Relay-aided Interference Alignment and Neutralization for 3-cellular Interference Channels

Yuquan Shu†, Qiang Wang†, Jianhua Zhang†, Tao Sun†
†Key Laboratory of Universal Wireless Communications,
Wireless Technology Innovation Institute, (Beijing Univ.of Posts and Telecom.), Ministry of Education, China
Email: shuyuquan@bupt.edu.cn, wangq@bupt.edu.cn

***Abstract*—As everyone knows, the energy of signals diminishes with increasing distance, so users at the edge of hexagon cells have lower signal-to-noise ratio (SNR) than users at the center, which means they are more easily interfered by signals from other base stations. So we consider the application of relay to the cellular case to help users at the edge to eliminate interferences and obtain higher sum-rate. In this paper we don't only use relay to neutralize interferences but also use interference alignment to reduce the number of antennas needed in destinations. To this end, we propose a relay-aided interference alignment and neutralization scheme. The relay knows all channel state information(CSI) and can solve out all signals transmitted by sources. To align and neutralize interferences, the relay needs to retransmit the signals solved out by using proper transmitting vectors. The transmitting vectors in sources and relay are carefully designed to achieve higher sum-rate and simulation results will prove it.**

***Index Terms*—Interference Alignment, Neutralization, Relay, multiple-in multiple-out (MIMO), sum-rate.**

I. Introduction

With the standardization and the gradual commercial of 4G wireless communications, multi-cell interference, as a key constraint, gets more attention to further enhance the overall system capacity. Interference alignment technique that by restricting all interference at every receiver into a small space a higher degree-of-freedom (DoF) than traditional methods can be achieved [1] is agreed as an important way to solve the multi-cell interference effectively. However, the limitations of high signal-to-noise ratio needed by the interference alignment technology, and the design of practical transceiver algorithm, both become the key problem in the multi-cell interference network. Relay can not only enhance the cell coverage and improve signal-to-noise ratio, which is necessary for users at the edge of hexagon cells but also help to neutralize some interferences at receivers. So the investigation on relay-helped interference alignment is necessary.

In the relay network, there is another idea to deal with interference called interference neutralization (IN) [2]. Interference neutralization is a technique which neutralizes interference signals by a careful selection of forwarding strategies when the signal travel through relay nodes before reaching the destination. The general idea has been applied to deterministic channel [3] and in two-hop relay channels [4,5].

There have been some investigations on relay-helped interference management already. In [6], relay was utilized to neutralize the interference and studied achievable rate region, however interference alignment is not taken into consideration. [7] and [8] only investigates relay-aided single input single output (SISO) system. [9] requires more than one intelligent relay which must have the capacity to obtain global channel knowledge. However, in the view of practice, it is not easy to keep all relays so intelligent to know global channel knowledge and instantaneous relay system is hard to realize. There is no investigation that combines interference alignment and interference neutralization at transmitters and receivers. Furthermore the system in [9] is constrainted to two pairs of transmitters and receivers. However, it is probably that there are more than two users in wireless system. It is necessary to investigate multiple users system.

A. Main Contributions

In this paper we explore users at the junction of 3 cellular. We not only use relay to enhance the cell coverage and improve signal-to-noise ratio(SNR) but also neutralize interference and utilize interference alignment to manage the rest interference at users. We give the procode matrix that satisfies the alignment and neutralization constrains, and the matching decode filters for the system. The sum-rate of the proposed algorithm is given by the simulation result. Because relay can increase the channel gain, we can achieve a higher sum-rate through designing proper transmitting vectors.

B. Organization and Notation

The remainder of this paper is organized as follows. In section II, we describe system model and signal model with relay. In section III we introduce how the retransmit vector is designed in relay and how to use interference neutralization and alignment to suppress interferences in users. Finally, numerical result and conclusions are shown in section IV.

We employ uppercase boldface letters for matrices and lowercase boldface for vectors. span(X), rank(X), vec(X) denotes the column space, rank, vector obtained by stacking the columns of matrix X respectively. Superscripts $(.)^T$, $(.)^H$, $(.)^{-1}$ stand for transpose, Hermitian transpose, and matrix inversion respectively. I_n denotes n-D identity matrix respectively.

II. System Model

In a wireless cellular system, users at the edge of hexagon cells are easier to receive signals sent from 3 cells. So we

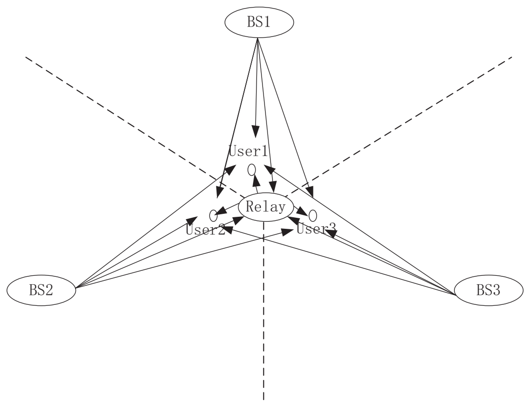

Fig. 1. Relay Helped System Scheme

consider 3-cellular network with the help of a relay. There is one user in each cell and they are all at the junction of three cells. The distance from BS_l to user_k is denoted by $r_{[kl]}$. Global channel knowledge is assumed at all nodes. As shown in Fig. 1, it's a two hop system. At first hop each base station (BS) sends data symbols to its corresponding user and relay. The relay can receive and solve all symbols sent by BS, then retransmit them to users through proper transmitting vectors at second hop. There are M_R, M_S and M_D antennas at the relay and each transmitter and receiver, respectively.

The transmit signal from any BS_l to user_k is impaired by flat fading and attenuation due to the environment, modelled as Rayleigh fading with path loss: Let $\mathrm{H}^{[k,l]}$ be a channel matrix with i.i.d. circularly symmetric complex Gaussian $\sim CN(0,1)$ distributed elements. Taking also the path loss $pl_{kl} = \sqrt{\beta r_{[kl]}^{-\alpha}}$ [10] into account, the channel is $\mathrm{G}^{[k,l]} = =pl_{kl}\mathrm{H}^{[k,l]}\alpha\beta=\sqrt{\beta r_{[kl]}^{-\alpha}}\mathrm{H}^{[k,l]}\alpha\beta$, with α and β depending on the channel model at hand.

At first hop, during the process of transmission, BS i sends signal vector $\mathrm{x}^{[i]}$ to its corresponding user and relay. $\mathrm{x}^{[i]}$ is precoded as $\mathrm{x}^{[i]} = \mathrm{V}^{[i]}\mathrm{s}^{[i]}$, where $\mathrm{V}^{[i]}$ represents the beamforming matrix $\mathrm{V}^{[i]} = [v_1^{[i]}\ v_2^{[i}\ \cdots v_d^{[i]}]$ and $\mathrm{s}^{[i]}$ denots the d data symbol vectors $\mathrm{s}^{[i]} = [\mathrm{s}_1^{[i]}\ \mathrm{s}_2^{[i]} \cdots \mathrm{s}_d^{[i]}]^T$. We assume that each data symbol has unit variance and each user has the average power constrain $E\left\{\mathrm{tr}[\mathrm{x}^{[i]}\mathrm{x}^{[i]^H}]\right\} \leq P$, where P is the transmit constraint power. The received signal at the relay and user i is obtained by

$$\mathrm{y}^{[R]}=\sum_{i=1}^{3}\mathrm{G}^{[R,i]}\mathrm{x}^{[i]}+\mathrm{n}^{[R]} \tag{1}$$

$$\mathrm{y}^{[i]} = \sum_{j=1}^{3}\mathrm{G}^{[i,j]}\mathrm{x}^{[j]} + \mathrm{n}^{[i]} \tag{2}$$

where $\mathrm{G}^{[R,i]}$ denotes the $M_R \times M_S$ channel matrix from user i to relay and $\mathrm{G}^{[i,j]}$ denotes the $M_D \times M_S$ channel matrix from transmitter j to receiver i. They all include path loss. $\mathrm{n}^{[R]}$ and $\mathrm{n}^{[i]}$ stands for the the additive white Gaussian noise (AWGN) vector with zero mean and unit variance at relay.

Since we assume that relay has the ability to decode and forward all the receive signals. After receiving the signal $\mathrm{y}^{[R]}$, relay generates new transmit signal $\mathrm{x}^{[R]}$ as $\mathrm{x}^{[R]} = \mathrm{V}^{[R]}\mathrm{s}^{[R]}$, where $\mathrm{V}^{[R]} = [v_1^{[R]}\ v_2^{[R]}\ \cdots v_d^{[R]}]$ and $\mathrm{s}^{[R]} = [\mathrm{s}_1^{[R]}\ \mathrm{s}_2^{[R]} \cdots \mathrm{s}_d^{[R]}]^T$. $\mathrm{x}^{[R]}$ must satisfy the relay power constrain $E\left\{\mathrm{tr}[\mathrm{x}^{[R]}\mathrm{x}^{[R]^H}]\right\} \leq P$. At second hop, relay retransmits symbols to users and we combine them with the symbols received at first hop:

$$\mathrm{y}_2^{[i]} = \sum_{j=1}^{3}\mathrm{G}^{[i,j]}\mathrm{x}^{[j]} + \mathrm{H}^{[i,R]}\mathrm{x}^{R} + \mathrm{n}_1^{[i]} + \mathrm{n}_2^{[i]} \tag{3}$$

where $\mathrm{G}^{[i,j]}$ denotes the $M_D \times M_S$ channel matrix from transmitter j to receiver i. $\mathrm{H}^{[i,R]}$ denotes the $M_D \times M_R$ channel matrix from relay to receiver i. Because the distance is short, path loss is not considered between users and relay. $\mathrm{n}_1^{[i]}$ and $\mathrm{n}_2^{[i]}$ stands for the the additive white Gaussian noise (AWGN) vector produced at the first and second hop at destination i with zero mean and unit variance.

We assume the decoding matrix of receiver i is $\mathrm{W}^{[i]}$, the receive filter output signals are

$$\hat{\mathrm{x}}^{[i]} = \mathrm{W}^{[i]}\mathrm{y}_2^{[i]} \tag{4}$$

III. Interference Alignment and Neutralization

To explain our scheme, we assume that M_S=M_D=3 and M_R= 6. Each BS sends two independent data symbols to its corresponding user, i.e. BS i transmits data symbol $\mathrm{s}_1^{[i]}, \mathrm{s}_2^{[i]}$ along linear beamforming vectors $v_1^{[i]}$, $v_2^{[i]}$ to user i. Because of the broadcast feature of interference channels, each user receives signals that sent by not only its corresponding BS but also other interfering BS. In the given system, user i receives 6 data symbols totally, two are desired symbols that sent by BS i and the rest four are interfering symbols sent by the other two BS. If we do not introduce the relay, it is hard to work out desired signals because of interference from other cells and low SNR caused by path loss. However, in our proposed scheme, by introduce a multiple antennas relay, users have the ability to decode desired data symbols with only has 3 antennas.

Firstly, we will propose the decoding scheme which aimed at neutralizing half of interference signals and aligning the other into a smaller subspace. Then, based on decoding scheme, the transmitting vector design at relay which aims at improving the sum-rate and reduce the computational complexity will be proposed.

A. The interference neutralization and alignment in destinations

Relay can hear all signals sent by transmitters. Then the received signal at relay is

$$y^{[R]} = \sum_{i=1}^{2}\mathrm{G}^{[R,1]}v_i^{[1]}s_i^{[1]}+\sum_{i=1}^{2}\mathrm{G}^{[R,2]}v_i^{[2]}s_i^{[2]} + \sum_{i=1}^{2}\mathrm{G}^{[R,3]}v_i^{[3]}s_i^{[3]}+\mathrm{n}^{[R]} \tag{5}$$

Since relay has 6 antennas, it can estimate all six data symbols from three sources, which are $s_1^1,\ s_2^1\ s_1^2,\ s_2^2,\ s_1^3,\ s_2^3$. Use the estimated symbols, we generate transmit data symbols at relay, which are

$$\begin{array}{ll} s_1^{[R]} = s_1^{[1]} & s_2^{[R]} = s_2^{[1]} \\ s_3^{[R]} = s_1^{[2]} & s_4^{[R]} = s_2^{[2]} \\ s_5^{[R]} = s_1^{[3]} & s_6^{[R]} = s_2^{[3]} \end{array} \tag{6}$$

Due to the existence of relay, users can hear not only signals transmitted by direct links but also signals transmitted by relay links. The channel output signal at user_k at the first hop can be given as follow:

$$y_1^{[k]} = \underbrace{\sum_{i=1}^{2} \mathrm{G}^{[k,k]} v_i^{[1]} s_i^{[1]}}_{desired-signal} + \underbrace{\sum_{i=1}^{2} \sum_{j=1, j\neq k}^{3} \mathrm{G}^{[k,j]} v_i^{[j]} s_i^{[j]}}_{\mathrm{inter}ferences} + \underbrace{\mathrm{n}_1^{[k]}}_{noise} \tag{7}$$

Apparently, there are 4 interferences at user_k, we will use relay-signal to neutralize two of them and align the other two. Because of the existence of path loss, the energy of desired signal is low, so we amplify the signal received at first hop through multiplying the reciprocal of the path loss pl_{kk}. Because user_k is at the edge, the distance between user_k to each BS is nearly equal. Thus, we assume the path loss $pl_{kl} = pl_{kk}$. After amplifying the signal received at first hop can be:

$$\hat{y}_1^{[k]} = \sum_{i=1}^{2} \mathrm{H}^{[k,k]} v_i^{[1]} s_i^{[1]} + \sum_{i=1}^{2} \sum_{j=1, j\neq k}^{3} \mathrm{H}^{[k,j]} v_i^{[j]} s_i^{[j]} + \mathrm{n}_1^{[k]} / pl_{kk} \tag{8}$$

Relay transmits data symbols $s_i^{[R]}$ along beamforming vector $v_i^{[R]}$. In order to neutralize interference at users we design $v_i^{[R]}$ to achieve following goals.

- Design $v_1^{[R]}$ to neutralize $s_1^{[1]}$ at user_2.
- Design $v_2^{[R]}$ to neutralize $s_2^{[1]}$ at user_3.
- Design $v_3^{[R]}$ to neutralize $s_1^{[2]}$ at user_1.
- Design $v_4^{[R]}$ to neutralize $s_2^{[2]}$ at user_3.
- Design $v_5^{[R]}$ to neutralize $s_1^{[3]}$ at user_1.
- Design $v_6^{[R]}$ to neutralize $s_2^{[3]}$ at user_2.

Then beamforming vector $v_i^{[R]}$ we choose must satisfy following constrains.

- At user_1.

$$\mathrm{H}^{[1,R]} v_3^{[R]} = -\mathrm{H}^{[1,2]} v_1^2 \tag{9}$$
$$\mathrm{H}^{[1,R]} v_5^{[R]} = -\mathrm{H}^{[1,3]} v_1^3 \tag{10}$$

- At user_2.

$$\mathrm{H}^{[2,R]} v_1^{[R]} = -\mathrm{H}^{[2,1]} v_1^{[1]} \tag{11}$$
$$\mathrm{H}^{[2,R]} v_6^{[R]} = -\mathrm{H}^{[2,3]} v_2^{[3]} \tag{12}$$

- At user_3.

$$\mathrm{H}^{[3,R]} v_2^{[R]} = -\mathrm{H}^{[3,1]} v_2^{[1]} \tag{13}$$
$$\mathrm{H}^{[3,R]} v_4^{[R]} = -\mathrm{H}^{[3,2]} v_2^{[2]} \tag{14}$$

By making the transmitting vector of the relay $\mathrm{V}_i^{[R]}\ i = 1, 2 \cdots 6$ satisfy the (9)-(14), we can neutralize 2 interferences in every user. So here is what is received by users after interference-neutralizing:

$$\begin{aligned} &\mathrm{y}^{[1]} = (\mathrm{H}^{[1,1]}\mathrm{V}_1^{[1]} + \mathrm{H}^{[1,R]}\mathrm{V}_1^{[R]})s_1^{[1]} + (\mathrm{H}^{[1,1]}\mathrm{V}_2^{[1]} + \\ &\mathrm{H}^{[1,R]}\mathrm{V}_2^{[R]})s_2^{[1]} + (\mathrm{H}^{[1,2]}\mathrm{V}_2^{[2]} + \mathrm{H}^{[1,R]}\mathrm{V}_4^{[R]})s_2^{[2]} \\ &+(\mathrm{H}^{[1,3]}\mathrm{V}_2^{[3]} + \mathrm{H}^{[1,R]}\mathrm{V}_6^{[R]})s_2^{[3]} + n_1 \end{aligned} \tag{15}$$

$$\begin{aligned} &\mathrm{y}^{[2]} = (\mathrm{H}^{[2,2]}\mathrm{V}_1^{[2]} + \mathrm{H}^{[2,R]}\mathrm{V}_3^{[R]})s_1^{[2]} + (\mathrm{H}^{[2,2]}\mathrm{V}_2^{[2]} + \\ &\mathrm{H}^{[2,R]}\mathrm{V}_4^{[R]})s_2^{[2]} + (\mathrm{H}^{[2,1]}\mathrm{V}_2^{[1]} + \mathrm{H}^{[2,R]}\mathrm{V}_2^{[R]})s_2^{[1]} \\ &+(\mathrm{H}^{[2,3]}\mathrm{V}_1^{[3]} + \mathrm{H}^{[2,R]}\mathrm{V}_5^{[R]})s_1^{[3]} + n_2 \end{aligned} \tag{16}$$

$$\begin{aligned} &\mathrm{y}^{[3]} = (\mathrm{H}^{[3,3]}\mathrm{V}_1^{[3]} + \mathrm{H}^{[3,R]}\mathrm{V}_5^{[R]})s_1^{[3]} + (\mathrm{H}^{[3,3]}\mathrm{V}_2^{[3]} + \\ &\mathrm{H}^{[3,R]}\mathrm{V}_6^{[R]})s_2^{[3]} + (\mathrm{H}^{[3,1]}\mathrm{V}_1^{[1]} + \mathrm{H}^{[3,R]}\mathrm{V}_1^{[R]})s_1^{[1]} \\ &+(\mathrm{H}^{[3,2]}\mathrm{V}_1^{[2]} + \mathrm{H}^{[3,R]}\mathrm{V}_3^{[R]})s_1^{[2]} + n_3 \end{aligned} \tag{17}$$

where n_i denotes the noise vector which is combined by noises produced at two hops, which are independent and identically distributed(i.i.d.) complex Gaussian noise vector with zero mean and $\mathrm{E}\left(NN^H\right) = \sigma_N^2 I$. So, n_i is also identically distributed(i.i.d.) complex Gaussian noise vector with zero mean. But at first hop the noise is amplified by multiplying $1/pl_{kk}$, so $\mathrm{E}\left(n_i {n_i}^H\right) = \sigma_{n_i}^2 I$, $\sigma_{n_i}^2 = (1 + (1/pl_{kk})^2)\sigma_N^2$.

As the equations show, there are two interferences left in every user which are the last two parts of each equation except the noise part. As we have set up, the user node has 3 antennas, which means we must align the two interferences to work out the desired signals. So we have this:

$$span(\mathrm{H}^{[1,2]}\mathrm{V}_2^{[2]} + \mathrm{H}^{[1,R]}\mathrm{V}_4^{[R]}) = span(\mathrm{H}^{[1,3]}\mathrm{V}_2^{[3]} + \mathrm{H}^{[1,R]}\mathrm{V}_6^{[R]}) \tag{18}$$

$$span(\mathrm{H}^{[2,1]}\mathrm{V}_2^{[1]} + \mathrm{H}^{[2,R]}\mathrm{V}_2^{[R]}) = span(\mathrm{H}^{[2,3]}\mathrm{V}_1^{[3]} + \mathrm{H}^{[2,R]}\mathrm{V}_5^{[R]}) \tag{19}$$

$$span(\mathrm{H}^{[3,1]}\mathrm{V}_1^{[1]} + \mathrm{H}^{[3,R]}\mathrm{V}_1^{[R]}) = span(\mathrm{H}^{[3,2]}\mathrm{V}_1^{[2]} + \mathrm{H}^{[3,R]}\mathrm{V}_3^{[R]}) \tag{20}$$

Bring (9)-(14) into the upper equations, after simplifying them, we can get:

$$\begin{aligned} &span((\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,2]}(\mathrm{H}^{[3,2]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_4^{[R]}) \\ &= span((\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,3]}(\mathrm{H}^{[2,3]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_6^{[R]}) \end{aligned} \tag{21}$$

$$\begin{aligned} &span((\mathrm{H}^{[2,R]} - \mathrm{H}^{[2,1]}(\mathrm{H}^{[3,1]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_2^{[R]}) \\ &= span((\mathrm{H}^{[2,R]} - \mathrm{H}^{[2,3]}(\mathrm{H}^{[1,3]})^{-1}\mathrm{H}^{[1,R]})\mathrm{V}_5^{[R]}) \end{aligned} \tag{22}$$

$$\begin{aligned} &span((\mathrm{H}^{[3,R]} - \mathrm{H}^{[3,1]}(\mathrm{H}^{[2,1]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_1^{[R]}) \\ &= span((\mathrm{H}^{[3,R]} - \mathrm{H}^{[3,2]}(\mathrm{H}^{[1,2]})^{-1}\mathrm{H}^{[1,R]})\mathrm{V}_3^{[R]}) \end{aligned} \tag{23}$$

Then, we make the precoder vector $\mathrm{V}_i^{[j]}\ i = 1, 2; j = 1, 2, 3$ disappear, so at the user we need not to know the precoding information from the BS and we only need to design the $\mathrm{V}_i^{[R]}$ $i = 1, 2 \cdots 6$ from the relay to align the interferences and improve the sum-rate, which is simplified.

We take $\mathrm{y}^{[1]}$ at user_1 for example and the computational process of the other signals received by the other users are the

$$y_1 = [\mathrm{M}_1(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,1]}(\mathrm{H}^{[2,1]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_1,\ \mathrm{M}_1(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,1]}(\mathrm{H}^{[3,1]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_2] \times \begin{pmatrix} p_1 & 0 \\ 0 & p_2 \end{pmatrix} \times \begin{pmatrix} s_1^{[1]} \\ s_2^{[1]} \end{pmatrix} + \mathrm{M}_1 n_1 \quad (27)$$

same to user$_1$. After eliminating the $\mathrm{V}_i^{[j]}$ $i = 1,2; j = 1,2,3$ we can write $\mathrm{y}^{[1]}$ as follows:

$$\begin{aligned}\mathrm{y}^{[1]} = &(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,1]}(\mathrm{H}^{[2,1]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_1 p_1 s_1^{[1]} + \\ &(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,1]}(\mathrm{H}^{[3,1]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_2 p_2 s_2^{[1]} + \\ &(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,2]}(\mathrm{H}^{[3,2]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_4 p_4 s_2^{[2]} + \\ &(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,3]}(\mathrm{H}^{[2,3]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_6 p_6 s_2^{[3]} + n_1\end{aligned} \quad (24)$$

Where we make $\mathrm{V}_i p_i$ $i = 1, 2 \cdots 6$ to take the place of $\mathrm{V}_i^{[R]}$ $i = 1, 2 \cdots 6$. The transmitting vector $\mathrm{V}_i^{[R]}$ must satisfy the transmit power constraint $Tr((\mathrm{V}_i^{[R]})^H \mathrm{V}_i^{[R]}) \leq p_i$, so we let V_i be unit vector and ${p_i}^2$ be the transmit power of the relay.We will use zero-forcing to solve out the desired signal. Because of (21), the $(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,2]}(\mathrm{H}^{[3,2]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_4$ and the $(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,3]}(\mathrm{H}^{[2,3]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_6$ span the same space. Thus, we choose one of these matrices arbitrarily and denote the singular value decomposition (SVD) of this as

$$(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,2]}(\mathrm{H}^{[3,2]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_4 = [\mathrm{U}^1\mathrm{U}^0]\Lambda\mathrm{V} \quad (25)$$

where the matrix U^0 is composed of the last two left singular vectors and U^1 holds the first one left singular vectors. Then from (25) the interference nulling matrix at D_1 which completely eliminates the interference becomes $\mathrm{M}_1 = (\mathrm{U}^0)^H$. Multiplying this to (24) the non-interfering received signal vector y_1 can be written as:

$$\begin{aligned}y_1 &= \mathrm{M}_1\mathrm{y}^{[1]} \\ &= \mathrm{M}_1(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,1]}(\mathrm{H}^{[2,1]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_1 p_1 s_1^{[1]} \\ &+ \mathrm{M}_1(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,1]}(\mathrm{H}^{[3,1]})^{-1}\mathrm{H}^{[3,R]})\mathrm{V}_2 p_2 s_2^{[1]} + \mathrm{M}_1 n_1\end{aligned} \quad (26)$$

Let's combine the two signal to a 2×1 vector, and reshape (26) as (27).

We make H_1,P_1,S_1 denote the corresponding part:

$$y_1 = \mathrm{H}_1\mathrm{P}_1 S_1 + \mathrm{M}_1 n_1 \quad (28)$$

where P_1 can be get by using the water-filling solution to improve capacity.

Apparently, interferences are restrained totally and we can solve the desired signals. The same method can be used to the other users.

B. The transmitting vectors design in relay

Then, we will discuss the method to get the transmit vector V_i aimed to simplify the complexity of calculation and improve the sum-rate. V_i is also restricted to interference alignment equation (21)-(23) (because $\mathrm{V}_i p_i = \mathrm{V}_i^{[R]}$). We take D_1 for example as usual. Review (21), to simplify the calculation-complexity we let V_4 be the eigenvectors of $(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,2]}(\mathrm{H}^{[3,2]})^{-1}\mathrm{H}^{[3,R]})$ and strengthen the constraint by getting rid of "span":

$$\mathrm{V}_4 = (\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,3]}(\mathrm{H}^{[2,3]})^{-1}\mathrm{H}^{[2,R]})\mathrm{V}_6 \quad (29)$$

so we can solve out V_6. Then expand this method to the other destinations we can get V_i.We normalize V_i through $\mathrm{V}_i/|\mathrm{V}_i|$.

Apparently, matrix $(\mathrm{H}^{[1,R]} - \mathrm{H}^{[1,2]}(\mathrm{H}^{[3,2]})^{-1}\mathrm{H}^{[3,R]})$ has not only one eigenvector and which one to choose can influence the capacity of the system. Here, we using a maximum chordal distance criterion [14] to determine the best V_i. Let us define the chordal distance between an $m \times n_1$ matrix X_1 and $m \times n_2$ matrix X_2 for $m \geq n_1, n_2$ as:

$$\begin{aligned}d_{cd}(X_1, X_2) &= \tfrac{1}{\sqrt{2}}\left\| Q(X_1)Q(X_1)^H - Q(X_2)Q(X_2)^H \right\|_F \\ &= \sqrt{\tfrac{n_1+n_2}{2} - \left\| Q(X_1)^H Q(X_2) \right\|_F^2}\end{aligned} \quad (30)$$

Where $Q(X)$ is defined as a matrix which consists of the orthonormal basis vectors that span the column space of X and we can get it from QR-decomposition. Note that, since the interference signal spaces are all aligned, we only consider the chordal distance between the desired signals and one of interference signal. We take user$_k$ for instance. Let $\mathrm{H}_j^{[i]}$ denote channel coefficient of $s_j^{[i]}$ in (24) and the chordal distance between desired signal and interference of user$_k$ can be:

$$\mathrm{P}_{\mathrm{D},1} = d_{cd}(\mathrm{H}_1^{[1]}\mathrm{V}_1^{[R]}, \mathrm{H}_2^{[2]}\mathrm{V}_4^{[R]}) + d_{cd}(\mathrm{H}_2^{[1]}\mathrm{V}_2^{[R]}, \mathrm{H}_2^{[2]}\mathrm{V}_4^{[R]}) \quad (31)$$

We expand this method to the other users we can get maximum chordal distance expression:

$$\mathrm{P} = \max_{\mathrm{V}_1^{[R]}, \mathrm{V}_2^{[R]}, \mathrm{V}_4^{[R]}} (\mathrm{P}_{D,1}, \mathrm{P}_{\mathrm{D},2}, \mathrm{P}_{D,3}) \quad (32)$$

Using the maximum chordal distance method we can choosing better transmitting vectors that can enlarge the chordal distance between desired signals and interferences, which can improve the capacity of the system.

IV. Numerical result

This section evaluates the sum rate performance of various data transmission strategies over multi-cell systems. In all simulations, we consider the case of 3 pair of user and BS. Because the relay exists we assume the transmission power in relay is equal with the total transmission power of the 3 BS. And also we assume that the elements of the channel matrix $\mathrm{H}^{[i,j]}$ have an i.i.d. complex Gaussian distribution with zero mean and unit variance.

We compare the sum-rate between our relay scheme and without-relay network (fig.2). In the without-relay network, we also consider the 3 cell scenario, which include 3 cells and

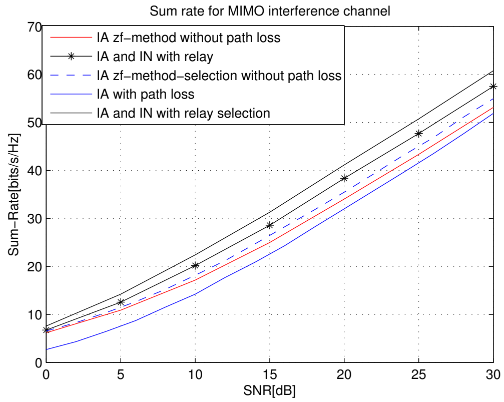

Fig. 2. Comparison of the sum rate ($\beta = 10^{-0.33}$ $\alpha = 3.76$ $r = 1km$).

there is one user in each cell. This is similar to 3-user system. We will use interference alignment to manage the interference and work out the desired signal [14]. As to [14], every BS and user node has 4 antennas and there are 24 antennas totally in this scenario which is equal to our relay scheme. Without the consideration of path loss we can get the red full line and the blue imaginary line, and the blue one optimizes precoder and decoder using a selecting strategy [14]. Then, the path loss is considered and we get the blue full line. Because of path loss, the SNR at users is low which leads to the reduce of sum-rate. At last, our proposed scheme is considered and we get the two black lines. The black full line uses the method mentioned in section III-B which apparently improves the sum-rate.

As to fig.2, our relay scheme overcome the path loss and improve the sum-rate. The path loss exists in the direct passing link from BS to user and the amplification in users at the first hop can not change the SNR because it amplifies the noise too. But at the second hop, the signals transmitted by relay neutralize some interferences and align the other interference signals and increase the power of desired signal which leads to the increase of SNR.

V. Conclusion

The sum-rate of 3-Cellular fading and interference channels with a relay is discussed in this paper. Also a scheme of interference neutralization and alignment is proposed. The relay and interference alignment are used to neutralize half of the interferences and align the other half separately. By using interference alignment the signal dimensions of residual interferences are reduced. Then destinations can decode the desired data symbols from receiving signals. Relay can also increase the SNR at the receivers. Finally, we demonstrate that relay can increase the sum-rate. A beamforming vectors design scheme using a maximum chordal distance criterion at relay is proposed, which aims at improving sum-rate further. At last, the numerical result is shown. Through the numerical result, we verified that the sum-rate achieved by the proposed scheme is improved compare with the 3-user scheme without relay.

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