{FTCA-LS} = \mathbf{F}\hat{\mathbf{g}}_{LS} = \mathbf{F}\left(\mathbf{F}^H_{\mathbf{P}}\mathbf{F_P}\right)^{-1}\mathbf{F}^H_{\mathbf{P}}\hat{\mathbf{H}}_{LS,\mathbf{P}} \quad (28)$$

IV. Simulations and Analyses

In this section, simulations are performed to investigate the performances of the FTCA estimators over multi-path slow-fading channels.

In the simulated OFDM system, the carrier frequency is 1 GHz, the signal bandwidth B_S is 2.5 MHz, N is 1024, and N_g is 32, where N_g is normally chosen according to N_D. In this paper, N_g is chosen as 32 for the purpose of simplicity, which can fit in with all the simulation cases in the paper. Therefore, the sampling interval T is 0.4 us and the total symbol duration is 422.4 us, 12.8 us of which is contained in the guard interval. In order to generate the $\mathbf{R}_{\mathbf{g},\mathbf{g}}$ for the FTCA channel model, Monte-Carlo simulations are performed. Then, the $\mathbf{R}_{\mathbf{g},\mathbf{g}}$ together with the actual noise variance σ^2 is used in the FTCA-MMSE estimator.

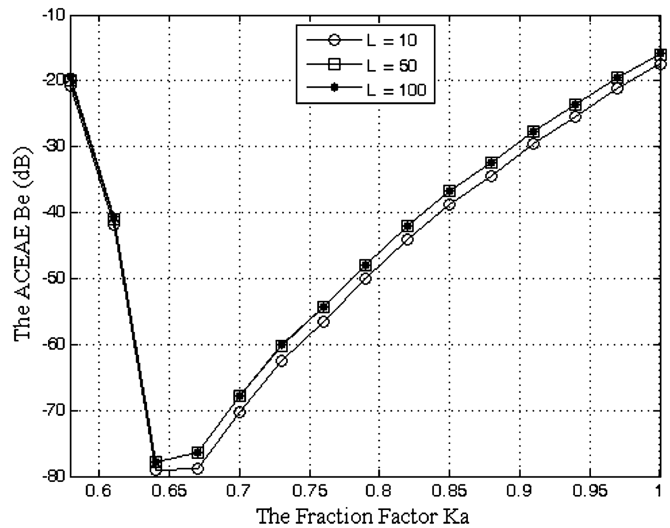

Fig. 3. The ACEAE B_e related to different values of K_a, where τ_{max} = 6.4 us.

The rest of this section is arranged as follows. In Section IV-A, the influence of the channel approximation error is investigated and the selection of the fraction factor K_a is discussed. And then, the performances of the FTCA estimators are evaluated over multi-path slow-fading channels with known τ_{max} in Section IV-B and with mismatched τ_{max} in Section IV-C, respectively. In this paper, these channel estimation methods, LS estimate of pilots with linear interpolation (LS-Linear), the low-rank MMSE (MMSE-5) [7], and low-rank LS (LS-5) [7] estimators, are also simulated and compared in Sections IV-B, C and D. Moreover, block-type pilot arrangement is used for the low-rank estimators while the comb-type pilot arrangement is used for the other estimators, where $S = 128$ and $D_f = 8$.

A. The Channel Approximation Error and the Selection of the Fraction Factor K_a

1) The Channel Approximation Error: When the FTCA channel model $h_F(\tau)$ is used to approximate the real channel $h(\tau)$, the approximation is biased and has channel approximation error $\mathbf{H}_e$ that described in (17), which equivalently adds noise to the FTCA channel $h_F(\tau)$. Here, the influence of $\mathbf{H}_e$ is defined as

$$\begin{aligned} B_e &= \frac{1}{N}\mathrm{E}\left(\mathbf{H}^H_e\mathbf{H}_e\right) \\ &= \frac{\mathrm{E}\left(\left(\mathbf{H} - \mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathbf{H}\right)^H\left(\mathbf{H} - \mathbf{F}(\mathbf{F}^H\mathbf{F})^{-1}\mathbf{F}^H\mathbf{H}\right)\right)}{N} \end{aligned} \quad (29)$$

where B_e denotes the average channel energy approximation error (ACEAE) and is related to the fraction factor K_a. Fig. 3 shows the relation between B_e and K_a, where L = 10, 50 and 100 are examined.

The results show that the number of the propagation paths L in the real channel $h(\tau)$ affects B_e little. This is expected

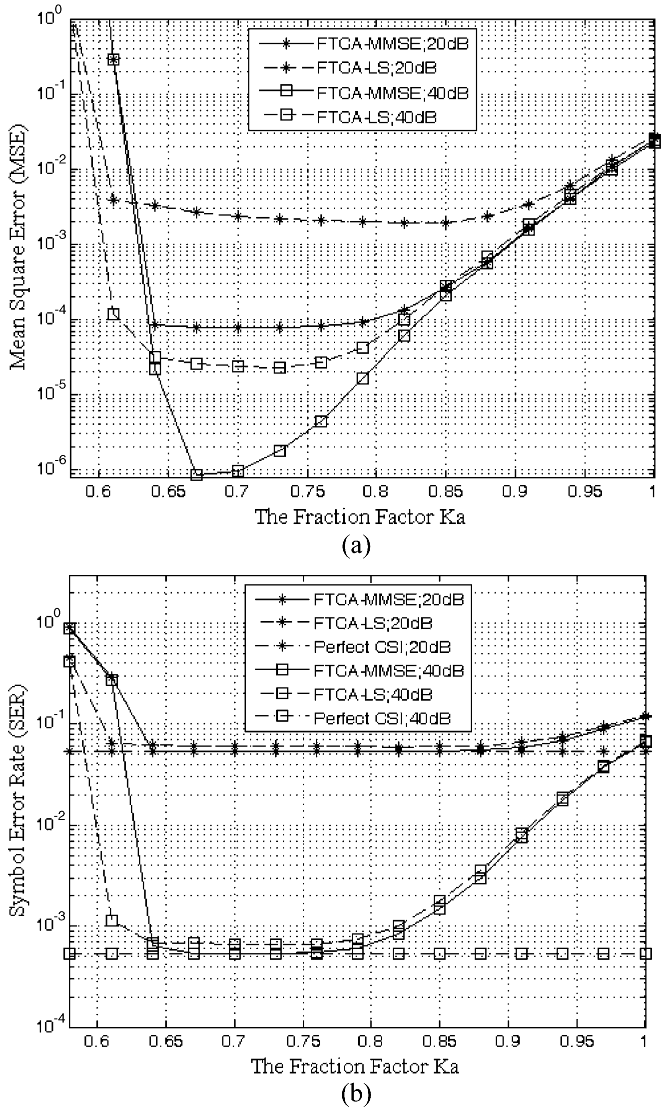

Fig. 4. The (a) MSE and (b) SER performances related to different values of K_a over multi-path slow-fading channels, where $\tau_{max} = 6.4$ us, and $L = 10$.

since the FTCA channel model $h_F(\tau)$ uses the finite impulse response on its definitive delay taps to approximate the real channel $h(\tau)$ and relates little to $h(\tau)$. Moreover, when K_a is chosen from [0.64, 0.76], B_e is well below -50 dB and thus the real channel $h(\tau)$ can be viewed to be well approximated by the FTCA channel model $h_F(\tau)$. This also proves that the assumption that the real channel $h(\tau)$ can be well approximated by the FTCA channel model $h_F(\tau)$ with K_a properly chosen is reasonable. However, when K_a decreases from 0.64 to 0.58, B_e increases sharply. The main reason is that the columns in the matrix $\mathbf{F}$ are not orthonormal with each other and $(\mathbf{F}^H\mathbf{F})^{-1}$ becomes a morbid matrix due to deep cross correlations among the approximation taps in the FTCA channel model $h_F(\tau)$ when K_a is chosen too small. The cross correlations grow severer when K_a decreases from 0.64 to 0.58. Therefore, the performance of the channel approximation degrades rapidly and consequently, B_e increases sharply. Otherwise, when K_a increases from 0.67 to 1, B_e increases. This is mainly because the number of the approximation taps M used to approximate the real channel $h(\tau)$

TABLE I
THE PROPER RANGES OF K_a ACCORDING TO DIFFERENT VALUES OF τ_{max}

The maximum delay spread τ_{max} in the channel (us)	The normalized maximum delay spread N_D	Proper range of the fraction factor K_a	Number of approximation taps used (M)
1.6	4	[0.36,0.57]	[9,13]
3.2	8	[0.50,0.68]	[13,17]
6.4	16	[0.67,0.79]	[22,25]
9.6	24	[0.74,0.82]	[31,34]
12.8	32	[0.79,0.85]	[39,42]

TABLE II
SIMULATION CONDITIONS 1 ($\tau_{MAX} = 6.4$ us & $N_D = 16$ & $K_A = 0.72$ & $L = 10$)

Estimator	Taps used (T)	Number of pilots used
FTCA-MMSE	[0…23]×0.72	128
FTCA-LS	[0…23]×0.72	128
MMSE-5	0…47, 992…1023	1024
LS-5	0…47, 992…1023	1024
LS-Linear	N.A.	128

decreases and more channel approximation error is introduced when K_a increases.

2) The Selection of the Fraction Factor K_a: When the FTCA estimators are used, a proper value should be chosen for K_a. The performance of the ACEAE B_e can be used to decide the proper K_a. However, in channel estimations, the performances of the mean square error (MSE) and symbol error rate (SER) are more concerned and thus more suitable for deciding the proper K_a. Fig. 4 shows the MSE and SER performances of the FTCA estimators related to the fraction factor K_a, where $\tau_{max} = 6.4$ us, $L = 10$, and the value x dB in the legends denotes the average SNR in the simulations.

The results show that the selection of the proper K_a is related to the channel noise condition. However, the SNR in the channel is always varying and hard to be obtained. Then, a high SNR is preferable to being used to design the FTCA estimators, where the channel approximation error is commonly small compared to the channel noise and affects little the performances of the FTCA estimators. Moreover, the selection of the proper K_a is also related to the maximum delay spread τ_{max} in the channel. Table I shows the ranges of the proper K_a making the FTCA estimators work normally (bring no error floor) for SNRs up to 40 dB according to different values of τ_{max}, where $B_S = 2.5$ MHz and $N = 1024$. It can be also seen from Table I that the proper K_a increases with τ_{max}. This is desirable for it leads to a slowing-down in the increase of M, which mainly determines the computational complexity of the FTCA estimators.

B. Performance Simulations With Known τ_{max}

When the FTCA estimators are used, the value of τ_{max} is needed to decide the proper K_a. In practice, τ_{max} is hard to be obtained directly but it can be achieved by obtaining the value of the root mean square (rms) delay spread τ_{rms} in the channel [28]–[30].

In this subsection, simulations are performed to evaluate the performances of the FTCA estimators designed with known τ_{max} over the multi-path slow-fading channels. In these simulations, $\tau_{max} = 6.4$ us and $N_D = 16$ are assumed to be known

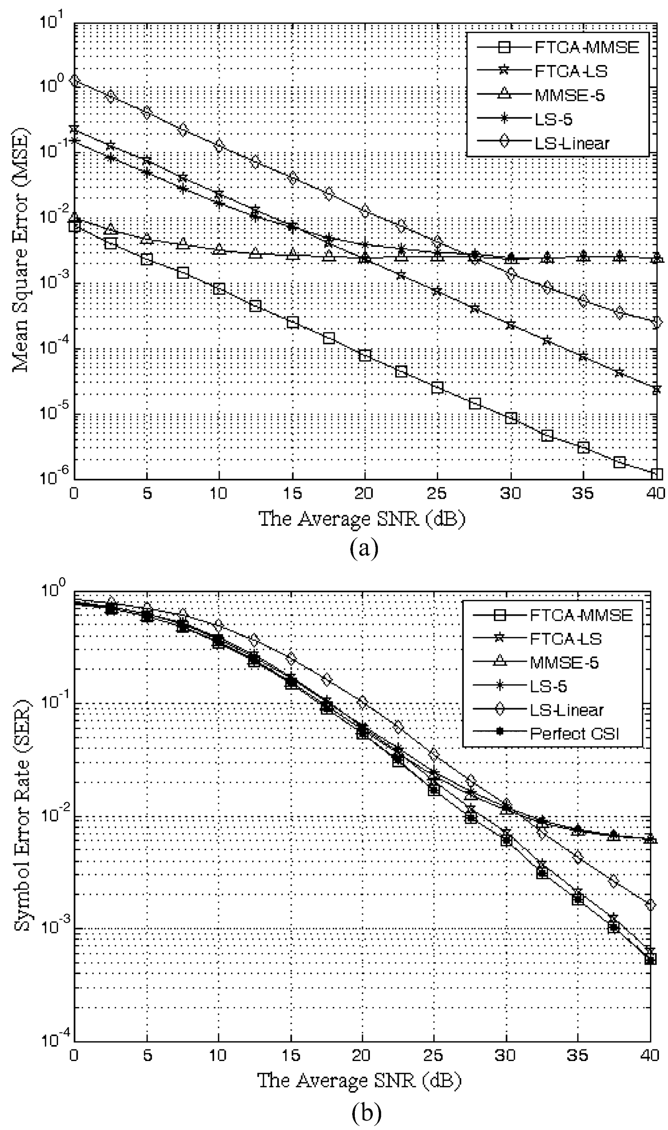

Fig. 5. Comparison of the (a) MSE and (b) SER performances over multi-path slow-fading channels, where $\tau_{max} = 6.4$ us, $L = 10$, and $K_a = 0.72$.

in advance and then $K_a = 0.72$ is chosen properly. Table II details some other simulation conditions, where $L = 10$.

Fig. 5 shows the MSE and SER performances of the compared estimators over multi-path slow-fading channels. The results show that the FTCA estimators work normally for SNRs up to 40 dB while the low-rank estimators perform well in low SNRs but achieve limited performances and even suffer error floors in high SNRs. This is expected and can be explained as follows. The low-rank estimators are based on the observed channel model $h(n)$ and use only the taps with significant energy, where the channel energy of the non-sample-spaced pulses leaks to all taps in $\{h(n)\}$. This introduces approximation error due to the channel energy loss in the excluded taps. The approximation error is small compared to the channel noise in low SNRs. However, it becomes dominant over the channel noise and strongly affects the performances of the low-rank estimators in high SNRs. Then, the error floors appear. Otherwise, the FTCA estimators are based on the FTCA channel model $h_F(\tau)$

TABLE III
SIMULATION CONDITIONS 2 ($\tau_{max,A} = 1.6$ us & $K_{a,A} = 0.51$ & $N_{D,A} = 4$ & $\tau_{max,E} = 6.4$ us & $K_{a,E} = 0.72$ & $N_{D,E} = 16$ & $L = 10$)

Estimator	Taps used (*T*)	Number of pilots used
FTCA-MMSE-A	[0…8]×0.51	128
FTCA-LS-A	[0…8]×0.51	128
FTCA-MMSE-E	[0…23]×0.72	128
FTCA-LS-E	[0…23]×0.72	128
MMSE-5	0…47,992…1023	1024
LS-5	0…47,992…1023	1024
LS-Linear	N.A.	128

and use all the approximation taps. Moreover, the channel approximation error is always very small compared to the channel noise and affects little the performances of the FTCA estimators. Therefore, the FTCA estimators perform normally and bring no error floors.

The FTCA-MMSE estimator outperforms the MMSE-5 estimator and achieves the best performance among all these compared estimators. It yields almost perfect channel state information (CSI) in SER and acquires 3 dB SER performance gain over the MMSE-5 estimator at a SER of 10^{-2}. At the same time, it achieves a MSE well below 10^{-3} whereas the MMSE-5 estimator never reaches. Moreover, when compared with the LS-Linear estimator, it achieves performance improvement of about 24 dB in MSE and almost 3.8 dB in SER.

The FTCA-LS estimator suffers performance loss relative to the FTCA-MMSE estimator and the performance degradation is almost 15 dB in MSE and about 0.6 dB in SER. However, it outperforms the LS-Linear estimator and achieves performance improvement of almost 8 dB in MSE and about 3.2 dB in SER. Moreover, when compared with the LS-5 estimator, the FTCA-LS estimator suffers certain performance loss in a low SNR while it completely outperforms the LS-5 estimator in high SNRs. The performance difference is inconspicuous in SNRs lower than 15 dB while it becomes obvious for SNRs larger than 25 dB.

C. *Performance Simulations With Mismatched* τ_{max}

In the OFDM system, the value of the maximum delay spread τ_{max} can be obtained by some methods. However, it brings large computational complexity and leads to unpractical. Considering that the maximum delay spread $\tau_{max,E}$ will be expected when an OFDM system is scheduled. The expected $\tau_{max,E}$ usually denotes the worst case delay spread in the channel, which appears rarely or even never appears. Therefore, the actual maximum delay spread $\tau_{max,A}$ in the channel is commonly less than the expected $\tau_{max.E}$. When the expected $\tau_{max,E}$ instead of the actual $\tau_{max,A}$ is used to decide the proper K_a, it introduces τ_{max} mismatch.

In this subsection, simulations are made to evaluate the performances of the FTCA estimators designed with the mismatched τ_{max} over multi-path slow-fading channels. At the same time, in order to examine the influence aroused by the mismatched τ_{max}, the performances of the FTCA estimators designed with the matched τ_{max} are also simulated. Table III details some other simulation conditions, where $\tau_{max,E} = 6.4$ us, $N_{D,E} = 16$, $K_{a,E} = 0.72$, $\tau_{max,A} = 1.6$ us, $N_{D,A} = 4$, $K_{a,A} = 0.51$, and $L = 10$. In Table III, the x-A estimator denotes the FTCA estimator based on the FTCA

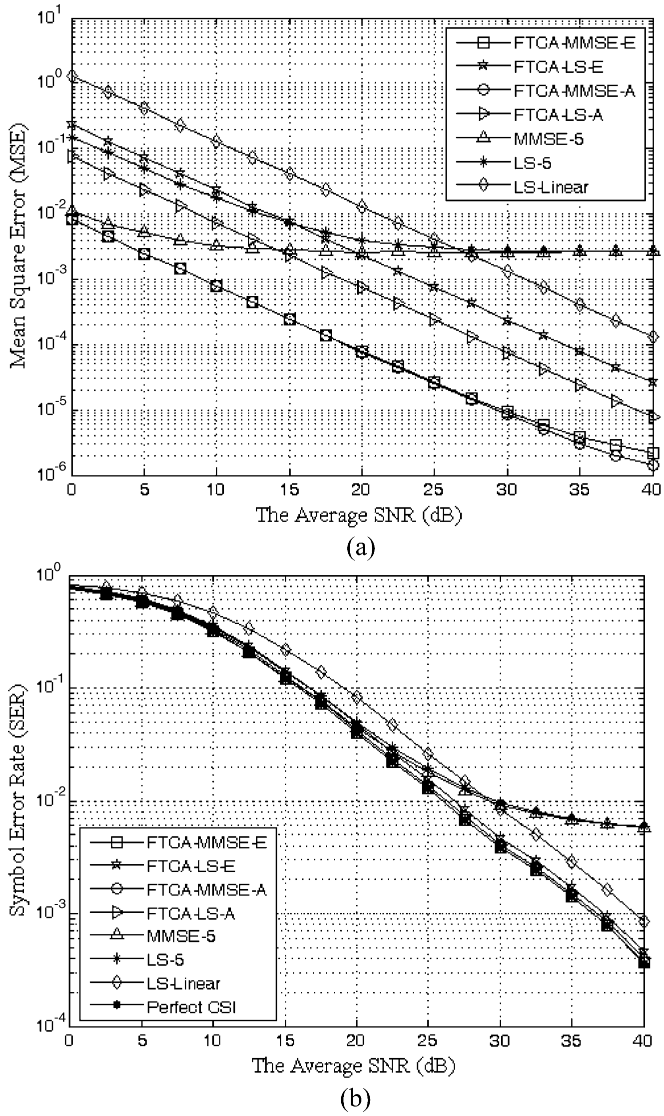

Fig. 6. Comparison of the (a) MSE and (b) SER performances over multi-path slow-fading channels, where $\tau_{max,A} = 1.6$ us, $K_{a,A} = 0.51$, $\tau_{max,E} = 6.4$ us, $K_{a,E} = 0.72$, and $L = 10$.

channel model $h_{F,A}(\tau)$ with parameters $\tau_{max,A} = 1.6$ us and $K_{a,A} = 0.51$, and the x-E estimator is derived from the FTCA channel model $h_{F,E}(\tau)$ with parameters $\tau_{max,E} = 6.4$ us and $K_{a,E} = 0.72$.

Fig. 6 shows the MSE and SER performances of the compared estimators over multi-path slow-fading channels. The FTCA-MMSE-A and FTCA-MMSE-E estimators achieve almost the same performances. This is expected and can be explained as follows. In the two FTCA-MMSE estimators, the real channel is both well approximated by the two FTCA channel models and the approximations errors are both small compared to the channel noise and affect little the performances of the two FTCA-MMSE estimators. Therefore, the expected $\tau_{max,E}$ can be used to decide the proper K_a when the FTCA-MMSE estimator is used. However, the FTCA-LS-E estimator suffers performance loss of almost 5 dB in MSE and about 0.5 dB in SER relative to the FTCA-LS-A estimator. This is mainly because less approximation taps are used in the FTCA channel model $h_{F,A}(\tau)$ and more channel noise is suppressed in the FTCA-LS-A estimator. And thus, the FTCA-LS-A estimator outperforms the FTCA-LS-E estimator. Therefore, when the FTCA-LS estimator is used, it is better to obtain the true value of τ_{max} for the better performance. However, the FTCA-LS-E estimator still well outperforms the LS-Linear estimator. It achieves performance improvement of 8 dB in MSE and about 2.8 dB in SER relative to the LS-Linear estimator.

V. Conclusions

In this paper, improved channel estimation methods based on comb-type pilot arrangement are proposed. These estimators are based on the FTCA channel model, where it uses some definitive delay taps that have a fraction tap delay spacing relative to the sampling interval to approximate the real channel. The use of the FTCA channel model provides two key advantages. First, it eliminates the problem of multi-path delay estimation and can be adopted in a channel not restricted to a sparse multi-path fading. Secondly, as compared to the observed channel model, its dimension is reduced, where the full-rank estimators using pilot tones can be adopted, and consequently, improves the channel estimation performance.

The FTCA-MMSE estimator achieves almost actual CSI but assumes a prior knowledge of the channel covariance and noise variance, which needs a large mount of information to be collected. The FTCA-LS estimator suffers about 0.6 dB performance degradation in SER but needs no supplementary information, such as the channel statistics and SNR, and can be widely used. Moreover, although these proposed estimators are derived based on the comb-type pilot arrangement, they can be easily adopted for any type of pilot arrangement and provide efficient channel estimations.

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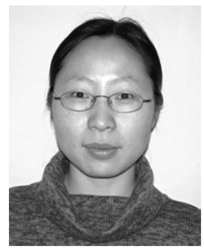

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