

# A Generalized Algorithm for the Generation of Arbitrary Correlated Nakagami Fading Channels

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**Abstract**—In the last decade, Nakagami fading shows good performance in matching measurement data under many scenarios, i.e., vehicle-to-vehicle (V2V), mobile-to-mobile (M2M) and indoor. However, little research has been done for generating of spatial-temporal correlated MIMO Nakagami fading. In this paper, we propose a generalized and accurate algorithm to generate Nakagami- $m$  envelope channels with arbitrary temporal autocorrelation and spatial cross-correlation. Specifically the temporal autocorrelation of Nakagami channels is derived using rank matching technology, then spatial cross-correlation is introduced by Sim's algorithm. Finally, the proposed algorithm is validated with theoretical data. This algorithm is valuable for communication system simulation and emulation.

## I. INTRODUCTION

Transmission channel is one of the most important part in the mobile communication systems. How to generate an accurate and effective channel model is a bottleneck for the testing of communication systems.

Nakagami channel model has been proven to be best fitting one with measurement data under many scenarios, i.e., vehicle-to-vehicle (V2V), mobile-to-mobile (M2M) and indoor. In this model, parameter  $m$  has to be decided to model different fading channels. For the typical highway traffic, the parameter  $m$  lies between 0.5 to 1, particularly when  $m=1$ , Nakagami fading is identical to Rayleigh fading, while for the open area, the value of  $m$  is in the intervals (1,4) [1].

Many models have been proposed in the literature to generate Rayleigh correlation fading channels. The work in [2] is based on sum-of-sinusoid (SoS) Method design a simple and accurate correlation Rayleigh fading simulation models. In [3], temporal-spatial correlation Rayleigh fading channels models is presented. As for [4] and [5], the authors propose different methods for generating arbitrary correlation Rayleigh fading channels. But little research has been done to generate of spatial-temporal correlated multiple-input multiple-output (MIMO) Nakagami fading, especially under the condition of non-isotropic scattering. An efficient approach for generating temporal correlation Nakagami channel models with specific values of  $m$  has been derived in [6] and [7]. Another useful method of calculating the autocorrelation of Nakagami sequences has been presented by [8] and [9]. Reference [10] has presented a universal method based on inverse fast Fourier transform (IFFT) filter to simulate Nakagami correlation channel model. In [11], the hardware implementation of arbitrary autocorrelation Nakagami model has been generated

by mapping Rayleigh sequences into Nakagami varieties. In [12], based on decomposition technique, a simple procedure has been derived for generating spatial correlated Nakagami channels.

All methods mentioned above require for high computational complexity and lack flexibility to generate different arbitrary Nakagami correlation fading. Therefore, in this paper, we propose a generalized algorithm to do this work, which proves to be of much lower computational complexity and higher precision.

The rest of this paper is organized as follows: Section II explains the motivation of our research. Section III explains the whole construction of the algorithm and gives how to use the rank matching technology to generate temporal autocorrelation Nakagami distribution sequence, then presents the Sim's algorithm [10] to generate spatial cross-correlation Nakagami fading channels. Numerical results and analysis are presented in Section IV. Finally, the conclusions are drawn in Section V.

## II. MOTIVATION

In accordance with [9], we will use the Kronecker based stochastic model (KBSM) for modeling channel, and the spatial correlation matrices of channel  $\mathbf{R}_H$  can be written as:

$$\mathbf{R}_H = \mathbf{R}_{T_x} \otimes \mathbf{R}_{R_x} \quad (1)$$

where  $\otimes$  means Kronecker Product(KP),  $\mathbf{R}_{T_x}$  and  $\mathbf{R}_{R_x}$  represent the transmit and receiver correlation matrices, respectively, they are defined as:

$$\mathbf{R}_{T_x} = E\{\mathbf{H}^H \cdot \mathbf{H}\} \quad (2)$$

$$\mathbf{R}_{R_x} = E\{\mathbf{H} \cdot \mathbf{H}^H\} \quad (3)$$

where  $E\{\cdot\}$  denotes the expectation over the variable, and  $\mathbf{H}$  is the Channel Impulse Response (CIR), with the superscript  $(\cdot)^H$  being the Hermitian operator.

Taking the multiple uncorrelated flat narrow-band Rayleigh fading into consideration, the MIMO channel correlation matrices can be written as:

$$\mathbf{H} = \mathbf{R}_H^{1/2} \text{vec}(\mathbf{G}) = (\mathbf{R}_{T_x} \otimes \mathbf{R}_{R_x})^{1/2} \text{vec}(\mathbf{G}) \quad (4)$$

where  $\text{vec}(\cdot)$  denotes the Vectorization operator matrix, i.e.,  $\text{vec}(\mathbf{A}) = [a_1, a_2, \dots, a_n]$ ,  $a_n$  is the  $n$ -th row vector in matrix  $\mathbf{A}$ , and  $\mathbf{G}$  denotes the independent and identically distributed variance matrix of the MIMO channels.

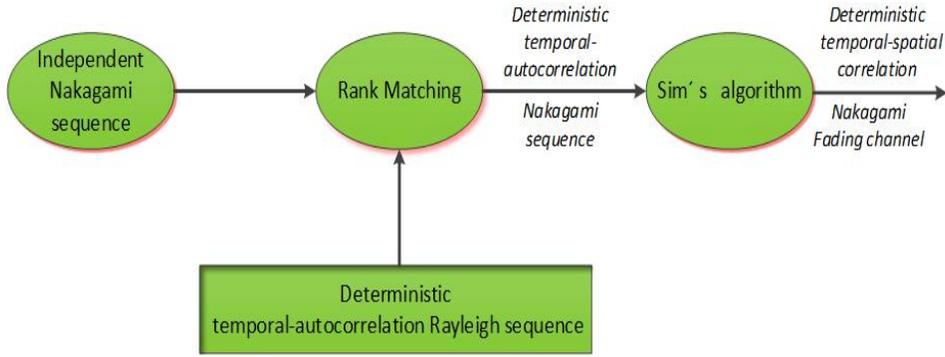


Fig. 1: Design of the proposed scheme.

According to the mathematical equations  $vec(\mathbf{ABC}) = (\mathbf{C}^H \otimes \mathbf{A}) \cdot vec(\mathbf{B})$ , we reshape the matrix  $\mathbf{H}$  into the classical KBSM:

$$\mathbf{H} = \mathbf{R}_{\mathbf{R}_x}^{1/2} \cdot \mathbf{G} \cdot \mathbf{R}_{\mathbf{T}_x}^{1/2} \quad (5)$$

The core of producing this KBSM is how to generate multiply spatial-temporal correlation Rayleigh complex channels [5], which means the envelop of channel model is a temporal-autocorrelation Rayleigh complex random process. At the same time, we use the Cholesky decomposition on the channel correlation matrix:

$$\mathbf{R}_H = \mathbf{L} \cdot \mathbf{L}^H, \mathbf{L} = \mathbf{R}_H^{1/2} \quad (6)$$

Then the CIR can be derived from:

$$\mathbf{H} = \mathbf{L} \cdot \mathbf{G} \quad (7)$$

It's not difficult to verify that  $\mathbf{H}$  is the matrix with zero-mean unit variance circularly-symmetric complex Gaussian distributed entries. And the covariance matrix can be written as:

$$E[\mathbf{H} \cdot \mathbf{H}^H] = \mathbf{R}_H \quad (8)$$

It's very clear that correlated Rayleigh fading channels has a close relationship with a complex Gaussian process. But unfortunately, this is not the case for Nakagami fading because there is no general theoretical link between the Nakagami variables and correlated Gaussian variables [12]. Therefore we develop a generalized algorithm for generating arbitrary spatial-temporal correlation Nakagami channel models.

The probability density function (PDF) of Nakagami distribution is given in [10] as:

$$f_R(r) = \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m r^{2m-1} \exp\left(-\left(\frac{m}{\Omega}\right)r^2\right), r \geq 0 \quad (9)$$

where  $\Gamma(\cdot)$  is the gamma function, and  $\Omega$  is the mean power,  $m = \frac{\Omega^2}{E\{(r - \Omega)^2\}}$  are known as fading figure representing the diversity degree of the fading in the propagation field.

In addition, the square root of Gamma distribution variables  $\gamma$  follows the Nakagami random variables (RVs)  $R$ , i.e.,

$$\mathbf{R} = \sqrt{\gamma} \quad (10)$$

On the other hand, the relationship between Rayleigh sequences  $Y_m$  and Nakagami variables can be constructed as:

$$\mathbf{R} = \sqrt{\mathbf{Y}_1^2 + \mathbf{Y}_2^2 + \dots + \mathbf{Y}_m^2} \quad (11)$$

### III. METHODOLOGY

As shown in Fig. 1, the general algorithm can be divided into three steps: (1).generate determined temporal autocorrelation Rayleigh sequences based on SoS method. (2).utilize the Rank matching technology between Nakagami and Rayleigh sequences to rearrange the independent Nakagami sequences .(3). produce arbitrary spatial-temporal correlation Nakagami fading which generated combining with Sim's algorithm.

#### A. ACF Rayleigh variables

As proposed by [10], the baseband signal of the classic clarke's two-dimension (2D) isotropic scattering Rayleigh fading channel model is given by

$$h(t) = \lim_{N \rightarrow +\infty} \frac{1}{\sqrt{N}} \sum_{n=1}^{\infty} \exp[j(\omega_d \cdot t \cdot \cos\alpha_n + \phi_n)] \quad (12)$$

where  $\omega_d$  is the maximum Doppler frequency,  $N$  is the number of scattering paths,  $\alpha_n$  and  $\phi_n$  are the angle of arrival (AoA) and initial phase of the  $n$ -th paths, respectively.

According to the central limit theorem, we could find that the real and imaginary part of  $h(t)$  can be approximated as complex Gaussian distribution. Then the normalized autocorrelation function (ACF) can be derived as:

$$R_{hh}(\tau) = E[h(t)h^*(t+\tau)] = E[\exp(j(\omega_d \cdot \tau \cdot \cos\alpha_n))] \quad (13)$$

The correspond Doppler Power Profile Spectrum is given by:

$$S_{hh}(\omega) = \int_{-\infty}^{\infty} R_{hh}(\tau) \exp^{-j\omega\tau} d\tau = 2\pi E[\sigma(\omega - \omega_d \cos\alpha_n)] \quad (14)$$

Especially, when the  $\alpha_n$  and  $\phi_n$  are uniformly independent over  $[-\pi, \pi)$ , mutually independent for all  $n$ . The ACF can be written as :

$$R_{hh}(\tau) = J_0(\omega_d \tau) \quad (15)$$

where  $J_0(\cdot)$  denotes the zero-order the first Bessel function. It is worth mentioning that Rayleigh fading channel simulator is

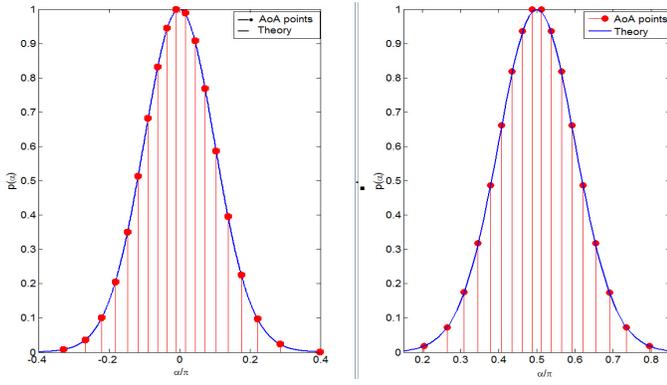


Fig. 2: The value of AoAs resulted by both symmetrical and asymmetrical PSD, where  $N = 20, k = 10, \alpha_0=0$  for asymmetrical PSD and  $\alpha_0 = \pi/2$  for symmetrical PSD.

based on SoS method for non-isotropic propagation scenarios [13]. Hence the utility of the von-Mises Probability Distribution Function (PDF) as a parametric model for the distribution of the AoA has been adopted in many different literatures which focus on non-isotropic scattering environment.

Then the normalized SoS extended Rayleigh fading channel is defined by:

$$h_i(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{\infty} \cos[j(\omega_d \cdot t \cdot \cos\alpha_{i,n} + \phi_n)] \quad (16)$$

$$h_q(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{\infty} \cos[j(\omega_d \cdot t \cdot \cos\alpha_{q,n} + \phi_n + \Omega_n)] \quad (17)$$

The main difference between the extended model and traditional model is the values of  $\Omega_n$  and  $\alpha_{i/q,n}$ , which resulted by both symmetrical and asymmetrical Power Spectral Density (PSDs). The PDF of AoA follows the von-Mises distribution and can be defined by:

$$p(x) = \frac{\exp[k \cos(\alpha - \alpha_0)]}{2\pi I_0(k)} \quad (18)$$

where  $k \geq 0$  is a concentration parameter, which controlling the angular spread of the AoA, when  $k = 0$ , the PDF of the AoA is approaching to the uniform distribution. And  $\alpha_0$  denotes the mean of the AoA.

According to [14], we present the following correlation of this model:

$$R_{hh}(\tau) = \frac{I_0(\sqrt{k^2 - (2\pi f_d \tau)^2} + j4\pi k f_d \cos(\alpha_0) \tau)}{I_0(k)} \quad (19)$$

At last, based on [9], we utilize the Method-of-Equal Areas (MEA) method to calculate the value of incoming plane wave's AoA (as shown in Fig. 2).

### B. Rank matching technology

In this step, the Nakagami distribution sequences are firstly generated by the rejection-method [9] and then rearranged to match the rank statistics of an underlying Rayleigh sequences

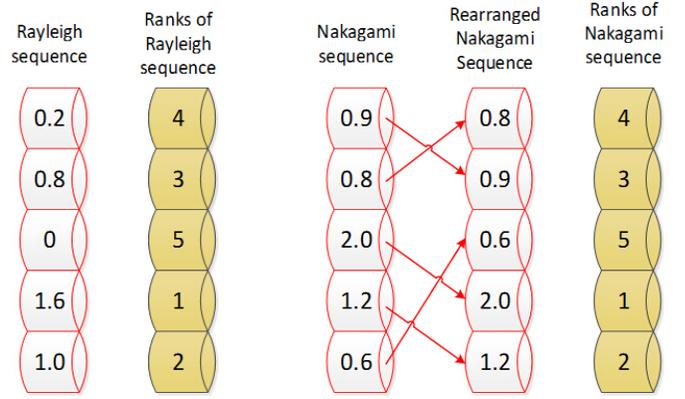


Fig. 3: The scheme of Rank matching.

generated by Step A. After that, we can derive the desired temporal autocorrelation Nakagami sequences.

Meanwhile, the ACF is used to measure the association strength of a sequence between different timeslots. Therefore, rank represents the position relationship between different data in a sequence. Moreover, according to the conclusion of [12] and equation [11], the ACF of Nakagami sequence approximately equal to the underlying Rayleigh samples when they have the same rank statistics.

As shown in Fig. 3, the principle of the rank matching technology is as follows: produce the Nakagami sequence by rejection method [15] [14] and generate a new Nakagami sequence which is a rearranged version of Rayleigh sequence, consequently its dates have the same ranking position as the corresponding samples of Rayleigh sequence. Namely each Nakagami dates should be place in the same position occupied by the Rayleigh sequence. Furthermore, they have the same statistical properties in time domain.

### C. Sim's algorithm

Based on the work of [13], the algorithm is used for transforming independent Gamma random variables into cross-correlated Nakagami distributions from the equation(11). As shown in Fig. 4, the core of this algorithm is to utilize known parameter  $m_k, P_k$  and  $\mathbf{C}_\gamma$  to acquire the desired correlated Gamma vector, which is written by:

$$\boldsymbol{\gamma} = \mathbf{T} \cdot \mathbf{g} = \mathbf{T}[g_1, g_2, \dots, g_N]^T \quad (20)$$

where  $g_k \sim G(m_{g,k}, \Omega_{g,k})$  is a Gamma random variables and the parameters are set as follows:

$$\mu_k = \frac{m_k}{\Omega_k} \quad (21)$$

$$x_{k,l} = \frac{\mu_l}{\mu_k} \quad (22)$$

Then the matrix  $\mathbf{T}$  is obtained by:

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ x_{21}y_{21} & 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & 0 \\ x_{(N-1)1}y_{(N-1)1} & \dots & \dots & \dots & 0 \\ x_{N1}y_{N1} & \dots & \dots & \dots & 1 \end{bmatrix} \quad (23)$$

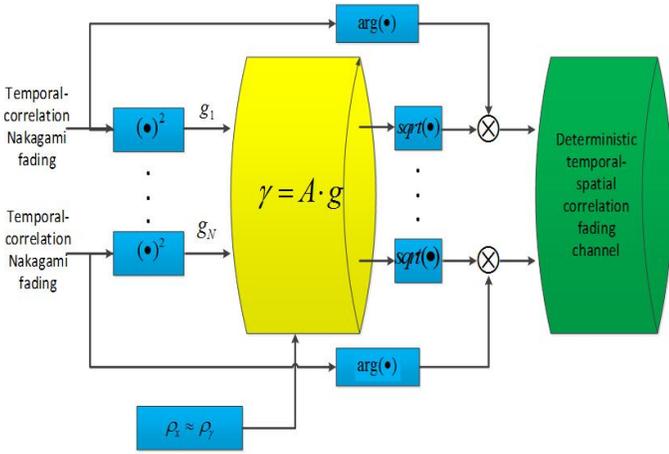


Fig. 4: The scheme of Sim's algorithm.

where  $y_{kl}$  follows the Beta-distribution like this:

$$F(t, a, b) = \frac{\Gamma(a+b) \cdot t^{a-1} \cdot (1-t)^{b-1}}{\Gamma(a)\Gamma(b)} \quad (24)$$

Then  $y_{kl}$  is calculated by:  $y_{kl} \sim \beta(x_{kl}, m_{gk} - x_{kl})$  where the parameters  $x_{kl}$  and  $m_{g,k}$  are determined from the following formulations:

$$m_{g,1} = m_1 \quad (25)$$

$$x_{k1} = \mu_1 \cdot \mu_k \mathbf{C}_\gamma(1, k) \quad (26)$$

$$x_{kl} = \mu_k \mu_l \cdot \mathbf{C}_\gamma(k, l) - \sum_{n=1}^{l-1} \left( \frac{a_{kn} a_{ln}}{m_{g,n}} \right) \quad (27)$$

$$m_{g,k} = m_i - \sum_{n=1}^{l-1} a_{kn} \quad (28)$$

In this case,  $\mathbf{c}_\gamma(\mathbf{k}, \mathbf{l})$  means the covariance between  $\gamma_k$  and  $\gamma_l$ . The relationship between  $\mathbf{c}_\gamma(\mathbf{k}, \mathbf{l})$  and  $\rho_{\mathbf{R}}(k, l)$  is obtained by:

$$\mathbf{C}_\gamma(k, l) = \rho_{\mathbf{R}}(k, l) \cdot \sqrt{\left( \frac{\Omega_{\gamma,k}^2}{m_k} \cdot \frac{\Omega_{\gamma,l}^2}{m_l} \right)} \quad (29)$$

where  $\rho_{\mathbf{R}}(k, l)$  is the cross-correlation parameter of Nakagami varieties. Finally, based on the mathematical equations of mentioned above, the desired RVs  $\gamma_i$  will be derived as:

$$\gamma_1 = g_1 \quad (30)$$

$$\gamma_k = g_k + \sum_{l=1}^{k-1} a_{kl} \cdot b_{kl} \cdot g_l \quad (31)$$

By using the relationship formulas (10) and (11), the spatial cross-correlation Nakagami distribution sequences are generated. More related details about this algorithm can be found in [15].

#### D. Statistical properties of the proposed method

After the fading envelop PDF and correlation function, the most important of statistical properties about channel model is the second-order statistics, i.e., the level crossing rates (LCR) and average fade duration (AFD). Therefore, in this section, we discuss the LCR and AFD properties of correlated Nakagami fading channel for non-isotropic scattering environment. For a fading signal, the LCR is by definition the average expected rate at which the signal envelop  $R$  crosses a specific level in the positive direction. Its expressions  $N_R$  is given by:

$$N_R = \int_0^\infty \dot{r} P(R, \dot{r}) d\dot{r} \quad (32)$$

where  $P(R, \dot{r})$  means the joints PDF of the envelop  $R$  and its time derivative  $\dot{r}$ . For correlated Nakagami fading channel in the von-Mises distribution scattering environments,  $N_R$  is depicted as follows:

$$N_{R-NK}(\rho) = \sqrt{B} \frac{(m\rho)^{m-0.5} \cdot e^{-m\rho^2}}{\sqrt{2\pi}\Gamma(m)} \quad (33)$$

where

$$B = \frac{2\pi^2 f_d^2 \cdot [I_0(k) + \cos(2\alpha_0)I_2(k)]}{I_0(k)} \quad (34)$$

and  $\rho$  denotes the normalized envelope level, which is the ratio between envelope threshold and envelope root mean square value. On the other hand, the AFD, is by definition that the average time over which the signal envelope  $r$  keeps below some certain value  $R$  and can be denoted as:

$$F = \frac{P(r < R)}{N_R} \quad (35)$$

where  $P(r < R)$  is the CDF of the given channel envelope when  $r < R$ . According to [16], the AFD of Nakagami fading channel can be written as:

$$F_{NK}(\rho) = 1 - Q(\sqrt{2k}, \sqrt{2(k+1)\rho^2}) \quad (36)$$

where  $Q(\cdot)$  stands for the generalized Marcum- $Q$  functions. When the von-Mises distribution scattering is taken into considered, the AFD of correlation Nakagami channels can be calculated as:

$$F_{NK}(\rho) = \frac{\Gamma(m, m\rho^2)}{\Gamma(m)} \quad (37)$$

where  $\Gamma(\cdot)$  is the incomplete Gamma function.

#### IV. TESTING AND VALIDATION

As for validation examples, Fig. 5 and Fig. 6 present the temporal-ACF of and the error between proposed model and theoretical model via Monte Carlo Simulations for varying parameters (*i.e.*,  $a_0 = 0, \pi/8, \pi/4$ ;  $k = 0, 5, 10, 20$ ). The simulation results indicate that the ACF of our proposed Nakagami fading are sufficiently close to the theoretical values, and the maximum margin of error is less than 0.025.

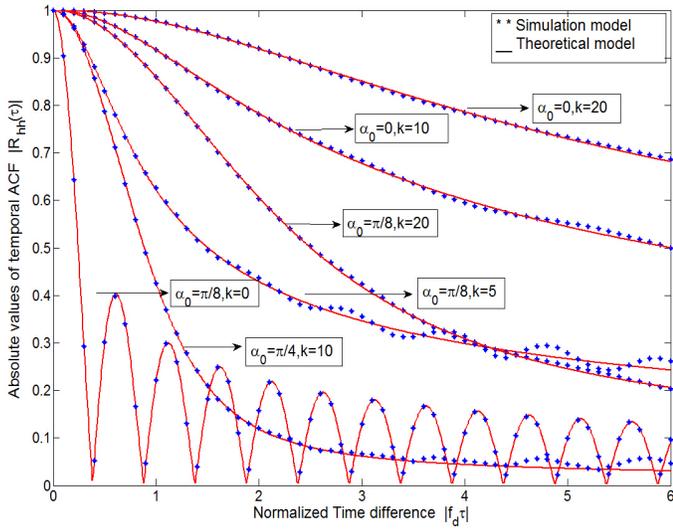


Fig. 5: Absolute of temporal-ACF  $|R_{hh}(\tau)|$  based on 100000 samples, where  $\alpha_0 = 0, \pi/8, \pi/4$ ,  $k=0,5,10,20$

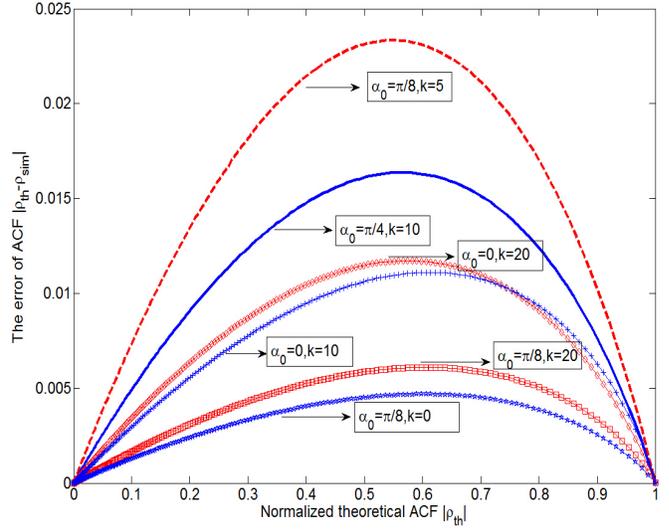


Fig. 6: The error of temporal-ACF between the proposed scheme and theoretical

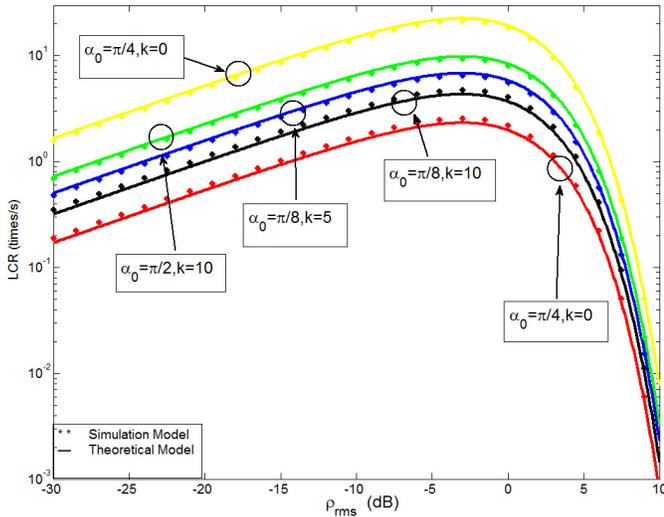


Fig. 7: LCR comparison results of different parameters between proposed method and theoretical model

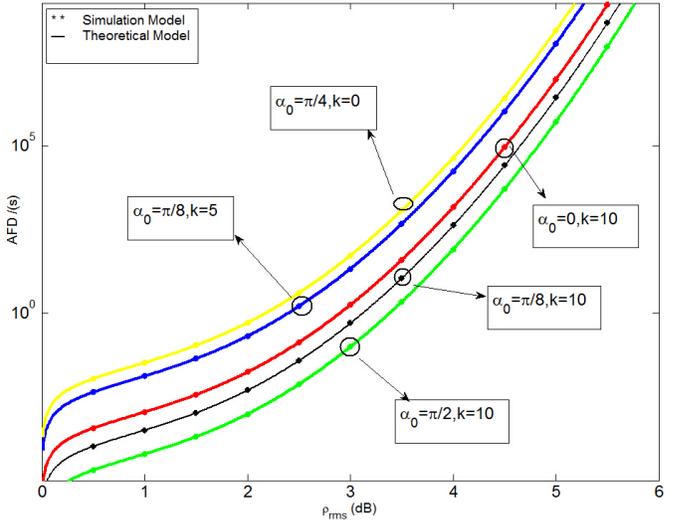


Fig. 8: AFD comparison results of different parameters between proposed method and theoretical model

As an estimation illustration, we consider a  $2 \times 2$  MIMO system, the Nakagami fading characteristic parameters of 4 sub-channels are designed as follows:

$$\mathbf{m} = [2.08 \quad 1.98 \quad 2.18 \quad 2.28] \quad (38)$$

$$\mathbf{\Omega} = [14.7907 \quad 20.0930 \quad 30.8837 \quad 25.8604] \quad (39)$$

and the desired spatial-CCF matrix is:

$$\rho_{\mathbf{R}} = \begin{bmatrix} 1 & 0.775 & 0.624 & 0.382 \\ 0.775 & 1 & 0.775 & 0.624 \\ 0.624 & 0.775 & 1 & 0.775 \\ 0.382 & 0.624 & 0.775 & 1 \end{bmatrix} \quad (40)$$

Fig. 9 shows the generated PDF and theoretical PDF given by sub-channels of Nakagami fading model. The PDFs for all 4 sub-channels match very well with the theoretical ones. And the corresponding characteristic parameters are calculated by output samples:

$$\mathbf{m}' = [2.05 \quad 2.01 \quad 2.13 \quad 2.24] \quad (41)$$

$$\mathbf{\Omega}' = [14.7280 \quad 20.0014 \quad 30.6527 \quad 25.6112] \quad (42)$$

$$\rho'_{\mathbf{R}} = \begin{bmatrix} 1 & 0.746 & 0.651 & 0.361 \\ 0.746 & 1 & 0.746 & 0.651 \\ 0.651 & 0.746 & 1 & 0.746 \\ 0.361 & 0.651 & 0.746 & 1 \end{bmatrix} \quad (43)$$

Comparing to (38) - (40), it is obviously that all parameters match very closely with the desired ones.

In addition, Fig. 7 and 8 depict the LCR and AFD properties of correlation Nakagami fading channel for different parameters  $k$  and  $\alpha_0$ , respectively. In analysing Fig. 7, the LCRs for correlation channels show that the fading envelope generated by the proposed method is very similar with the theoretical values. The same result can be observed in Fig. 8, the proposed method simulation results of AFD match very well with the theoretical results. In a word, it can be clearly observed from those simulations: 1) the more concentrated of AoA distribution, the slower of the channel fading; 2) When the  $k$  value increased, the LCR of each threshold level decreased and the AFD became longer; 3) when  $\alpha_0$  is closing to  $\pi/2$ , the more faster of channel fading and LCR increases but AFD decreases under the condition of equal  $k$  value. Those results validate the utility of the proposed method.

## V. CONCLUSION

This paper discusses a general algorithm to generate correlated Nakagami fading channels with arbitrary spatial-temporal correlation and fading parameters. Moreover, the proposed algorithm is low of complexity and capable for large-scale and real-time simulation. The simulation result shows that the proposed generation algorithm has match well with the theoretical channel model. At the same time, this method can be applied for hardware implementation in channel model emulation.

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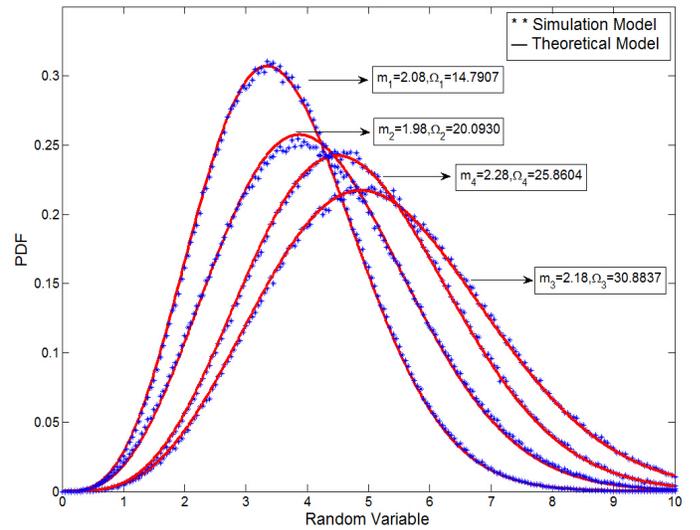


Fig. 9: The PDFs based on 10000 samples so generated for 4 sub-channels. where  $m_1 = 2.08, \Omega_1 = 14.7907, m_2 = 1.98, \Omega_2 = 20.0930, m_3 = 2.18, \Omega_3 = 30.8837, m_4 = 2.28, \Omega_4 = 25.8604$

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