Received November 9, 2017, accepted December 12, 2017, date of publication December 27, 2017, date of current version February 14, 2018.

Digital Object Identifier 10.1109/ACCESS.2017.2786681

Dimension Reduction of Channel Correlation Matrix Using CUR-Decomposition Technique for 3-D Massive Antenna System

XIONGWEN ZHAO[1], (Senior Member, IEEE), ADAM MOHAMED AHMED ABDO[1], CHEN XU[1], (Member, IEEE), SUIYAN GENG[1], JIANHUA ZHANG[2], (Senior Member, IEEE), AND IMRAN MEMON[3], (Member, IEEE)

[1]School of Electrical and Electronic Engineering, North China Electric Power University, Beijing 102206, China
[2]State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China
[3]College of Computer Science, Zhejiang University, Hangzhou 310027, China

Corresponding author: Adam Mohamed Ahmed Abdo (adam_ma83@yahoo.com)

This work was supported in part by the open research fund of National Mobile Communications Research Laboratory, Southeast University under Grant 2016D09, in part by the National Key Laboratory of Electromagnetic Environment, China Research Institute of Radio wave Propagation under Grant 201600012, in part by the Key program of Beijing Municipal Natural Science Foundation under Grant 17L20052, and in part by the National Nature Science Foundation of China under Grant 61771194.

ABSTRACT Millimeter wave (mm-Wave) communications are emerging to meet the increasing demand for high transmission data rate in high user density areas. Meanwhile, the mm-Wave base station (BS) needs to employ a large number of antenna elements to increase the gain as well as serve a huge number of users. However, a vast number of antenna elements causes dimensionality problem in channel correlation matrix (rotation matrix). Therefore, we propose a novel codebook construction design based on CUR-decomposition technique to reduce the dimensionality problem. In this paper, the original correlation matrix is decomposed to the product of three low dimension matrices ($\mathbf{C}$, $\mathbf{U}$, and $\mathbf{R}$). The new rotated codebook is then constructed by the new rotation matrix. Moreover, we evaluate the new decomposition matrix with the original matrix in terms of compression ratio and mismatch error. We also provide the achievable sum rate capacities for singular value decomposition, zero forcing, and a matched filter techniques to compare with the proposed method. Furthermore, the system capacity enhancement related to the number of antenna elements and the required feedback bits are analyzed. Simulation results show that the proposed method achieves much better system performance since the dimensionality problem is solved. The proposed method can be applied in the fifth generation massive antenna multi-user system with over a hundred antenna elements.

INDEX TERMS Millimeter-wave, CUR-decomposition, matrix dimension, massive antenna system, compression ratio (CR), mismatch error, sum-rate capacity, multi-user, and Ricean channel.

I. INTRODUCTION

Fifth generation (5G) millimeter wave (mm-Wave) communications are emerging to meet the increasing demand for high data rate transmission in high user density areas. The mm-Wave band has been recently considered as a key solution to achieve a significant performance on frequency spectrum utilization and provide high-speed multimedia data services [1], [2]. Furthermore, the performance of a three dimensional (3-D) massive antenna multi-user system highly depends on the channel state information (CSI) at a base station (BS). In frequency division duplexing (FDD) system, limited feedback is used to perform quantization of the CSI at the mobile end, and then feedback to the BS. However, due to a large number of antenna elements applied to a massive antenna system, the feedback overhead is unacceptable [3]. The common problems mm-Wave communications face include the high path loss and signal attenuation which make mm-Wave signal very weak after arriving at the receiver [4]–[6], [7]. In fact, the path loss between two antenna elements separated by a certain distance depends on its effective area which is proportional to the wavelength of the carrier signal [5]. Meanwhile, the larger the antenna aperture, the larger the antenna gain and vice versa [8]. Therefore, to maximize the gain in mm-Wave with the small effective area, a large number of antenna elements should be packed into an equivalent antenna aperture zone to implement transceiver beamforming enabling high array gain [9]. In the mm-Wave band, the inter-element spacing is normally half a wavelength which is extremely short. Hence, we can integrate many antenna elements into small form factor. Consequently, high directional beamforming using massive antenna system can compensate huge path loss [9], [10].

However, integration of a massive number of antenna elements causes a large dimension of correlation channel matrix, which is called dimensionality problem that brings high computational complexity in matrix operation. Moreover, it is much difficult to determine an accurate correlation matrix required by the rotation codebook. In recent years, some works have been done to reduce the dimension problem. In [11], [12], a joint spatial division and multiplexing (JSDM) method were proposed to divide all users into groups with same channel covariance eigenspace. For a multi-user (MU) massive antenna downlink system, the structure of channel vector correlation allows a large number of antenna elements to be deployed to reduce the dimensionality of channel state information at the BS. In [13], it was proposed to use a compressive sensing (CS) technique to reduce the CSI feedback load for multi-user multiple-input multiple-output (MU-MIMO) system making the transmitter obtains the CSI with an acceptable level of accuracy. Based on singular value decomposition (SVD), an adaptive beamforming technique to generate the beam weight by considering the dominant eigenvector of the covariance matrix and discarding the mono-dominant was proposed in [14]. The dominant eigenvectors are used to construct the new channel weight. The method updates the new reduced dimension channel weight at any time. Antenna group scheduling (AGS) is a method proposed by [15] to reduce the dimension of channel vector. The idea of AGS algorithm is based on a set of predetermined antenna group, and each user has to select the best one with the maximum gain and then quantizes the channel vector corresponding to the selected group. Tucker decomposition was proposed by [3], which is applied to a rotation matrix and decomposes it into three low dimension matrices using SVD method. These matrices are multiplied together to form the rotation matrix which is used to construct the rotation codebook where each user uses the codebook to quantize its CSI. Therefore, a CUR-decomposition technique can be applied in the rotation matrix to construct its rotation codebook. The CUR is a multiplication of the three matrices $\mathbf{C}$, $\mathbf{U}$, and $\mathbf{R}$ corresponding to the matrices $\mathbf{U}$, $\boldsymbol{\lambda}$, and $\mathbf{V}$ in SVD method [3]. In order to reduce the matrix dimension, feedback overhead and computation complexity, the decomposition of the original rotation matrix are obtained by applying SVD on the channel correlation matrix.

In this study, we propose a novel codebook construction design technique based on the CUR- decomposition method to reduce the dimensionality, in which a 3D-MIMO channel model based on Kronecker product of two array responses is applied with a uniform rectangular array antenna (URA) used at the BS. The main contributions of this work are as follows: we propose an algorithm to design a new codebook rotation matrix from the original correlation matrix, and an expression for the compression ratio (CR) is derived, which is inversely proportional to the mismatch error; we design the precoding vectors, which are quantized into B bits and fedback to the BS. Moreover, we derive an analytical model regarding the required number of feedback bits. A large number of bits is required when a large number of antenna elements and users are deployed. Finally, comparisons between our proposed method and conventional ones are carried out regarding average system capacity.

The organization of this work is as follows. In section II, we introduce a 3D-MIMO system model. The concepts of rotation matrix and codebook are presented in section III. In section IV, our proposed method and algorithm are described. The simulation results with discussions are shown in section V. Finally; the conclusion is outlined in section VI.

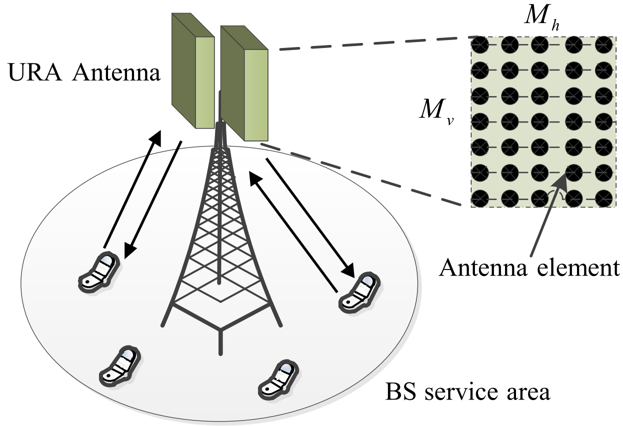

FIGURE 1. A 3D-MIMO system in which the BS equipped with a uniform rectangular array antenna (URA).

II. SYSTEM MODEL

Fig. 1 shows a 3D massive antenna multi-user system, where the BS is equipped with URA antennas, in which M_v is the number of vertical antenna elements and M_h is the number of horizontal antenna elements. We assume that the system serves K users, each with a single antenna. For the sake of simplicity, assume that $M_v = M_h$, and the total number of URA antennas M is given by $M = M_v{}^*M_h$. We use d_v and d_h to denote the spacing between elements in the vertical and horizontal direction respectively. In addition, in this study the path loss is ignored by assuming all the users are closely distributed around the BS. Let's assume that the channel undergoes Ricean Fading which takes the consideration of line-of-sight (LOS) component. This means that the two-channel components are deterministic LOS component and Rayleigh-distributed random component. The random version of the channel is accounted for the diffused multipath signals which are modeled by Random variable distribution (RVD). Then the channel vector becomes:

$\mathbf{h}_{w,k} = \sqrt{1/(\kappa_k+1)}\mathbf{h}_k^{\text{rand}} + \sqrt{\kappa_k/(\kappa_k+1)}\mathbf{h}_k^{\text{det}}$ where κ_k, $\mathbf{h}_k^{\text{det}}$ and $\mathbf{h}_k^{\text{rand}}$ are respectively the Ricean K-factor, a deterministic component and the random component of k^{th} user. The elements of $\mathbf{h}_k^{\text{rand}}$ being independent and identically distributed complex Gaussian distribution with zero mean and unit variance i.e $\mathbf{h}_k^{\text{rand}} \approx \mathbb{CN}(0,1)$. The deterministic component of URA channel matrix is constructed by applying Kronecker product to vertical and horizontal array responses as follow [16]

$$\mathbf{h}_k^{\text{det}} = \mathbf{a}(\theta,\phi) = \mathbf{a}_h(\theta) \otimes \mathbf{a}_v(\phi), \tag{1}$$

where the vectors $\mathbf{a}_h(\theta)$, $\mathbf{a}_v(\phi)$ are $M_h \times 1$ and $M_v \times 1$ dimensional array responses in the horizontal and vertical directions, respectively. The array response vectors are given by [17].

$$\mathbf{a}_h(\theta) = \left[1, e^{-jk_h\cos\theta}, \ldots, e^{-jk_h(M_h-1)\cos\theta}\right]^T, \tag{2}$$

$$\mathbf{a}_v(\phi) = \left[1, e^{-jk_v\cos\phi}, \ldots, e^{-jk_v(M_v-1)\cos\phi}\right]^T, \tag{3}$$

where $k_h = 2\pi d_h/\lambda$, $k_v = 2\pi d_v/\lambda$. Suppose $d_h = d_v = d$, λ is the wavelength, θ and ϕ are the angle-of-departure (AoD) for a horizontal and vertical array antenna respectively. Consequently, the channel vector $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ for each user can be written as

$$\mathbf{h}_k = \bar{\mathbf{R}}_k^{1/2}\mathbf{h}_{w,k}, \tag{4}$$

where $\bar{\mathbf{R}}_k \in \mathbb{C}^{M\times M}$ is the correlation matrix for each user, and can be given as $\bar{\mathbf{R}}_k = \mathrm{E}\left\{\mathbf{h}_{w,k}\mathbf{h}_{w,k}^H\right\}$. Accordingly, the multi user MIMO channel matrix $\mathbf{H} \in \mathbb{C}^{M\times K}$ can be written in concatenating form as $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K] \in \mathbb{C}^{M\times K}$.

III. ROTATION MATRIX AND ROTATED CODEBOOK

A 3D massive antenna multi-user system performance is highly dependent on the accuracy of the CSI obtained at the BS. The CSI is given by $\bar{\mathbf{h}}_k = \mathbf{h}_k/\|\mathbf{h}_k\|$ which is a unit-norm of the channel vector. In the feedback procedure, each user quantizes the CSI using codebook known by the BS, and then feedback the optimal code-word to the BS for beamforming [3]. The rotated codebook can be formed from the pre-constructed code-words. The rotation matrix can be expressed as in (5).

$$\bar{\mathbf{c}}_i = \bar{\mathbf{R}}_k^{\frac{1}{2}}\mathbf{w}_i \Big/ \left\|\bar{\mathbf{R}}_k^{\frac{1}{2}}\mathbf{w}_i\right\|, \quad i = 1, 2, \ldots, 2^B, \tag{5}$$

where $\mathbf{w}_i$ is the $M \times 1$ code vector, and the i^{th} code word in B-bits discrete Fourier transform (DFT) codebook can be constructed by [18], [19]

$$\mathbf{w}_i = \mathbf{w}(\varphi_i) = \frac{1}{\sqrt{M}}\left[1, e^{-j2\pi\varphi_i}, \ldots, e^{-j2\pi(M-1)\varphi_i}\right]^T, \tag{6}$$

where $\varphi_i = \frac{i}{2^B}(i = 1, 2, \ldots, 2^B)$ and $2\pi\varphi_i$ is the i^{th} sample in angular domain $[0, 2\pi]$. Under a massive 3D-MIMO antenna system, high dimensionality problem arises. Large dimension in $\bar{\mathbf{R}}_k$ increases not only the complexity of MIMO, but also the difficulty in application of rotated codebook. In [3], it was proposed that a solution is found by obtaining new $\hat{\mathbf{R}}_k$ from SVD matrices to reduce the dimension issue. The new rotation matrix is given by

$$\hat{\mathbf{R}}_k = (\mathbf{U}\otimes\mathbf{V})\, diagv\,(\lambda)\,(\mathbf{U}\otimes\mathbf{V})^H, \tag{7}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices obtained by SVD from the correlation matrix, and $\boldsymbol{\lambda}$ is a non-negative diagonal element. Thus, the new rotated codebook for a given decomposition is [20]

$$\hat{\mathbf{c}}_i = \hat{\mathbf{R}}_k^{\frac{1}{2}}\mathbf{w}_i \Big/ \left\|\hat{\mathbf{R}}_k^{\frac{1}{2}}\mathbf{w}_i\right\|, \quad i = 1, 2, \ldots, 2^B. \tag{8}$$

In fact, if the matrix $\hat{\mathbf{R}}_k$ has a large dimension, we need to represent this matrix by its SVD components $\mathbf{U}$, $\mathbf{V}$ and Σ $(\boldsymbol{\lambda})$. However, these components have large dimensions and demand large and costly storage. Therefore, the best way to reduce the dimensions of the three matrices is to set the smallest singular value of $\boldsymbol{\lambda}$ to zero, and then remove the corresponding eigenvectors in matrices $\mathbf{U}$ and $\mathbf{V}$. The disadvantage of this procedure is that it reduces the root-mean-square-error (RMSE) which in turn results in signal power reduction.

IV. THE PROPOSED METHOD USING CUR-DECOMPOSITION

In fact, CUR matrix decomposition technique is less accurate than SVD if the SVD is fully used without dimension reduction. However, the CUR technique offers the key advantages. *Firstly*, the calculation of CUR is of lower asymptotic time complexity compared with that of SVD. *Secondly*, the calculated matrices are more interpretable, i.e. the meanings of rows and columns in the decomposed matrix are the same as their meanings in the original matrix. Additionally, in this approach, if the matrix $\mathbf{A}$ is sparse, then the two large matrices called $\mathbf{C}$ (columns) and $\mathbf{R}$ (rows) analogous to the two SVD matrices ($\mathbf{U}$ and $\mathbf{V}$) are also sparse. Only the middle matrix similar to Σ in SVD is dense, but it is with a small dimension. Therefore, its density does not affect too much on the density of CUR matrix [21], [22].

A. CUR-DECOMPOSITION AND FRAMEWORK

In the following discussions, let's use a matrix $\mathbf{A}$ to replace the original correlation matrix $\bar{\mathbf{R}}$, because there is a matrix $\mathbf{R}$ in the algorithm which is important to be kept as it is. Let $\mathbf{A}$ be a matrix with dimension $m \times n$, and then pick up a target number of c columns and r rows to be used in CUR-decomposition. The CUR-decomposition construction of the matrix $\mathbf{A}$ is randomly chosen as a set of c columns of $\mathbf{A}$ to form the matrix $\mathbf{C}$ with dimension $m \times c$, and randomly selected as r rows of the matrix $\mathbf{A}$ to form a matrix $\mathbf{R}$ with dimension $r \times n$. The rotation matrix obtained by the channel matrix $\mathbf{h}$ as in (6) is a square matrix assumed $M_v = M_h$. Therefore, the number of rows m and columns n in matrix $\mathbf{A}$ are equal $(m = n)$. To simplify the construction of new approximated matrix, we consider $c = r$. Furthermore, the middle square matrix $\mathbf{U}$ with dimension $c \times c$ can be constructed from $\mathbf{C}$ and $\mathbf{R}$ as explained in the ***CUR decomposition Algorithm***. The CUR-decomposition mainly consists of three steps:

- ***Step 1***, build a matrix $\mathbf{C}$ by randomly sampling columns of $\mathbf{A}$ with c identical independent trials, and in each trial, the column of the matrix $\mathbf{A}_{:,j}$ is sampled with a probability of $p_j = \|\mathbf{A}_{:,j}\|^2 / \|\mathbf{A}\|_F^2$. After the sampling, the matrix $\mathbf{A}$ is re-scaled by $1/\sqrt{cp_j}$.

- ***Step 2***, construct a matrix $\mathbf{R}$ by randomly sampling $r(r = c)$ rows of $\mathbf{A}$, and in each trial, the row $\mathbf{A}_{i,:}$ is sampled with a probability of $q_i = \|\mathbf{A}_{i,:}\|^2 / \|\mathbf{A}\|_F^2$, and then the matrix $\mathbf{A}$ is re-scaled by $1/\sqrt{rq_i}$.
- ***Step 3***, construct the middle matrix $\mathbf{U}$ at the intersection of $\mathbf{C}$ and $\mathbf{R}$, and then we can write the approximated matrix $\mathbf{A}$ as: $\mathbf{A} \simeq \mathbf{CUR}$. The specific steps of CUR decomposition are summarized in the ***CUR-decomposition Algorithm***.

The computation cost of the CUR-decomposition versus SVD can be described as follows: For a given matrix $\mathbf{A}_{m\times n}$, the CUR decomposition requires a minimum storage space as $mc + nc + c^2$, while SVD requires a space for three matrices that can be given by $m^2 + mn + n^2$, noting that $m, n \gg c$. Regarding the computation cost, the CUR-decomposition has a matrix of dimension $c \times c$, so it has an asymptotic cost of $O(mnc)$, but for SVD, it has a cost of $O\left((m+n)\,c^2\right)$. Moreover, we provide Fig. 2 for the comparison of two technique, by assuming the cost function (the number of computations) is a function in sample matrix dimension (c) with fixed the original matrix dimensions (m & n), the result shows the required number of computation for the proposed and SVD methods. We observe that the cost of CUR-decomposition is linear increasing with c. However, in SVD method it is non-linear increasing with c. Therefore, we can conclude that the proposed method is much efficiently can enhance the system performance than SVD. Particularly, when the sample matrix dimension $c > 4$.

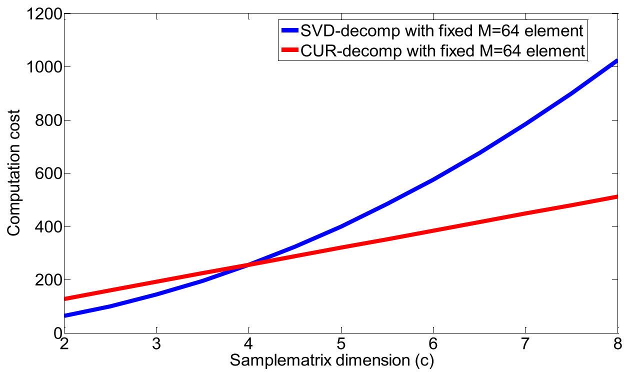

FIGURE 2. Computation cost of proposed method and SVD technique regarding the sample matrix dimension (c).

B. PERFORMANCE EVALUATION OF CUR-DECOMPOSITION

It is necessary to evaluate the proposed method regarding the amount of compression in the matrix, as well as mismatch error between the original correlation matrix and decomposed ones. One of the evaluation factors is the percent-root-mean square distortion ratio (PDR), which is calculated as follows

$$\mathrm{PDR}(\%) = \frac{\|\mathbf{A} - \mathbf{CUR}\|_F}{\|\mathbf{A}\|_F} = \frac{\left\|\mathbf{A} - \tilde{\mathbf{A}}\right\|_F}{\|\mathbf{A}\|_F}. \tag{9}$$

The numerator in (9) is the Frobenius norm of mismatch error which is divided by the Frobenius norm of the original matrix $\mathbf{A}$. We can calculate the reduction or compression ratio which is the ratio of a total number of antenna elements in the original matrix and in the decomposed matrix. In CUR-decomposition, the matrix $\mathbf{A} \in \mathbb{C}^{m\times n}$ is decomposed into matrices $\mathbf{C} \in \mathbb{C}^{m\times c}$, $\mathbf{U} \in \mathbb{C}^{c\times r}$ and $\mathbf{R} \in \mathbb{C}^{r\times n}$, thus the compression ratio is given by:

$$\mathrm{CR} = \frac{mn}{mc + cr + rn}. \tag{10}$$

In fact, we store a subset of the original columns and rows of the matrix $\mathbf{A}$. We assume that the value of c and r are restricted by $c, r \geq 2$ to avoid missing of the original data. Moreover, in approximation, $\mathbf{C}$ and $\mathbf{R}$ have c^*r elements. To improve the compression efficiency, these matrices can be stored in $c + r$ positions instead of c^*r redundant elements. Therefore, the improved compression ratio in (10) can be re-written as

$$\mathrm{CR}_{\text{improved}} = \frac{mn}{mc + cr + (rn - cr) + c + r}, \tag{11}$$

Consequently, as assumed that the rotation matrix is a square matrix, $m = n$ and $c = r$, and (10) and (11) can be re-written as in (12) and (13), respectively.

$$\mathrm{CR} = \frac{m^2}{c^2 + 2mc}, \tag{12}$$

$$\mathrm{CR}_{\text{improved}} = \frac{m^2}{2c(m+1)}. \tag{13}$$

C. CONSTRUCTION OF PRECODING MATRIX

In this subsection, we will describe the channel feedback mechanism as well as precoding matrix design based on CUR-decomposition. In the limited feedback, each user knows its channel vector $\mathbf{h}_k$, which should be quantized into B bits, and then the quantized version feedback to the BS. We set a uniform generated codebook $\mathbf{W} = \left[\mathbf{w}_1, \mathbf{w}_2, \ldots \mathbf{w}_{2^B}\right]$, and the unit norm vector $\mathbf{w}_i \in \mathbb{C}^{M\times 1}$ given by (5). Then, the quantization vector $\bar{\mathbf{c}}_{k,i} \in \mathbb{C}^{M\times 1}$ in the k^{th} user's codebook $\bar{\mathbf{C}}_k = \left[\bar{\mathbf{c}}_1, \bar{\mathbf{c}}_2, \ldots \bar{\mathbf{c}}_{2^B}\right]$ can be obtained by multiplying the vector $\mathbf{w}_i$ by the square root of the correlation matrix $\bar{\mathbf{R}}$ and taking the normalized code vector as given in (5). However, the objective is to use the proposed decomposed matrix, which is defined by the matrix $\tilde{\mathbf{A}}$. Therefore, the new quantized code vector is constructed as follows;

$$\tilde{\mathbf{c}}_{k,i} = \frac{\tilde{\mathbf{A}}_k^{1/2}\mathbf{w}_i}{\left\|\tilde{\mathbf{A}}_k^{1/2}\mathbf{w}_i\right\|} = \frac{\sqrt{\tilde{\mathbf{A}}_k}\mathbf{w}_i}{\left\|\sqrt{\tilde{\mathbf{A}}}\mathbf{w}_i\right\|}. \tag{14}$$

As the quantization vector is calculated in (14), the distribution of the quantization vector is close to the actual channel $\mathbf{h}_k$ which is shown in (4).

Algorithm 1 CUR-Decomposition ***Algorithm***

Input: $\mathbf{A} \in \mathbb{C}^{n\times d}$, $1 \le c \le d$, $1 \le r \le n$, $1 \le k \le \min(n, d)$
Output: $\mathbf{C} \in \mathbb{C}^{n\times c}$, $\mathbf{U} \in \mathbb{C}^{c\times r}$, $\mathbf{R} \in \mathbb{C}^{r\times d}$
For $s = 1$ *to c do*
Pick $j \in \{1, 2, \ldots, d\}$ with probability $p_j = \left\|\mathbf{A}_{:,j}\right\|^2/\|\mathbf{A}\|_F^2$
Set $\mathbf{C}_{:,s} = \mathbf{A}_{:,j}/\sqrt{cp_j}$
Set $k = min\left(k, \operatorname{rank}\left(\mathbf{C}^T\mathbf{C}\right)\right)$
End
For $s = 1$ *to r do*
Pick $i \in \{1, 2, \ldots, n\}$ with probability $q_i = \left\|\mathbf{A}_{i,:}\right\|^2/\|\mathbf{A}\|_F^2$
Set $\mathbf{R}_{s,:} = \mathbf{A}_{i,:}/\sqrt{rq_i}$
Set $\mathbf{\Psi}_{s,:} = \mathbf{C}_{i,:}/\sqrt{rq_i}$
Let $\mathbf{U} = \left(\left[\mathbf{C}^T\mathbf{C}\right]\right)^{-1}\mathbf{\Psi}^T$
End
Output $\mathbf{C}, \mathbf{U}, \mathbf{R}$
Finally we have $\tilde{\mathbf{A}} = \mathbf{C}^*\mathbf{U}^*\mathbf{R}$, multiplication of three matrices. Anywhere we will use $\tilde{\mathbf{A}}$ instead of **CUR**.
End of the algorithm

Therefore, the quantization vector closest to the actual channel can be measured by the angle between the two vectors, so that the k^{th} user computes the quantization index F_k as follows

$$F_k = \underset{i\in[1,2^B]}{\arg\max}\left|\tilde{\mathbf{h}}_k^H\tilde{\mathbf{c}}_{k,i}\right|^2, \tag{15}$$

where $\tilde{\mathbf{h}}_k = \mathbf{h}_k/\|\mathbf{h}_k\|$ is denoted as the direction of the channel vector (feedback channel vector).

Remark: Only the direction of channel vector is quantized. However, the channel magnitude $\|\mathbf{h}_k\|$ is not quantized by codebook $\bar{\mathbf{C}}$ because its magnitude is a scalar, and it can be easily feedback to the BS. From the received quantization vector index F_k, the BS can easily obtain the feedback channel vector as:

$$\hat{\mathbf{h}}_k = \|\mathbf{h}_k\|\,\tilde{\mathbf{c}}_{k,F_k}. \tag{16}$$

Similarly, the new concatenation of the feedback channel vectors can be written as $\hat{\mathbf{H}} = \left[\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2, \ldots, \hat{\mathbf{h}}_K\right] \in \mathbb{C}^{M\times K}$ [23]. Hence, we can construct the precoding matrix $\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K] \in \mathbb{C}^{M\times K}$, which consists of K different M dimensional unit norm vector $\mathbf{v}_i \in \mathbb{C}^{M\times 1}$, and the normalized precoding vector can be obtained by a normalized i^{th} column of the matrix $\hat{\mathbf{U}}$ as follows:

$$\hat{\mathbf{U}} = \hat{\mathbf{H}}\left(\hat{\mathbf{H}}^H\hat{\mathbf{H}}\right)^{-1}, \tag{17}$$

$$\mathbf{v}_i = \hat{\mathbf{U}}(:, i)/\left\|\hat{\mathbf{U}}(:, i)\right\|, \quad i = 1, 2, \ldots, K. \tag{18}$$

Results of (18) are considered as practical precoding vectors. However, these vectors for an ideal case are known by zero forcing (ZF). Consequently, the precoding matrix can be obtained by calculating the matrix $\hat{\mathbf{U}} = \mathbf{H}\left(\mathbf{H}^H\mathbf{H}\right)^{-1}$, where $\mathbf{H}$ is the channel matrix described in section II. Furthermore, the matched filter (MF) precoding vector can be achieved directly from (16) as $\mathbf{v}_i^{MF} = \hat{\mathbf{h}}_i$. Hence, the precoding matrix is the same as the composite channel matrix $\mathbf{V} = \hat{\mathbf{H}}$. Moreover, to compare the precoding matrix obtained by the proposed method and the original correlation matrix $\bar{\mathbf{R}}_k$, we can easily substitute the matrix $\tilde{\mathbf{A}}_k$ in (14) by using the original matrix $\bar{\mathbf{R}}_k$ in (4).

D. ACHIEVABLE SUM RATE CAPACITY

Sum rate capacity is a key factor used to evaluate the performance of a multi-user system, which can be obtained by signal to interference plus noise ratio (SINR). SINR of the k^{th} user can be calculated as [23]

$$SINR_k = \frac{\frac{p}{K}\left|\mathbf{h}_k^H\mathbf{v}_k\right|^2}{\sigma_k^2 + \frac{p}{K}\sum\limits_{\substack{i=1\\i\neq k}}^{K}\left|\mathbf{h}_k^H\mathbf{v}_i\right|^2}, \tag{19}$$

where p is the total transmit power, K is the number of users, and the total transmit power is divided equally among all users. Moreover, in (19) a random distribution complex white Gaussian noise with zero mean and unit norm variance $\left(\sigma_k^2 = 1\right)$ is assumed. Therefore, the corresponding sum rate capacity is given by

$$R_{practical} = \mathrm{E}\left[\log_2\left(1 + \frac{\frac{p}{K}\left|\mathbf{h}_k^H\mathbf{v}_k\right|^2}{1 + \frac{p}{K}\sum\limits_{\substack{i=1\\i\neq k}}^{K}\left|\mathbf{h}_k^H\mathbf{v}_i\right|^2}\right)\right]. \tag{20}$$

Clearly, the user rate depends on the precoding matrix vectors $\mathbf{v}_i$, which are calculated from feedback channel quality $\hat{\mathbf{H}}$. The sum rate capacity computed by (20) is considered as practical sum rate $\left(R_{practical}\right)$. Additionally, as demonstrated in (18), the ideal case of precoding matrix is called ZF precoding matrix with corresponding to precoding vector of $\mathbf{v}_{ideal}^{ZF}$. Hence, the achievable sum rate capacity for ZF becomes R_{ideal}^{ZF}, which can be written as:

$$R_{ideal}^{ZF} = E\left[\log_2\left(1 + \frac{p}{K}\left|\mathbf{h}_k^H\mathbf{v}_{ideal,k}^{ZF}\right|^2\right)\right]. \tag{21}$$

The gap loss capacity ΔR is defined by the difference between the capacities R_{ideal} achieved by ideal channel precoding and $R_{practical}$ which is obtained by limited feedback. Accordingly, it can be described as $\Delta R = R_{ideal} - R_{practical}$. Moreover, by using Jensen's inequality, the gap loss capacity is written in (22) [24].

$$\Delta R \le \log_2\left(1 + \frac{p}{K}(K-1)\,E\left[\left|\mathbf{h}_k^H\mathbf{v}_i\right|^2\right]\right), \tag{22}$$

where $\mathrm{E}\left[\left|\mathbf{h}_k^H\mathbf{v}_i\right|^2\right]$ is the upper bound of the multi-user interference, which depends on the channel quantization error

$E\left[\sin^2\left(\angle\left(\tilde{\mathbf{h}}_k, \hat{\mathbf{h}}_k\right)\right)\right]$ as in (23)

$$E\left[\left|\mathbf{h}_k^H \mathbf{v}_i\right|^2\right] \leq E\left[\|\mathbf{h}_k\|^2\right] E\left[\sin^2\left(\angle\left(\tilde{\mathbf{h}}_k, \hat{\mathbf{h}}_k\right)\right)\right]. \quad (23)$$

Furthermore, the rate gap loss which is mentioned in (22), is bounded by the number of feedback bits (B) per user, and assuming the number of antenna elements employed by ZF at the BS is equal to the number of users ($M = K$). In fact, the gap loss increases with increased transmit power p. We write the gap loss as a function of feedback bits as

$$\Delta R(p) \leq \log_2\left(1 + C(M, d, \lambda, r)\, 2^{-2B} p\right), \quad (24)$$

where

$$C(M, d, \lambda, r) = \frac{(M-1)\left(M^2-1\right)\pi^2 d^2 r^2}{12\lambda^2}. \quad (25)$$

Moreover, r is the rank of the channel correlation matrix to maintain the gap loss not more than $\log_2(i)$ bps/Hz per user, where i is the code vector index mentioned in (5). Thus, the approximate number of bits can be obtained by solving (24) regarding B as follows

$$B \approx \frac{P_{dB}}{6} + \frac{1}{2}\log_2 C(M, d, \lambda, r) - \frac{1}{2}\log_2(i-1). \quad (26)$$

From (26), we set $i = 2$ and keep the SNR loss between the sum rate capacity achieved under ideal precoding and limited channel feedback at 3 dB. Then, assume that inter-element spacing d is equal to $\lambda/2$ and setting $r = 2$, we have:

$$B \approx \frac{P_{dB}}{6} + \frac{1}{2}\log_2\left[\frac{(M-1)\left(M^2-1\right)\pi^2}{12}\right]. \quad (27)$$

V. SIMULATION RESULTS AND DISCUSSION

In this section, numerical results and discussions are presented to validate the CUR-decomposition technique. The proposed method is evaluated in terms of the CR and mismatch error with reference to the original correlation matrix. The number of feedback bits B and the dimension of the original correlation matrix is also analyzed. Additionally, we compare the technique with SVD, ZF, and MF methods in terms of sum rate capacity.

Fig. 3 shows the effect of CR as sampling matrix dimension varies with original correlation matrix dimension $M = 75$ and $M = 100$. It is observed that both the CR and improved CR decrease with increasing value of c, where c is defined as in section III-A. Moreover, the CR has a significant enhancement when the value of c is greater than 8. Explicitly, the storage cost and computation complexity can be minimized as c is a small value. On the other hand, more compressed data distort the meaning of original data.

Fig. 4 shows the mismatch error $|\mathbf{A} - \mathbf{CUR}| = \left|\mathbf{A} - \tilde{\mathbf{A}}\right|$ as a function of CR with 5, 20 and 35 iterations. It has shown that the mismatch error increases as CR increases. However, in the CR ranges $2 - 10$ dB, the mismatch errors for different iteration numbers are nearly the same, while in the ranges of $10 - 25$ dB, the mismatch errors have some variations.

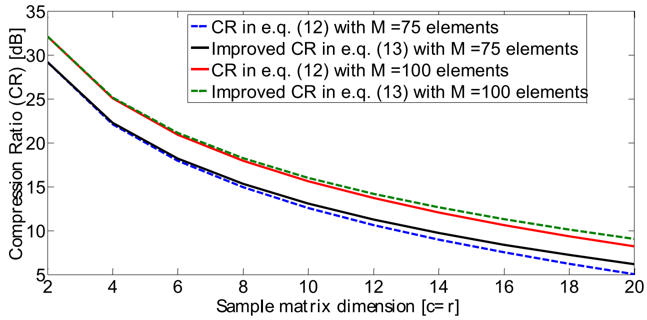

FIGURE 3. Compression ratio & improved compression ratio for the proposed method with sampling matrix dimension ($c \geq 2$) regarding the original matrix dimension.

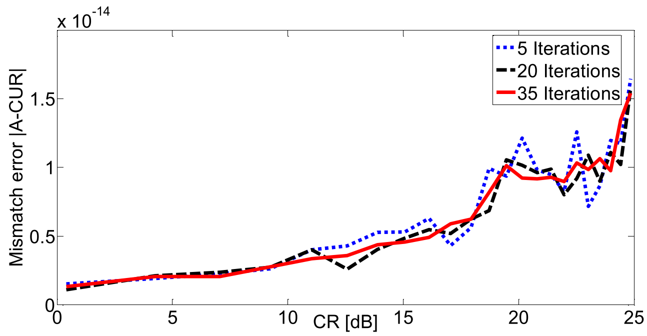

FIGURE 4. Mismatch error (the error between the proposed CUR decomposition matrix and the original correlation matrix) with respect to compression ratio regarding the number of iterations.

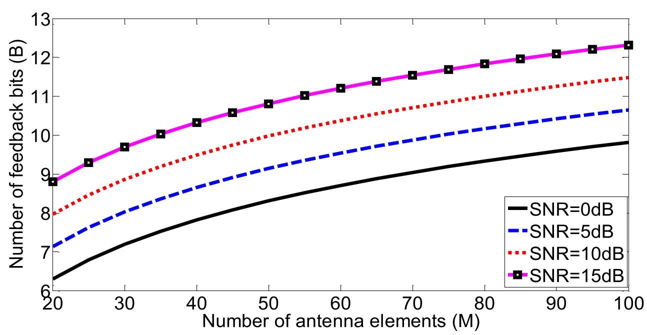

FIGURE 5. The required number of feedback bits per user vs. number of antenna elements regarding the SNR.

Meanwhile, the number of iterations does not have any effect on minimizing the mismatch error.

Fig. 5 shows the required number of feedback bits B to maintain the SNR loss of 3dB with respect to the number of antenna elements M. The required number of feedback bits B increases with M. We assume that the number of users is large and equal to the number of antenna elements ($M = K$). It is observed that with a small number of antenna elements, the required number of feedback bits per user is smaller. Additionally, at a higher level of SNR, it requires more feedback bits, because the gap loss is higher with increasing of SNR.

Fig. 6 shows the ergodic sum-rate capacities by different methods. We set the SNRs in the ranges of $-10 \sim 20$ dB

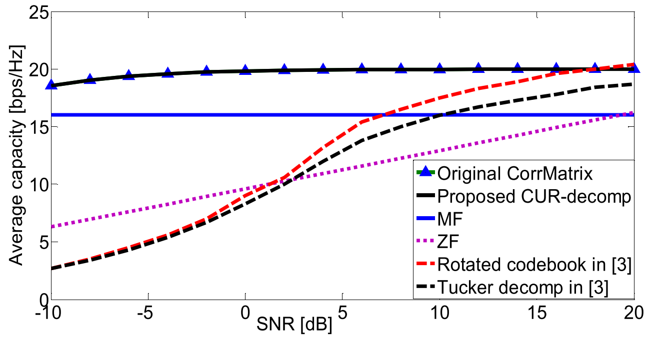

FIGURE 6. Ergodic sum rate capacities for the proposed method and other techniques with respect to the SNR.

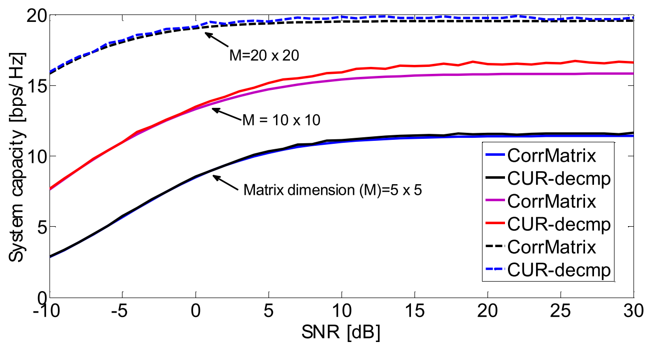

FIGURE 7. Ergodic sum-rate capacity of proposed and original correlation matrix vs. SNR with respect to different number of antenna elements (original matrix dimension).

and fixed number of users of 32. Moreover, the sum rate capacity achieved by MF method is not affected by SNR, it is constant with 16 bps/Hz. However, ZF has the sum rate capacity increase with increased SNRs. Therefore, the proposed method achieves much higher sum rate capacity than the conventional methods.

Particularly, the superiority is more pronounced at low levels of SNRs.

Fig. 7 shows the system capacities versus the SNRs for a different number of antenna elements. In fact, as the number of antenna elements increases, the system capacity also increases. Moreover, it can be seen that the CUR- decomposition method achieves almost the same performance as the original correlation matrix.

VI. CONCLUSION

Nowadays, reducing the high dimension of channel correlation matrix is a hot topic in the application of a vast number of antennas in 5G mm-Wave massive antenna system at the BS. In this paper, we proposed a novel method to reduce the matrix dimension issue based on matrix decomposition technique. A model for rotation codebook design is formulated based on CUR-decomposition to generate the new rotation matrix from the original correlation matrix. The proposed CUR-decomposition algorithm is evaluated in terms of CR, mismatch error and the required number of feedback bits. Furthermore, the proposed method is compared with the conventional ones (SVD, ZF, and MF) in terms of ergodic sum-rate capacity.

The simulation results show that the CUR-decomposition method has achieved a higher CR at higher original correlation matrix dimension and lower sample matrix dimension. At the given SNR, as the number of antenna elements increases, the required feedback bits increase. Comparatively, this method achieves higher performance than the conventional methods in terms of average system capacity. In this work, the dimensionality problem is significantly reduced, hence can be applied in 5G massive antenna multi-user systems deployed with over hundred antenna elements.

REFERENCES

[1] J. H. Yoo, J. Bae, S. H. Lim, S. Kim, J. W. Choi, and B. Shim, "Sampling-based tracking of time-varying channels for millimeter wave-band communications," in *Proc. IEEE Int. Conf. Commun.*, May 2017, pp. 1–6.

[2] T. S. Rappaport *et al.*, "Millimeter wave mobile communications for 5G cellular: It will work!" *IEEE Access*, vol. 1, pp. 335–349, 2013.

[3] F. Yuan, "Tucker decomposition for rotated codebook in 3-D MIMO system under spatially correlated channel," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 7813–7818, Sep. 2016.

[4] W. Feng, Z. Xiao, D. Jin, and L. Zeng, "Circular-antenna-array-based codebook design and training method for 60 GHz beamforming," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Apr. 2013, pp. 4140–4145.

[5] J. Wang *et al.*, "Beamforming codebook design and performance evaluation for 60GHz wideband WPANs," in *Proc. IEEE 70th Veh. Technol. Conf.*, Sep. 2009, pp. 1–6.

[6] Y. Niu, C. Gao, Y. Li, L. Su, D. Jin, and A. V. Vasilakos, "Exploiting device-to-device communications in joint scheduling of access and backhaul for mmWave small cells," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 10, pp. 2052–2069, Oct. 2015.

[7] J. Zhang, X. Xue, E. Björnson, B. Ai, and S. Jin, "Spectral efficiency of multipair massive MIMO two-way relaying with hardware impairments," *IEEE Wireless Commun. Lett.*, to be published, doi: 10.1109/LWC.2017.2750162.

[8] W.-X. Zou, G.-L. Du, B. Li, and Z. Zhou, "Step-wisely refinement based beam searching scheme for 60 GHz communications," *Wireless Pers. Commun.*, vol. 71, no. 4, pp. 2993–3010, 2013.

[9] Z. Pi and F. Khan, "An introduction to millimeter-wave mobile broadband systems," *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 101–107, Jun. 2011.

[10] S. Kutty and D. Sen, "Beamforming for millimeter wave communications: An inclusive survey," *IEEE Commun. Surveys Tuts.*, vol. 18, no. 2, pp. 949–973, 2nd Quart., 2016.

[11] A. Adhikary, J. Nam, J.-Y. Ahn, and G. Caire, "Joint spatial division and multiplexing—The large-scale array regime," *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6441–6463, Oct. 2013.

[12] J. Nam, A. Adhikary, J.-Y. Ahn, and G. Caire, "Joint spatial division and multiplexing: Opportunistic beamforming, user grouping and simplified downlink scheduling," *IEEE J. Sel Signal Processing.*, vol. 8, no. 5, pp. 876–890, Oct. 2014.

[13] P.-H. Kuo, H. T. Kung, and P.-A. Ting, "Compressive sensing based channel feedback protocols for spatially-correlated massive antenna arrays," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Apr. 2012, pp. 492–497.

[14] J.-Y. Ko and Y.-H. Lee, "Adaptive beamforming with dimension reduction in spatially correlated MISO channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 4998–5002, Oct. 2009.

[15] B. Lee, L. Ngo, and B. Shim, "Antenna group selection based user scheduling for massive MIMO systems," in *Proc. IEEE Global Conf. Signal Process. (GLOBECOM)*, Dec. 2014, pp. 3302–3307.

[16] X. Li, S. Jin, H. A. Suraweera, J. Hou, and X. Gao, "Statistical 3-D beamforming for large-scale MIMO downlink systems over rician fading channels," *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1529–1543, Apr. 2016.

[17] J. Zhang, L. Dai, Z. He, S. Jin, and X. Li, "Performance analysis of mixed-ADC massive MIMO systems over rician fading channels," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1327–1338, Jun. 2017.

[18] D. Yang, L.-L. Yang, and L. Hanzo, "DFT-based beamforming weight-vector codebook design for spatially correlated channels in the unitary precoding aided multiuser downlink," in *Proc. IEEE Int. Conf. Commun.*, May 2010, pp. 1–5.
[19] F. Yuan, S. Han, C. Yang, Y. Zhang, G. Wang, and M. Lei, "Weighted DFT codebook for multiuser MIMO in spatially correlated channels," in *Proc. IEEE Veh. Technol. Conf.*, May 2011, pp. 1–5.
[20] D. J. Love and R. W. Heath, "Limited feedback diversity techniques for correlated channels," *IEEE Trans. Veh. Technol.*, vol. 55, no. 2, pp. 718–722, Mar. 2006.
[21] M. A. Liebert and T. E. T. Al, "Dimensionality reduction," *J. Comput. Biol.*, vol. 10, pp. 617–634, 2003.
[22] H. Ye, Y. Li, and Z. Zhang, "A simple approach to optimal CUR decomposition," *Comput. Sci.*, vol. 42, pp. 1–17, Nov. 2015.
[23] N. Ravindran, N. Jindal, and H. C. Huang, "Beamforming with finite rate feedback for LOS MIMO downlink channels," in *Proc. IEEE Global Telecommun. Conf. (GLOBECOM)*, Nov. 2007, pp. 4200–4204.
[24] W. Shen, L. Dai, Y. Zhang, J. Li, and Z. Wang, "On the performance of channel-statistics-based codebook for massive MIMO channel feedback," *IEEE Trans. Veh. Technol.*, vol. 66, no. 8, pp. 7553–7557, Aug. 2017.

XIONGWEN ZHAO (SM'06) received the Ph.D. degree (Hons.) from the Helsinki University of Technology (TKK), Finland, in 2002. From 1992 to 1998, he was with the Laboratory of Communications System Engineering, China Research Institute of Radiowave Propagation, where he was the Director and a Senior Engineer. From 1999 to 2004, he was with the Radio Laboratory, TKK, as a Senior Researcher and the Project Manager in the areas of MIMO channel modeling and measurements at 2, 5, and 60 GHz as well as UWB. From 2004 to 2011, he was with Elektrobit Corporation (EB), Espoo, Finland, as a Senior Specialist at EB Wireless Solutions. From 2004 to 2007, he was with European WINNER project as a Senior Researcher in MIMO channel modeling for 4G radio systems. From 2006 to 2008, he was also involved in the field of wireless network technologies, such as WiMAX and wireless mesh networks. From 2008 to 2009, he was with mobile satellite communications for GMR-1 3G, DVB-SH RF link budget, and antenna performance evaluations. From 2010 to 2011, he was with the spectrum sharing and interference management between satellite and terrestrial LTE networks. He is currently a Professor in wireless communications at North China Electric Power University, Beijing, and chairs several projects by the National Science Foundation of China and the State Key Laboratories and Industries on channel measurements, modeling, and simulations. He has served as the TPC members, session chairs, and a keynote speaker at numerous international and national conferences. He is a reviewer for the IEEE transactions, journals, letters, and conferences. He was a recipient of the IEEE Vehicular Technology Society Neal Shepherd Memorial Best Propagation Paper Award in 2014.

ADAM MOHAMED AHMED ABDO was born in North Darfur, Sudan, in 1983. He received the bachelor's and master's degrees from the School of Electronic Engineering, Sudan University of Science and Technology in 2007 and 2014, respectively. He is currently pursuing the Ph.D. degree with North China Electric Power University. From 2014 to 2015, he was a Senior Lecturer with the Faculty of Engineering Science, Electrical and Electronic Department, University of Nyala, Sudan. His main field of interest is 5G in millimeter wave communications.

CHEN XU (S'12–M'15) received the B.S. degree from the Beijing University of Posts and Telecommunications in 2010, and the Ph.D. degree from Peking University, Beijing, in 2015. She is currently a Lecturer with the School of Electrical and Electronic Engineering, North China Electric Power University. Her research interests mainly include wireless resource allocation and management, game theory, optimization theory, heterogeneous networks, and smart grid communication. She served as a TPC member for the IEEE Globecom, the IEEE ICC, and the IEEE ICCC. She received the Best Paper Award at the 2012 International Conference on Wireless Communications and Signal Processing, and the IEEE Leonard G. Abraham Prize in 2016.

SUIYAN GENG received the M.Sc. (tech.) and Ph.D. degrees from the Helsinki University of Technology (TKK), Espoo, Finland, in 2003 and 2011, respectively. From 1992 to 1998, she was a Research Engineer with the China Research Institute of Radiowave Propagation, Xinxiang, China. From 2001 to 2011, she was a Research Engineer with the Radio Laboratory (Department of Radio Science and Engineering since the beginning of 2008), TKK. She is currently an Associate Professor at North China Electric Power University. Her research topics include millimeter-wave and ultra-wideband radio wave propagation and stochastic channel modeling for future-generation radio systems and technologies. She was a recipient of the IEEE Vehicular Technology Society Neal Shepherd Memorial Best Propagation Paper Award in 2014.

JIANHUA ZHANG (SM'12) received the Ph.D. degree in circuit and system from the Beijing University of Posts and Telecommunication (BUPT) in 2003. She is currently the Drafting Group Chairwoman of the ITU-R IMT-2020 channel model. She is a Professor with BUPT. She has authored over 100 articles in referred journals and conferences and 40 patents. Her current research interests include 5G, artificial intelligence, data mining, especially in massive MIMO and millimeter wave channel modeling, channel emulator, OTA test, and so on. She was awarded the 2008 Best Paper of the *Journal of Communication and Network*. In 2007 and 2013, she received two national novelty awards for her contribution to the research and development of Beyond 3G TDD demo system with 100Mbps@20MHz and 1Gbps@100MHz, respectively. In 2009, she received the Second Prize for Science Novelty from Chinese Communication Standards Association for her contributions to ITU-R 4G (ITU-R M.2135) and 3GPP Relay channel model (3GPP 36.814). From 2012 to 2014, she did the 3-D channel modeling work and contributed to 3GPP 36.873 and is also a member of 3GPP 5G channel model for bands up to 100 GHz.

IMRAN MEMON received the B.S. degree in electronics from the IICT University of Sindh Jamshoro, Sindh, Pakistan, in 2008, and the M.E. degree in computer engineering from the University of Electronic Science and Technology, Chengdu, China. He is currently pursuing the Ph.D. degree with the College of Computer Science and Technology, Zhejiang University. His current research interests include artificial intelligence system, network security, embedded system, information security, peer-to-peer networks, location-based services, and road network. He was a recipient of the Academic Achievement Award 2011–2012 and the Excellent Performance Award 2011–2012 from UESTC, China. He serves as an Editor-in-Chief for the *Journal of Network Computing and Applications*. He serves as an Editorial Board Member of the *Journal of Web Systems and Applications*. He has authored over 30 research papers in recent years. He serves as an organizing committee chair and TPC member more than 250 international conferences, as well as a reviewer for over 50 international research journals, including the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS FOR VIDEO TECHNOLOGY, the IEEE TRANSACTIONS ON IMAGE PROCESSING, the IEEE TRANSACTIONS ON SIGNAL PROCESSING, the IEEE TRANSACTIONS ON MULTIMEDIA, the IEEE MULTIMEDIA, the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE SIGNAL PROCESSING LETTERS, the *Journal of Electronic Imaging*, *Information Sciences Computer Vision and Image Understanding*, *Image and Vision Computing*, *EURASIP Journal on Advances in Signal Processing*, *Computer Standards & Interfaces*, *Circuits Systems and Signal Processing*, the *Journal of Information Science and Engineering*, *International Journal of Computers and Applications*, *Far East Journal of Experimental and Theoretical Artificial Intelligence*, *IEE Proceedings Vision, Image and Signal Processing*, *EURASIP Journal on Advances in Signal Processing*, IEE Proceedings - Information Security, the *Journal of Circuits, Systems and Signal Processing*, the *International Journal of Computers and Applications*, *LNCS Transactions on Data Hiding and Multimedia Security*, the *Signal Processing*, the *International Journal of Pattern Recognition and Artificial Intelligence*, the IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY, the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, *Transactions on Internet and Information Systems*, *Wireless Personal Communication*, *Computers & Electrical Engineering*, *Computer Networks*, *Wireless Networks*, *Telecommunication Systems*, and others.