Clustering in Wireless Propagation Channel with a Statistics-based Framework

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***Abstract*—In this article, we introduce a statistics-based clustering framework that is able to model the clustering problem corresponding to the channel propagation characteristics and evaluate the clustering results effectively. In the framework a Gaussian mixture model (GMM) is employed to model the channel multipaths. Then, we optimize the GMM parameters with the expectation-maximization (EM) algorithm. To evaluate the clustering results effectively, a compact index (CI) is devised, in which both the mean and variance of the clusters are considered. In the simulation, outdoor-to-indoor (O2I) channel measurement data is presented to demonstrate the effectiveness of the proposed framework.**

I. Introduction

With the increasing number of antennas and the application scenarios in the fifth generation (5G) mobile communication, the complexity of channel model increases rapidly. The geometry-based stochastic model (GBSM) [1] which is cluster-based is popularly used in the 5G. In the GBSM, a cluster indicates a group of multipath components (MPCs) with similar parameters. In the system evaluation aspect, it is convenient to model the propagation characteristics in terms of cluster rather than model the behavior of individual MPCs. The clustering can also help us analyze the channel propagation characteristics more accurately and intuitively. The clustering has a weighty impact on channel capacity [2]. What's more, in [3] a channel model incorporating the clustering with the artificial intelligence (AI) is proposed. Therefore an effective clustering algorithm corresponding to the MPCs propagation characteristics is necessary.

Numerous algorithms have been proposed to implement the channel multipath clustering, such as the clustering algorithm in the visual aspect [4], which has discovered that the deviation between the cluster angle spread (AS) and average tap AS becomes narrow with the decrease of channel bandwidth. Subsequently, many automatic clustering algorithms [5] are proposed, such as the clustering characteristics in [6] where a novel initialization is proposed, in [7] the elevation angle domain is considered for the clustering in the 3D MIMO channels, in [8] a modified definition of the multiple component distance (MCD) is proposed. For the clustering algorithms mentioned above, the Kmeans rule is employed to find the possible clustering and then the Calinski-Harabasz (CH) evaluation index [6] is applied to evaluate the clustering result. The task of Kmeans algorithm is to find the possible clustering result using the distances between the datapoints, then with the help of evaluation index the best result is pointed out. While, the Kmeans clustering method and its evaluation index are all distance-based. However, only the distance can not catch the propagation properties of the channel multipaths. Thus, the Kmeans framework can not accomplish the channel clustering reasonably.

In order to carry out the channel multipath clustering with more statistical characteristics, the Gaussian mixture model (GMM) [9] is firstly applied to the channel clustering in the context. The GMM supposes all the multipaths are generated from the mixture distributions, where the multipath belongs to a certain clustering in a strict probability meaning [10]. By taking linear integration of K Gaussian components, it can approximate to arbitrary continuous function by using an adequate number of Gaussian distributions and adjusting the means and covariances as well as their coefficients. The expectation-maximization (EM) algorithm [11], utilized to find the GMM parameters, is a preferable choice in the statistical signal processing. It iterates between calculating the log-likelihood expectation (E-step) and maximizing the log-likelihood (M-step).

On the other side, the validity criteria in the Kmeans community are mainly based on the distance, which is lack of sufficient statistical characteristics to evaluate the clustering results. Besides, the scattering property of the channel multipaths obeys Gaussian distribution which accords with the GMM clustering mechanism. Thus, the selected clusters under the distance-based criterions may not reflect the propagation properties of the channel multipaths. Based on the above analysis, a compact index (CI) which evaluates the clustering results based on the means and variances is proposed. As the propagation characteristics of the multipath parameters obey Gaussian distribution, which expects the clusters we get with relative small mean to variance ratio. Moreover, considering sufficient statistics characteristics, the CI can uncover the inherent information of the multipath parameters and provide appropriate explanation to the clustering result. Thus, more reasonable clustering results can be selected under

the CI criterion. At the moment, we combine the multipath scattering property, the GMM clustering mechanism and the CI evaluation index together. The CI can reveal the propagation characteristics of the channel multipaths and provide appropriate explanation of the clustering result. Therefore, the CI can select more reasonable clustering results.

The above work can be categorized into a statistics-based clustering framework consisting of three sections: (i) a new clustering model, the GMM, in the channel, (ii) an optimization method that can optimize the parameters of the GMM, (iii) a validation index that can select reasonable clusters. Detailed contributions of our work are listed as follows:

- In this article, it is the first time that the GMM has been employed to the channel multipath clustering to our knowledge.
- Taken the mean and variance of the dataset into consideration, the CI index is proposed which can evaluate the clustering results more reasonably.
- Benefiting from the combination of the GMM clustering mechanism, the multipath propagation properties and the CI evaluation index, a preferable clustering performance is expected. That is, the CI index is conformed to the GMM clustering mechanism and the preferable clustering results under the CI can reflect the multipath propagation property more effectively.

This paper is organized as follows. In section II, the clustering problem and the distance-based clustering framework is described. In section III, the statistics-based clustering framework is presented. Outdoor-to-indoor (O2I) channel measurement data are used to highlight the statistics-based clustering framework in section IV. Finally, the paper is concluded in section V.

Notation: $(\cdot)^T$ denotes the transpose of $(\cdot)$. $|\cdot|$ denotes the Euclidean norm. The upper-right corner marked with bracket represents the ith assessment of the variable. In this paper, when we write $p(\cdot;\theta)$ we deem it as the likelihood function of θ and we imply the θ are parameters. In contrary, we imply the θ are random variables when we write $p(\cdot|\theta)$.

II. Clustering Problem And the Distance-based Clustering Framework

A. *Clustering Problem*

Channel multipath parameters are extracted by the space-alternating generalized expectation-maximization (SAGE) [12] parameter estimation algorithm based on the O2I channel measurement data. In the channel community, a cluster is defined as a group of channel multipaths with similar parameters [5], including the delay (τ), azimuth angle of arrival (AOA), azimuth angle of departure (AOD), elevation angle of arrival (EOA), elevation angle of departure (EOD) [13] and so on. Fig. 1 shows the cluster phenomenon of the multipaths in the propagation. The left side is the base station (BS), the right side is the mobile station (MS). Each circle with several dots represents one scattering region causing one group of propagation multipaths with similar properties, called as cluster [14].

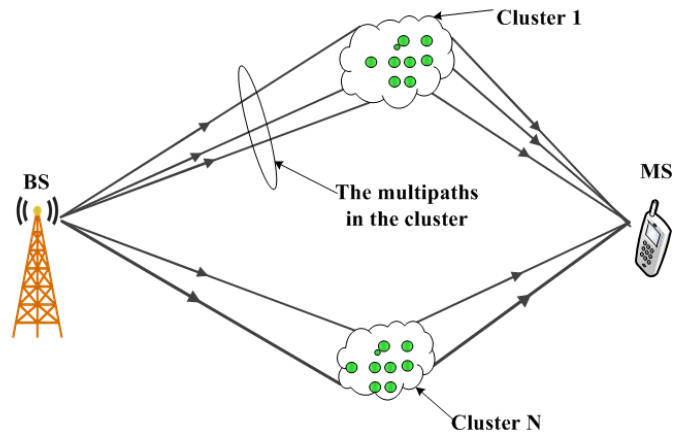


Fig. 1: The channel model between the BS and MS.

The channel of the non-line of sight (NLOS) for the nth cluster can be modeled as

$$H^{3d}_{n_r,n_t,l} = \sum_{m=1}^{M_l} F^{3d}_{Rx}(\theta_{Rx,l},\phi_{Rx,l}) A^{3d}_{l,m} F^{3d}_{Tx}(\theta_{Tx,l}a,\phi_{Tx,l}) e^{j2\pi f_{d,l,m}t}, \quad (1)$$

where $l \in \{1,2,...,L\}$, L is the number of the clusters, M_l is the number of multipaths in the lth cluster, Rx and Tx denote the receiving and transmitting ends respectively, F^{3d}_{Rx} and F^{3d}_{Tx} are respectively the receiving and the transmit antenna gain, $A^{3d}_{l,m}$ is the gain of the phase, $m = 1,2,...,M_l$ is the multipath index within the cluster, n_r and n_t represent respectively the receiving and transmitting antenna, $f_{d,l,m}$ is the Doppler frequency, and $H^{3d}_{n_r,n_t,l}$ is the sum of M_l channel multipaths within the lth cluster.

B. *The Kmeans-based Clustering Algorithm*

Traditionally, we use the Kmeans algorithm to find the possible clusters. The Kmeans finds K cluster centroids, and then it iteratively groups the multipaths so that the distance sum of the respective multipath is minimized over all clusters. The MCD, which denotes the similarity of the multipaths in the delay and angular aspects, is usually adopted in the Kmeans. The total MCD between the ith ($i = 1,2,...,N$) and the jth ($j = 1,2,...,N$) multipath is given by

$$MCD_{ij} = \sqrt{\|MCD_{Rx,ij}\|^2 + \|MCD_{Tx,ij}\|^2 + MCD_{\tau,ij}{}^2}, \quad (2)$$

where

$$MCD_{Tx/Rx,ij} = \frac{1}{2}\left|\begin{pmatrix}\sin(\theta_i)\sin(\varphi_i)\\ \sin(\theta_i)\cos(\varphi_i)\\ \cos(\varphi_i)\end{pmatrix} - \begin{pmatrix}\sin(\theta_j)\sin(\varphi_j)\\ \sin(\theta_j)\cos(\varphi_j)\\ \cos(\varphi_j)\end{pmatrix}\right|, \quad (3)$$

$$MCD_{\tau,ij} = \frac{|\tau_i - \tau_j|}{\Delta\tau_{\max}} \cdot \frac{\tau_{std}}{\Delta\tau_{\max}}, \quad (4)$$

where $\Delta\tau_{max}$ is the maximum difference of the delay, τ_{std} is the standard deviation of the delay, θ_i and ϕ_i are the azimuth and elevation angle respectively. Apart from the clustering mechanism of Kmeans, it evaluates the clustering results with the MCD which is detailedly described in section III-C. We can category the Kmeans and its evaluation index into the distance-based framework. Whereas, only the MCD can not reflect the similarity of the channel multipaths effectively. Therefore, a new clustering framework is introduced in next section.

III. The Statistics-based Clustering Framework

A. The Gaussian Mixture Model

To implement the clustering with the mean and covariance structure of channel multipaths, the GMM [9] is applied to the channel multipath clustering. In the GMM, the channel multipaths are described by a set of d-dimensional vector $\mathbf{X} = \{\mathbf{x}_i; i = 1, 2, ..., N\}$, where $\mathbf{x}_i \in R^d$ characterizes the ith multipath parameters of the channel. In the GMM, each $\mathbf{x}_i$ is assumed to be generated from one of the K Gaussian distributions, each of which models the multipaths as follows.

$$p(\mathbf{x}_i \,|\boldsymbol{\theta}_k\,) = \frac{1}{(2\pi)^{\frac{d}{2}}\,|\boldsymbol{\Sigma}_k|}\exp\left(\frac{(\mathbf{x}_i-\boldsymbol{\mu}_k)^T\Sigma_k^{-1}(\mathbf{x}_i-\boldsymbol{\mu}_k)}{-2}\right), \tag{5}$$

where each component is denoted by the parameters, $\boldsymbol{\theta}_k = \{\,\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k; k = 1, 1, ..., K\}$, $\boldsymbol{\mu}_k$ is the mean, and $\boldsymbol{\Sigma}_k$ is the covariance matrix. In the unsupervised learning, the parameters of the mixture model have to be extracted from the data. If one Gaussian component is assumed, then the estimation of parameters reduces to a maximum likelihood (ML) estimation.

Given a set of N channel multipath samples $\mathbf{X}$, the log-likelihood of the K-component mixture model is

$$L(\boldsymbol{X};\boldsymbol{\Theta}) = \sum_{i=1}^{N}\log\sum_{k=1}^{K}p(\boldsymbol{x}_i\,|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k\,)\pi_k, \tag{6}$$

where $\Theta = \{\pi_k, \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k\}_{k=1}^{K}$ is the assembly of all parameters involved in the GMM and the mixing coefficient $\pi_k \in [0, 1]$ denotes the prior probability of vectors drawn from class k and satisfies the constraint $\sum\limits_{k=1}^{K}\pi_k = 1$. The ML estimation of the GMM parameters which best models the data

$$\hat{\boldsymbol{\Theta}}_{ML} = \arg\max_{\Theta}\{L(\mathbf{X};\boldsymbol{\Theta})\}, \tag{7}$$

cannot be determined directed since we do not know which of the component has produced x_i. This issue can be solved by the EM algorithm using an iterative scheme [15].

B. Training the GMM with the EM

To estimate the GMM parameters, the maximum log-likelihood function is solved by the EM [11]. The maximum log-likelihood estimation can be obtained among posterior probability formula (6) and parameters updating formula (8)-(11).

$$\boldsymbol{\omega}_k^{(i)} = \frac{p(\boldsymbol{x}^{(i)}|z^{(i)} = k; \boldsymbol{\mu}, \boldsymbol{\Sigma})p(z^{(i)} = k; \pi)}{\sum\limits_{l=1}^{K}p(\mathbf{x}^{(i)}|z^{(i)} = l; \boldsymbol{\mu}, \boldsymbol{\Sigma})p(z^{(i)} = l; \pi)}, \tag{8}$$

$$\pi_k = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{\omega}_k^{(i)}, \tag{9}$$

$$\boldsymbol{\mu}_k = \frac{\sum\limits_{i=1}^{N}\boldsymbol{\omega}_k^{(i)}\mathbf{x}^{(i)}}{\sum\limits_{i=1}^{N}\boldsymbol{\omega}_k^{(i)}}, \tag{10}$$

$$\boldsymbol{\Sigma}_k = \frac{\sum\limits_{i=1}^{N}\boldsymbol{\omega}_k^{(i)}(\mathbf{x}^{(i)}-\boldsymbol{\mu}_k)(\mathbf{x}^{(i)}-\boldsymbol{\mu}_k)^T}{\sum\limits_{i=1}^{N}\boldsymbol{\omega}_k^{(i)}}. \tag{11}$$

The calculation of $\boldsymbol{\omega}_k^{(i)}$ is usually referred to as the E-step which guesses the values of the hidden variable $z^{(i)}$'s. Exactly known the $z^{(i)}$'s, we can update the parameters of our model in the M-step. The signal processing procedure is shown in Table I.

TABLE I: The operation procedure of the EM-GMM clustering.

Input data: The maximum iteration number $r_{\max}$ and the channel multipath matrix $\mathbf{X}$,
Initiation: The number of multipaths N, the dimensionality of feature vectors d, and the mixtures K used to generate data.
Loop body: 1. E-step: calculating the posterior probability $\boldsymbol{\omega}_k^{(i)}$ as formula (8).
2. M-step: Re-estimating the parameters using samples weighted by the posterior probabilities as formula (9)-(11).
3. If the maximum iteration number $r_{\max}$ or the termination condition is reached, jumped out, if not $r = r + 1$.
Output: The parameter sets of each GMM component.

C. Validity index of the clustering

Traditional clustering algorithm (Kmeans) assigns a data-point to a certain cluster according to the distance. Therefore, the validity index in the Kmeans are all distance-based such as the CH index, the Davies Bouldin (DB) index, Jaccard Coefficient (JC), Fowlkes and Mallows (FMI) index [16] and so on. Generally, these validity indexes will reach a uniform conclusion on evaluating the solution. Here we only analyze the CH index for example. The CH is defined as

$$CH(K) = \frac{tr(\mathbf{B})/(K-1)}{tr(\mathbf{W})/(L-K)}. \tag{12}$$

By means of the MCD formula (2), $tr(\mathbf{B})$ and $tr(\mathbf{W})$ are given as

$$tr(\mathbf{B}) = \sum_{k=1}^{K} L_k \cdot MCD(\mathbf{c}_k, \bar{\mathbf{c}})^2, \tag{13}$$

$$tr(\mathbf{W}) = \sum_{k=1}^{K} \sum_{j \in C_k} MCD(\mathbf{x}_j, \mathbf{c}_k)^2, \tag{14}$$

where L_k is the number of multipaths corresponding to the kth cluster and

$$\bar{\mathbf{c}} = \frac{\sum_{l=1}^{N} (P_l \cdot \mathbf{x}_l)}{\sum_{l=1}^{N} P_l}. \tag{15}$$

However, as the multipath propagation characteristics obey Gaussian distribution statistically, only the distance may not reflect the similarity of the channel multipaths effectively. In general, we expect the clusters with large mean and small variance [17]. Considering those, we put forward the CI which can evaluate the clustering result corresponding to the multipath propagation property.

$$CI = \frac{tr(\mathbf{B})/(K-1)}{tr(\mathbf{W})/(L-K)} \cdot \frac{1}{\sum_{k=1}^{K} \mathbf{V}_k^2}, \tag{16}$$

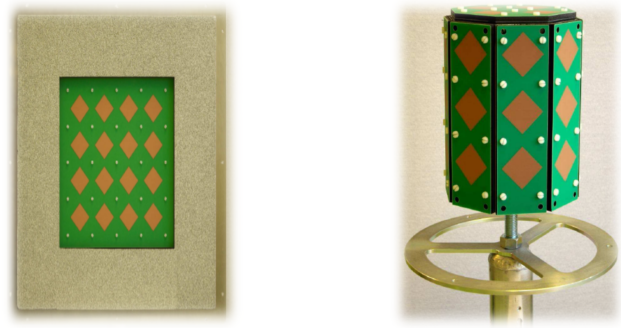
where $\mathbf{V}_k^2$ is the variance of the kth cluster. We can simply find that the former part is the CH. Both the means and variances of the clusters are considered in the CI. Considering sufficient statistics characteristics, CI can uncover the inherent information of the multipath parameters and provide appropriate explanation to the clustering result. At the moment, we can make a combination among the CI evaluation index, the multipath propagation property and the GMM clustering mechanism. When a multipath has the same distance with two cluster centroids by chance, it will cause confusion under CH. While, the CI index will choose the cluster within which a small variance obtains. Besides, the cluster result is expected to be as compact as possible, which reflects comprehensively with both mean and variance.

IV. Validation Results

To compare the clustering performance of GMM and K-means, O2I channel measurement data is used to illuminate the GMM over Kmeans.

TABLE II: Sounder parameters.

Parameter	Value
Carrier frequency [GHz]	3.5
Bandwidth [MHz]	50
Transmit power [dBm]	37
Chip frequency [MHz]	127
Code length [ns]	40
Cycle duration [ms]	9.28
Channel sampling rate [Hz]	26.983



(a)The TX antenna: UPA. (b)The RX antenna: ODA.

Fig. 2: The antenna layout in the measurement.

A. *Measurement Scenario*

In the measurement, the Elektrobit Prosound Sounder is used to detect the channel information [18]. The basic parameters are illustrated in Table II.

As is shown in Fig. 2, a dual-polarized uniform planar array (UPA) with 32 elements is employed at the transmitting side (Tx). At the receiving side (Rx), the dual-polarized omnidirectional antenna (ODA) with 56 elements is used. The layout of the antenna arrays at Tx and Rx side is illustrated in Fig. 2(a) and Fig. 2(b) respectively. The antenna's parameters are shown in Table III.

TABLE III: Antenna parameters.

	UPA	ODA
Number	32	56
Polarization	dual	dual
Space	0.5 wavelength	0.5 wavelength
Azimuth	$-70^\circ \sim 70^\circ$	$-180^\circ \sim 180^\circ$
Elevation	$-70^\circ \sim 70^\circ$	$-55^\circ \sim 90^\circ$

As is presented in Fig. 3, the measurement is conducted in the Beijing University of Posts and Telecommunications (BUPT). It is a O2I scenario where the Tx is fixed on a lower building covered with plasterboard on the surface. On the Rx side as is shown in Fig. 3(b), the antenna array is fixed on a trolley about 1.8m height.

B. *Parameter Settings*

After necessary signal processing using the SAGE, we get 74 multipaths and their corresponding parameters as

(a)The TX scenario at outdoor.

(b)The RX scenario at indoor.

Fig. 3: The measurement scenario.

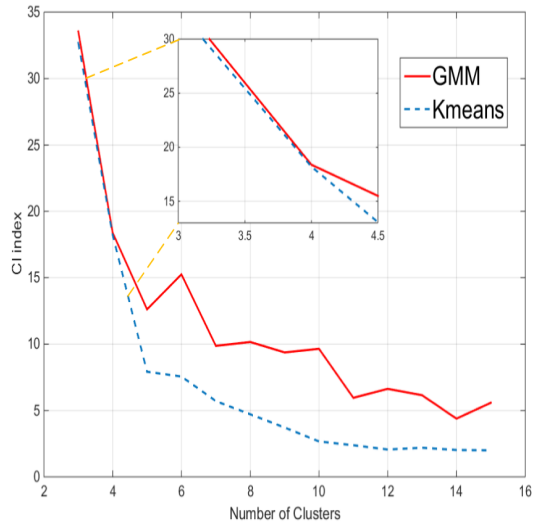
the input of the clustering. In the Kmeans-based and EM-GMM clustering, the data dimension is set to 5, that is $[\tau, \phi_{Rx}, \phi_{Tx}, \theta_{Rx}, \theta_{Tx}]$. As we mainly concern about the propagation characteristics of the multipahts, the power of the multipaths is set to unit value. In the EM algorithm, the input data is arranged in column as $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N)$, where $\mathbf{x}_j = (x_{j,1}, ..., x_{j,M})^{\mathrm{T}}$ indicates the data vector of the jth $(j = 1, 2, ..., N)$ multipath. The diagonal covariance matrix is designed for the Gaussian distributions. To avoid getting into local optimization, 30 Monte Carlo simulations are run.

C. *Clustering Comparison between EM-GMM and Kmeans*

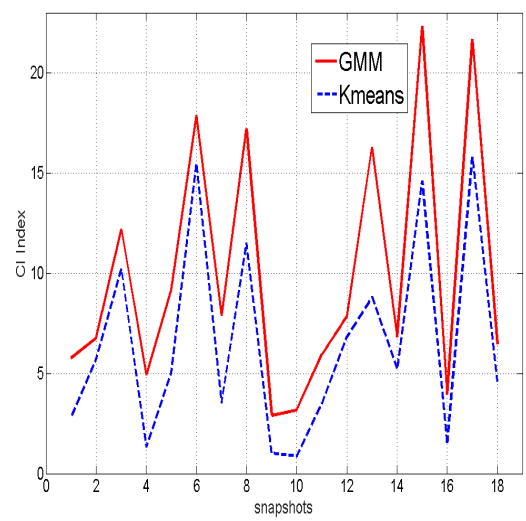
*1) **Clustering Comparison under the CI Index**:* In this section, we will compare the CI values of the EM-GMM clustering and Kmeans-based clustering. Firstly, we conduct a ergodic experiment at a fixed snapshot from 3 to 16 clusters. Then, we compare 18 different snapshots at the same sampling site. The experiment results are illustrated as Fig. 4.

In the channel clustering area, a high CI value corresponds to a preferable clustering result. The internal small block diagram, in Fig. 4(a), is the enlargement of the cluster from 3 to 4.5. From Fig. 4 we can see that the CI values of the EM-GMM clustering are mostly higher than that of Kmeans both in different clusters of a fixed snapshot and different snapshots at the same sampling site. Conclusion can be drawn that the EM-GMM clustering can get more favorable clusters with large mean to variance ratio. And a large CI corresponds to a compact clustering result which conforms to the scattering mechanism of the channel multipaths. The clustering mechanism of GMM accords with the propagation mechanism of channel multipaths, thus a evaluation index corresponding to the above mechanism can select favorable clustering result.

*2) **Clustering Comparison in the Visual Aspect**:* We choose 3 parameters with largest variance ($[\tau, \phi_{Rx}, \phi_{Tx}]$) for the visualization. Fig. 5 shows the visualization comparison in 6 clusters, where the same cluster is coloured the same.



(a)The CI index with different clusters.



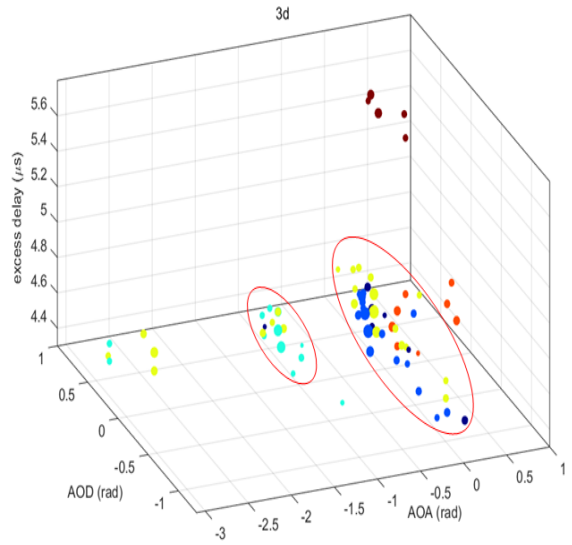
(b)The CI index with different clusters.

Fig. 4: The clustering results of the two algorithms.

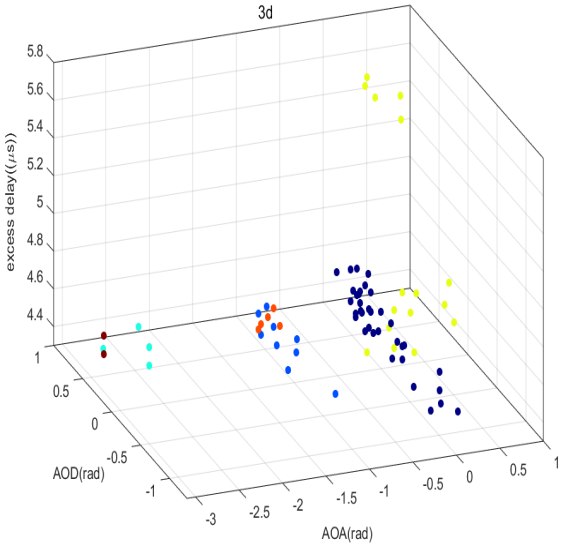
As can be seen from Fig. 5, the Kmeans-based technique gets chaotic clustering result, especially among $[-1, 1]$ in the AOA. The result can not reveal the inner structure characteristics of the channel multipaths clearly. On the contrary, the EM-GMM clustering obtains more clearly as well as compact clusters. From the visual aspects, we can see that the EM-GMM clustering can get more favorable results.

V. Conclusion

In this paper, we illustrated that distance-based clustering framework can not reveal the hidden information effectively or correspond to the propagation characteristics of the channel multipaths reasonably. To overcome the problem mentioned above, a statistics-based clustering framework is employed to model the channel multipaths. Initially, the EM is used



(a)The Kmeans result.



(b)The EM-GMM result.

Fig. 5: The clustering results in the visual aspect.

to optimize the GMM parameters. Considering the mean and variance of the multipaths, the EM-GMM clustering can grasp the propagation properties of the channel multipaths effectively. Furthermore, to select the results corresponding to the clustering mechanism, the CI evaluation index is proposed. Benefiting from the combination of GMM clustering mechanism, multipath propagation properties and the CI evaluation index, we can get a satisfactory clustering result. Validation results illustrate that the EM-GMM clustering can get more reasonable results in visual and quantitative analysis.

Acknowledgment

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