Practical Differential Quantization for Spatially and Temporally Correlated Massive MISO Channels

Yanliang Sun, Jianhua Zhang, Ping Zhang, Linyun Wu and Yuning Wang
Key Lab of Universal Wireless Communications, Ministry of Education
Beijing University of Posts and Telecommunications, Beijing, China
Email: YanliangSun@bupt.edu.cn

***Abstract*—In this paper, an implementable channel quantization scheme that can effectively exploit both spatial and temporal channel correlation for massive MIMO systems is proposed. In limited feedback systems, differential quantization conducted by applying skewing and rotations on a differential codebook is effective on improving the overhead efficiency practically. To apply these techniques in massive MIMO systems, we adopt noncoherent trellis coded quantization (NTCQ) for Rayleigh channel as a foundation. The inherent codebook of NTCQ is defined and investigated thoroughly. Then we propose a scheme that can produce a differential inherent codebook, which makes adaptive skewing and rotations applicable. In numerical simulations, compared to previous approaches on differential NTCQ, superiority of the proposed scheme is significant. It needs no prior statistical knowledge of channel correlation, while high overhead efficiency can also be achieved. The results reveal that in massive MIMO systems, if ideal channel state information at user terminals is assumed to be available, precisely feeding them back to base stations is practical within affordable overhead and computations.**

I. Introduction

Recently, scaling up the dimension of multiple-input, multiple-output (MIMO) systems, which is literally called massive MIMO, has gained a great deal of interest [1]. In massive MIMO, since transmit beamforming has been considered as the optimal transmission technique, the channel state information at the transmitter (CSIT) is important for guiding the beam-vector generation [2]. In frequency division duplexing (FDD) systems, due to the absence of channel reciprocity, CSIT is usually obtained by quantizing the channel direction vector at the user terminals (UTs) and feeding quantized bits back to the base station (BS). Quantization error may mislead the generation of beam-vector, causing degradation in beamforming gain. Therefore, the design of efficient and high quality limited feedback schemes is of great value in massive MIMO.

To take advantage of channel correlation in small-scale arrays, differential quantization schemes that based on skewing and rotating differential codebooks (DCBs), e.g. random vector quantization (RVQ) codebooks and Fourier-base codebooks, are well explored in [3], [4]. In [5], adaptive skewing is proposed so that both temporal and spatial channel correlation can be tracked and utilized directly without prior statistics. On the other hand, if independent and identically distributed (i.i.d.) Rayleigh channel is considered, the total of feedback bits should be increased linearly with the number of antennas [6], [7]. Based on this law, noncoherent trellis coded quantization (NTCQ) that is efficient for massive MIMO application is proposed in [8], [9] for i.i.d. Rayleigh channel. Utilizing trellis coded modulation (TCM), minimum Euclidean distance (ED) between codewords is maximized, and Viterbi algorithm can be used to ensure that the complexity scales linearly to the number of antennas. In [9], a differential NTCQ that need prior statistical knowledge of spatial channel correlation is mentioned. However, it is not convenient for implementation.

In this paper, we propose a differential quantization scheme for massive MIMO channel. A deep insight into the equivalence between ED based noncoherent codeword search and chordal distance (CD) based codeword search is taken, and the inherent codebook of NTCQ is defined. Then to make the inherent codebook of proposed scheme suitable for DCB, we introduce the definition of vertex dimension, and assign particular constellation for this dimension. For other dimensions, the trellis coded quantization (TCQ) is performed, while the structure of TCM and Viterbi decoding is adotped as the same as in NTCQ. Verified by average beamforming gain under both first-order Gaussin-Markov channel model and geometric-based channel models [10], [11], the proposed scheme is shown to have better quantization performance than differential NTCQ in [9], while the implementability is also attractive for massive MU-MIMO systems. It is also shown that, even with a small amount of antennas at BS, the proposed scheme is still superior compared to preceding techniques.

The paper is organized as followed. The system model of massive MIMO with limited feedback is given in Section II, and various quantization schemes including NTCQ are also briefly introduced in this part. In Section III, some analysis on NTCQ is given and the proposed differential quantization is detailed. Numerical results are provided in Section IV. Conclusions are given in Section V.

II. Preliminaries

A. System and Channel Model

We consider a block-fading massive MIMO downlink in a microcell. One base station (BS) has N_T transmit (Tx) antennas and K active UTs. Each UT have one receive (Rx) antenna, while $N_T \gg K$. For this multiple-input single-output (MISO) system, received signal for the kth UT at the ith fading block can be written as

$$r_{k,i} = \rho \mathbf{h}_{k,i}^H \mathbf{x}_i + n_{k,i}, \tag{1}$$

ρ is the Tx signal-to-noise ratio (SNR). $\mathbf{h}_{k,i} \in \mathbb{C}^{N_T \times 1}$ is the small scale fading channel from BS to the kth UT in the ith block.[1] $\mathbf{x}_i \in \mathbb{C}^{N_T \times 1}$ is the transmitted signal at all N_T Tx antennas. $n_{k,i}$ is assumed to be zero-mean additive white Gaussian noise with covariance $\mathbb{E}\left[|n_{k,i}|^2\right] = 1$. Since the quantization procedure is the same for all UTs, we omit the suffix k subsequently for convenience.

In this work, first-order Gaussian-Markov (FOGM) channel model is adopted for the spatially and temporally correlated channel, written as

$$\mathbf{h}_i = \eta \mathbf{h}_{i-1} + \sqrt{1-\eta^2} \mathbf{R}^{\frac{1}{2}} \mathbf{g}_i, \tag{2}$$

where η is the time correlation coefficient, and $\mathbf{R}$ is the spatial correlation matrix at the transmitter. $\mathbf{g}_i$ is the uncorrelated MISO channel vector with i.i.d. complex Gaussian entries. Realistically, if the carrier frequency f_c is set, η is mainly affected by UT speed v_{UT}, fading block interval T, and angles of departure (AoD) distribution, while $\mathbf{R}$ is mainly affected by the array manifold and AoD distribution.

B. Channel Quantization Schemes

To obtain optimal beamforming performance in massive MIMO, the channel direction vector $\bar{\mathbf{h}}_i = \frac{\mathbf{h}_i}{\|\mathbf{h}_i\|_2}$ should be quantized as $\hat{\mathbf{h}}_i$ and fed back to BS. Here $\|\mathbf{a}\|_2$ denotes the 2-norm of a vector $\mathbf{a}$. In the literature, differential quantization is preferred for correlated channel as in (2). One approach is to directly quantize $\mathbf{f}_i = \mathbf{g}_i$ by applying quantization schemes for i.i.d. Rayleigh channel. However, as shown in (2), this approach needs the assumption of perfect knowledge on η and $\mathbf{R}$ at both sides, which is impractical especially for the BS, who has only partial CSIT. Another approach is to quantize the *chordal differential vector* $\mathbf{f}_i = \Delta_C\left(\bar{\mathbf{h}}_i, \hat{\mathbf{h}}_{i-1}\right)$, which is defined as

$$\Delta_C\left(\bar{\mathbf{h}}_i, \hat{\mathbf{h}}_{i-1}\right) = \left[\hat{\mathbf{h}}_{i-1}, \hat{\mathbf{h}}_{i-1}^{\perp}\right]^H \bar{\mathbf{h}}_i. \tag{3}$$

$\mathbf{a}^{\perp}$ denotes the $N_T \times (N_T - 1)$ space orthogonal to the $N_T \times 1$ vector $\mathbf{a}$. Then after quantizing f_i as $\hat{f}_i$, $\hat{\mathbf{h}}_i$ is obtained as

$$\hat{\mathbf{h}}_i = \left[\hat{\mathbf{h}}_{i-1}, \hat{\mathbf{h}}_{i-1}^{\perp}\right] \hat{\mathbf{f}}_i. \tag{4}$$

Subsequently, we use

$$\mathbf{\Phi}_i = \left[\hat{\mathbf{h}}_{i-1}, \hat{\mathbf{h}}_{i-1}^{\perp}\right] \tag{5}$$

to denote the rotation matrix, and $\mathbf{\Phi}_i$ should be unitary. [2] In this approach, DCB is needed for the quantization of $\mathbf{f}_i$. Various definitions of DCB have been given in the literature. In this work, the set of DCB $\mathbf{\Omega}$ is defined as a set of codebook $\mathcal{W}_D$ that includes a vertex vector $\boldsymbol{\zeta} = [1, 0, ..., 0]^T$, written as $\mathbf{\Omega} = \{\mathcal{W}_D, \boldsymbol{\zeta} \in \mathcal{W}_D\}$. Here $\boldsymbol{\zeta}$ is defined as the first column in the identity matrix $\mathbf{I}$.

If arbitrary codebook $\mathcal{W} = \{\mathbf{w}_t, t = 1, ..., N_c\}$ is available for BS and each UT, in quantization, the codeword $\mathbf{w}_t$ that has the minimal distance to $\mathbf{f}_i$ is picked, written as

$$\hat{\mathbf{f}}_i = \arg\min_{\mathbf{w}_t \in \mathcal{W}} [d(\mathbf{f}_i, \mathbf{w}_t)], \tag{6}$$

and the corresponding index t is fed back in the form of binaries $\mathbf{b}_t = (t)_2$. Here $(a)_2$ denotes the binary form of a integer a. N_c is the codebook size, and $l_c = \log_2 N_c$ is the length of feedback bits.

Normally two kinds of distance are used for quantization in (6). The chordal distance (CD) d_C between two $N_T \times 1$ vectors $\mathbf{a}$ and $\mathbf{b}$ is defined as

$$d_C(\mathbf{a}, \mathbf{b}) = \sqrt{1 - |\mathbf{a}^H \mathbf{b}|^2}. \tag{7}$$

The Euclidean distance (ED) is defined as

$$d_E(\mathbf{a}, \mathbf{b}) = \|\mathbf{a} - \mathbf{b}\|_2. \tag{8}$$

For small BS arrays, CD is used in quantizing normalized vectors, i.e. $\bar{\mathbf{h}}_i$ or $\mathbf{f}_i$. The search for optimal $\mathbf{w}_t$ is conducted by enumerating all codewords in $\mathcal{W}$. This is not feasible for massive MIMO, because its complexity scales exponentially to N_c, while N_c should scale linearly to N_T.

For massive MIMO, ED is used as an equivalent of CD in NTCQ by introducing the noncoherent search [8]. The optimal codeword for quantizing $\bar{\mathbf{h}}_i$ in i.i.d Rayleigh channel is chosen as

$$\hat{\mathbf{h}}_i = \arg\min_{\mathbf{w}_t \in \mathcal{W}} \min_{\alpha \in \mathbb{R}^+} \min_{\theta \in [0, 2\pi)} \left[\left\|\bar{\mathbf{h}}_i - \alpha e^{j\theta} \mathbf{w}_t\right\|_2\right]. \tag{9}$$

Based on (9), TCM can be introduced for massive MIMO to set up a mapping from feedback binaries $\mathbf{b}_t = (t)_2$ to modulated symbols $\mathbf{w}_t$, as $\mathbf{b}_t = \mathbb{Q}(\mathbf{w}_t)$. The convolutional coded symbols for $\mathbf{b}_t$ is,

$$\mathbf{w}_t = \mathbb{Q}^{-1}(\mathbf{b}_t) = \mathbb{M}(\mathbb{T}(\mathbf{b}_t)), \tag{10}$$

where $\mathbb{T}$ and $\mathbb{M}$ denote the mappings from uncoded bits $\mathbf{b}_t$ to coded bits $\mathbf{d}_t$, and from $\mathbf{d}_t$ to modulated symbols $\mathbf{w}_t$, respectively. The structure of encoder $\mathbb{T}$ and Viterbi decoder $\mathbb{T}^{-1}$ are omitted in this work due to limited space. M-ary constellation $\mathbf{\Xi}_M = \left\{\frac{1}{\sqrt{N_T}} \xi_m, \mathbb{E}|\xi_m|^2 = 1, m = 1, ..., M\right\}$ is used in modulation $\mathbb{M}$, where ξ_m denotes a point in the M-ary constellation. The length of $\mathbf{b}_t$ depends on code rate r_c, state number S used in $\mathbb{T}$ and M in $\mathbb{M}$, as

$$l_c = r_c N_T \log_2 M + \log_2 S. \tag{11}$$

The quantization of $\bar{\mathbf{h}}_i$ can be viewed as the demodulation in Euclidean space. Therefore, (9) can be implemented by Viterbi algorithm and noncoherent search. In noncoherent search, discrete sets $\{\theta_p, p = 1, ..., N_p\}$ and $\{\alpha_q, q = 1, ..., N_q\}$ are set up, where $\theta_p \in [0, 2\pi)$ are uniformly chosen, and α_q are chosen according to $\mathbf{\Xi}_M$. Then (9) is implemented as

$$\hat{\mathbf{h}}_i = \arg\min_{\mathbf{w}_t \in \mathcal{W}} \min_{p,q} \left[\left\|\bar{\mathbf{h}}_i - \alpha_q e^{j\theta_p} \mathbf{w}_t\right\|_2\right]. \tag{12}$$

If N_T is large, N_p, N_q can be quite small as proved in [8].

[1] $\mathbf{h}_{k,i}$ is conjugate transposed in (1) for the convenience of notations in transmit beamforming.

[2] In practice, a convenient method for calculating $\mathbf{\Phi}_i$ is proposed in [4].

III. Analysis and Proposed Algorithms

A. On Euclidean Space and Inherent Codebook of NTCQ

Firstly we define the *inherent codebook* of NTCQ as

$$\mathcal{C}=\left\{\mathbf{c}_t=\frac{\mathbf{w}_t}{\left\|\mathbf{w}_t\right\|_2},t=1,...,N_c\right\}. \tag{13}$$

$N_c=2^{l_c}$ is the codebook size. The reason for normalizing $\mathbf{w}_t$ in (13) is given as follows. From [8, Eq. (4-7)], we have

$$\begin{aligned}&\arg\min_{\mathbf{w}_t\in\mathcal{W}}\min_{\alpha\in\mathbb{R}^+}\min_{\theta\in[0,2\pi)}\left[\left\|\bar{\mathbf{h}}_i-\alpha e^{j\theta}\mathbf{w}_t\right\|_2^2\right]\\ =&\arg\min_{\mathbf{w}_t\in\mathcal{W}}\min_{\alpha\in\mathbb{R}^+}\left[\left\|\bar{\mathbf{h}}_i\right\|_2^2+\alpha^2\left\|\mathbf{w}_t\right\|_2^2-2\alpha\left|\bar{\mathbf{h}}_i\mathbf{w}_t^H\right|\right] \quad (14)\\ =&\arg\max_{\mathbf{w}_t\in\mathcal{W}}\frac{\left|\bar{\mathbf{h}}_i\mathbf{w}_t^H\right|^2}{\left\|\mathbf{w}_t\right\|_2^2}. \quad (15)\end{aligned}$$

If we neglect the minimization on α and fix $\alpha=\frac{1}{\|\mathbf{w}_t\|_2}$ in (14), we can also have

$$\begin{aligned}&\arg\min_{\mathbf{w}_t\in\mathcal{W}}\left[\left\|\bar{\mathbf{h}}_i\right\|_2^2+\alpha^2\left\|\mathbf{w}_t\right\|_2^2-2\alpha\left|\bar{\mathbf{h}}_i\mathbf{w}_t^H\right|\right]\\ =&\arg\max_{\mathbf{w}_t\in\mathcal{W}}\frac{\left|\bar{\mathbf{h}}_i\mathbf{w}_t^H\right|}{\left\|\mathbf{w}_t\right\|_2}. \quad (16)\end{aligned}$$

The equivalence of (15) and (16) is obvious. Therefore, we have the following equivalence

$$\begin{aligned}\hat{\mathbf{h}}_i&=\arg\min_{\mathbf{w}_t\in\mathcal{W}}\min_{\alpha\in\mathbb{R}^+}\min_{\theta\in[0,2\pi)}\left[\left\|\bar{\mathbf{h}}_i-\alpha e^{j\theta}\mathbf{w}_t\right\|_2\right]\\ &=\arg\min_{\mathbf{c}_t\in\mathcal{C}}\min_{\theta\in[0,2\pi)}\left[\left\|\bar{\mathbf{h}}_i-e^{j\theta}\mathbf{c}_t\right\|_2\right]. \quad (17)\end{aligned}$$

(17) is generally not good for implementation, because constellations with unequal amplitudes, e.g. 16QAM can be used as $\mathbf{\Xi}_M$ in NTCQ, and in most cases, fixing α before quantizing $\bar{\mathbf{h}}_i$ by Viterbi algorithm is not feasible. However, for the purpose of analysis, since codewords in $\mathcal{C}$ have unit norm, (17) provides a visual explanation on the equivalence between ED and CD. On the surface of unit sphere in real space, the relation between ED and CD can be illustrated as the length of a chord between two points and the corresponding radian. In complex space, the relation is quite the same, except a phase differential θ that should be considered for parallel noncoherent search, as in (17). Besides, $M=4$ for QPSK and $M=8$ for 8PSK are two important exceptions satisfying $\|\mathbf{w}_t\|_2=1$ for arbitrary t. In practice, 1-bit/antenna or 2-bits/antenna are usually favorable for quantizing $\bar{\mathbf{h}}_i$, which can be obtained by setting $r_c=1/2$ for QPSK or $r_c=2/3$ for 8PSK in $\mathbb{T}$.

In Fig. 1, the inherent codewords of NTCQ with QPSK and 8PSK are illustrated. They all lie on the plain $|w_1|=|w_2|=...=|w_{N_T}|$, while w_{n_t} is the n_tth entry of $\mathbf{w}_t$. This is optimal for i.i.d. Rayleigh channel, since each entry of $\bar{\mathbf{h}}_i$ shares the same distribution. However, obviously, $\mathcal{C}\notin\mathbf{\Omega}$. For other M-ary constellations, the case would be the same. Therefore, NTCQ is not valid for the quantization of $\mathbf{f}_i$.

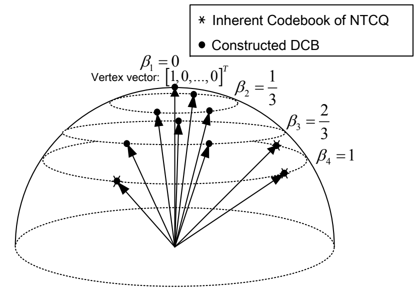

Fig. 1. Pre-skewing on NTCQ inherent codebook $\mathcal{C}$ to generate DCB $\mathcal{V}_D$. By codebook skewing, multiple plain are added, including the vertex vector $\boldsymbol{\zeta}=[1,0,...,0]^T$.

B. Differential Channel Quantization for Massive MIMO

In this section, we propose a practical differential channel quantization scheme for massive MIMO. Firstly, we define the *vertex dimension* τ as the non-zero dimension in the vertex vector $\boldsymbol{\zeta}$. Clearly, $\tau=1$ in this work. From (3), we have

$$\begin{aligned}\mathbb{E}\left[\left|f_{i,\tau}\right|^2\right]&=\mathbb{E}\left[\left|\hat{\mathbf{h}}_{i-1}^H\bar{\mathbf{h}}_i\right|^2\right] \quad (18)\\ &=\mathbb{E}\left[\left|\bar{\mathbf{h}}_{i-1}^H\bar{\mathbf{h}}_i\right|^2\right]+\mathbb{E}\left[\left\|\mathbf{e}_{i-1}\right\|_2^2\right], \quad (19)\end{aligned}$$

where $\mathbf{e}_i=\bar{\mathbf{h}}_i-\hat{\mathbf{h}}_i$ is the Euclidean estimation error at the ith block, which is uncorrelated to $\bar{\mathbf{h}}_i$. (19) is obtained from (18) by using $\mathbb{E}\left[\mathbf{e}_i\right]=\mathbf{0}$, and the entries of $\mathbf{e}_i$ are uncorrelated. Using results in [5], the first term in (19) can be written as

$$\mathbb{E}\left[\left|\bar{\mathbf{h}}_{i-1}^H\bar{\mathbf{h}}_i\right|^2\right]=\eta^2+\frac{\left(1-\eta^2\right)\sum\limits_{z=1}^{N_T}d_z^2}{2\sum\limits_{z=1}^{N_T}d_z^2+2\sum\limits_{z_1=1}^{N_T}\sum\limits_{z_2=z_1+1}^{N_T}d_{z_1}d_{z_2}}, \tag{20}$$

where $d_1,...,d_{N_T}$ are eigenvalues of $\mathbf{R}$. η and $\mathbf{R}$ are defined in (2). Therefore, it is a term indicating the strength of channel correlation. The second term in (19) is merely the uncertainty resulted from quantization error [3]. If $\mathbf{h}_{i-1}$ and $\mathbf{h}_i$ are highly correlated, $\mathbf{f}_i$ is close to the vertex vector $\boldsymbol{\zeta}$, which inspires designs of codebook skewing. We adopt codebook skewing as in [5], where for each codeword $\mathbf{v}_t\in\mathcal{V}_D\subset\mathbb{C}^{N_T\times1}$, while $\mathcal{V}_D\in\mathbf{\Omega}$, skewing operation $\mathbb{S}$ with factor ε_i is applied as

$$\mathbb{S}\left(\mathbf{v}_t,\varepsilon_i\right)=\begin{bmatrix}\sqrt{\dfrac{1-\varepsilon_i^2\left(1-\left|v_{t,1}\right|^2\right)}{\left|v_{t,1}\right|^2}}v_{t,1}\\ \varepsilon_i v_{t,2}\\ \vdots\\ \varepsilon_i v_{t,N_T}\end{bmatrix}. \tag{21}$$

According to (19), ε_i should be adaptively updated as [5]

$$\varepsilon_i=\bar{\mu}_{i-1}+\varepsilon_{i-1}2^{-\frac{l_c}{N_T-1}}, \tag{22}$$

TABLE I
PRE-SKEWING FACTOR β_v AND CONSTELLATION FOR VERTEX DIMENSION $\mathbf{\Xi}_V$ TO GENERATE $\mathcal{V}_D$ $V=16$ ($\gamma_v=\sqrt{1-\beta_v^2\left(N_T-1\right)/N_T}$)

β_v	Constellation $\mathbf{\Xi}_V$	Quantization Index v
0	$\{1\}$	$\{0\}$
1/3	$\left\{\gamma_v e^{j\frac{k\pi}{3}}, k=0,...,5\right\}$	$\{1,2,3,4,5,6\}$
2/3	$\left\{\gamma_v e^{j\frac{2k\pi}{5}}, k=0,...,4\right\}$	$\{7,8,9,10,11\}$
1	$\left\{\gamma_v e^{j\frac{k\pi}{2}}, k=0,...,3\right\}$	$\{12,13,14,15\}$

where $\varepsilon_1=1$. $\bar{\mu}_i$ is the estimation of $\mathbb{E}\left[d_C\left(\bar{\mathbf{h}}_{i-1},\bar{\mathbf{h}}_i\right)\right]$ at the ith block for both BS and UTs, which satisfies $\bar{\mu}_1=0$, and

$$\bar{\mu}_i=\frac{1}{i-1}\sum_{k=1}^{i-1}\sqrt{1-\left|\hat{\mathbf{h}}_k\hat{\mathbf{h}}_{k+1}^H\right|^2}, i>1. \tag{23}$$

Now we need to construct a codebook $\mathcal{V}_D=\{\mathbf{v}_t, t=1,...,N_v\}\in\mathbf{\Omega}$ for massive MIMO differential quantization. To obtain $\boldsymbol{\zeta}\in\mathcal{V}_D$, we firstly assign a specific constellation $\mathbf{\Xi}_V$ for quantizing the vertex dimension $f_{i,\tau}$, as

$$\mathbf{\Xi}_V=\left\{\xi_v=\gamma_v e^{j\varphi_v}, v=1,...,V\right\}, \tag{24}$$

where $\gamma_v\in\left[1/\sqrt{N_T},1\right]$ and $\varphi_v\in[0,2\pi)$ are amplitudes and phases for each symbol ξ_v. Then, utilizing TCM as the same as in NTCQ, the codeword $\mathbf{v}_t\in\mathcal{V}_D$ is constructed as

$$\mathbf{v}_t=\begin{bmatrix}\xi_v\\ \frac{\beta_v\tilde{\mathbf{w}}_t}{\left\|\tilde{\mathbf{w}}_t\right\|_2}\end{bmatrix}, \tag{25}$$

where β_v is given as

$$\beta_v=\sqrt{\frac{\left(1-\gamma_v^2\right)N_T}{N_T-1}}, \tag{26}$$

and $\tilde{\mathbf{w}}_t\in\tilde{\mathcal{W}}\subset\mathbb{C}^{(N_T-1)\times 1}$ are constructed by symbols in $\mathbf{\Xi}_M$. Finally the corresponding quantized bits $\tilde{\mathbf{b}}_t$ is constructed as

$$\tilde{\mathbf{b}}_t=\tilde{\mathbb{Q}}\left(\mathbf{v}_t\right)=\begin{bmatrix}(v)_2\\ \mathbb{Q}\left(\tilde{\mathbf{w}}_t\right)\end{bmatrix}, \tag{27}$$

where $\mathbb{Q}$ is defined in (10). Obviously, if $\xi_1=1$ is included in $\mathbf{\Xi}_V$, $\boldsymbol{\zeta}\in\mathcal{V}_D$ can be achieved. The length of $\tilde{\mathbf{b}}_t$ can be calculated as

$$\tilde{l}_c=r_c\left(N_T-1\right)\log_2 M+\log_2 S+\log_2 V, \tag{28}$$

and $N_v=2^{\tilde{l}_c}$.The construction of $\mathcal{V}_D$ can also be viewed as pre-skewing on the inherent codebook of NTCQ $\mathcal{C}$, where β_v is the pre-skewing factor. An example of this is illustrated in Fig. 1, where small-scale array $N_T=8$ and QPSK constellation $M=4$ is used for convenience. Mapping of β_v, $\mathbf{\Xi}_V$ and v for this example is given in Table I.

An important issue for DCB in massive MIMO is the constellations $\mathbf{\Xi}_V$ and $\mathbf{\Xi}_M$. We notice that if $\mathbf{h}_{i-1}$ and $\mathbf{h}_i$ are highly correlated, ε_i is quite small in (22). In this case, power in the rest N_T-1 dimensions of $\mathbf{f}_i$ is small, and wasting bits on these dimensions is not wise. Therefore, usually QPSK as $M=4$ or 8PSK as $M=8$ would be enough for $\mathbf{\Xi}_M$ in constructing $\mathcal{V}_D$. For the vertex dimension, however, from the perspective of implementation, we take noncoherent search for $\mathbf{f}_i$ similarly to (17), as

$$\hat{\mathbf{f}}_i=\arg\min_{\mathbf{v}_t\in\mathcal{V}_D}\min_p\left[\left\|\mathbf{f}_i-e^{j\theta_p}\mathbb{S}\left(\mathbf{v}_t,\varepsilon_i\right)\right\|_2\right], \tag{29}$$

where $\{\theta_p, p=1,...,N_p\}$ is a predefined set. Substituting (21), (25), (26) into (29), we have

$$\begin{aligned}&\left\|\mathbf{f}_i-e^{j\theta_p}\mathbb{S}\left(\mathbf{v}_t,\varepsilon_i\right)\right\|_2^2\\ &=\left|f_{i,\tau}-\sqrt{1-\frac{\varepsilon_i^2\beta_v^2\left(N_T-1\right)}{N_T}}e^{(\varphi_v+\theta_p)}\right|^2\\ &\quad+\left\|\mathbf{f}_{i,\bar{\tau}}-\varepsilon_i\beta_v e^{j\theta_p}\frac{\mathbf{w}_t}{\left\|\mathbf{w}_t\right\|_2},\right\|_2^2.\end{aligned} \tag{30}$$

Observing (30), we notice that β_v is comprised in both part of the squared ED, therefore, parallel search respected to β_v has to be performed. To minimize the branch number for β_v, the constellation $\mathbf{\Xi}_V$ can be designed as follows. Firstly a set of candidates for γ_v is given as

$$\left\{\bar{\gamma}_1=1,\bar{\gamma}_{N_s}=\chi,\bar{\gamma}_s\in(\chi,1), s=2,...,N_s-1\right\}, \tag{31}$$

where $\chi=\frac{1}{\sqrt{N_T}}$. Calculation of $\bar{\beta}_s$ is similar to (26). Then for each $\bar{\gamma}_s$, U_s candidates of phases are given as

$$\mathbf{\Psi}_s=\left\{\bar{\varphi}_{u_s}=e^{j\frac{2\pi(u_s-1)}{U_s}}, u_s=1,...,U_s\right\}. \tag{32}$$

Specifically, $\mathbf{\Psi}_1=\{\bar{\varphi}_1=1\}$, and $U_1=1$. Then, for the vth symbol in $\mathbf{\Xi}_V$, we have

$$\xi_v=\bar{\gamma}_s e^{j\bar{\varphi}_{u_s}}, v=\sum_{k=1}^{s-1}U_k+u_s, \tag{33}$$

and obviously, $V=\sum_{s=1}^{N_s}U_s$. Table I gives an example for this. The values for V and N_s are obtained by simulations. Generally, as shown in the numerical results, $V=16$ and $N_s=4$ is enough for 1 bit/antenna quantization.

Now we come back to the implementation of codeword search as in (29) and (30), with $\mathbf{\Xi}_V$ obtained from (31), (32), (33). For each pair (p,s), $f_{i,\tau}$ is quantized as

$$\hat{f}_{i,\tau,p,s}=\sqrt{1-\frac{\varepsilon_i^2\bar{\beta}_s^2\left(N_T-1\right)}{N_T}}e^{(\hat{\varphi}_{p,s}+\theta_p)}, \tag{34}$$

where

$$\hat{\varphi}_{p,s}=\arg\min_{\varphi_{u_s}\in\mathbf{\Psi}_s}\left|f_{i,\tau}-\sqrt{1-\frac{\varepsilon_i^2\bar{\beta}_s^2\left(N_T-1\right)}{N_T}}e^{(\varphi_{u_s}+\theta_p)}\right|. \tag{35}$$

The quantization for the rest N_T-1 dimensions $\mathbf{f}_{i,\bar{\tau}}$ is similar to the quantization in i.i.d Rayleigh channel in (12). With $\{\alpha_q, q=1,...,N_q\}$ defined as the same as in NTCQ, we have

$$\hat{\mathbf{f}}_{i,\bar{\tau},p,s}=\arg\min_{\tilde{\mathbf{w}}_t\in\tilde{\mathcal{W}}}\min_q\left[\left\|\mathbf{f}_{i,\bar{\tau}}-\alpha_q\varepsilon_i\beta_v e^{j\theta_p}\tilde{\mathbf{w}}_t\right\|_2\right], \tag{36}$$

TABLE II
ALGORITHM SUMMARY

Algorithm: Differential quantization with adaptive skewing for massive MIMO channel

Initialization in both BS and UT: $\varepsilon_1 = 1, \mathbf{\Phi}_1 = \mathbf{I}$.
Quantization for $\bar{\mathbf{h}}_i$ at the ith block in UT:
  Rotate $\bar{\mathbf{h}}_i$ as $\mathbf{f}_i = \mathbf{\Phi}_i^H \bar{\mathbf{h}}_i$.
  for $s = 1 : N_s$
    for $p = 1 : N_p$
      Quantize $f_{i,\tau}$ by (34) and (35).
      Quantize $\mathbf{f}_{i,\bar{\tau}}$ using (36).
      Obtain the quantized $\hat{\mathbf{f}}_{i,s}$ as (37).
    end
  end
  Obtain the quantized $\hat{\mathbf{f}}_i$ as (38).
  Obtain estimated channel as $\hat{\mathbf{h}}_i = \mathbf{\Phi}_i \hat{\mathbf{f}}_i$.
  Feedback $\tilde{\mathbf{b}}_i$ as (27).
Reconstruction for $\hat{\mathbf{h}}_i$ at the ith block in BS:
  Reconstruct $\mathbf{v}_t$ by (25).
  Skew $\mathbf{v}_t$ by (21) to obtain $\hat{\mathbf{f}}_i$.
  Obtain estimated channel as $\hat{\mathbf{h}}_i = \mathbf{\Phi}_i \hat{\mathbf{f}}_i$.
Updatings at the ith block in both BS and UT:
  Update ε_{i+1} by (22) and (23).
  Update $\mathbf{\Phi}_{i+1}$ by (5).

where Viterbi algorithm can be applied. Finally, the quantized result in the (p, s)th branch is obtained as

$$\hat{\mathbf{f}}_{i,p,s} = \left[\hat{f}_{i,\tau,p,s}, \frac{\varepsilon_i \bar{\beta}_s \hat{\mathbf{f}}^T_{i,\bar{\tau},p,s}}{\left\| \hat{\mathbf{f}}_{i,\bar{\tau},p,s} \right\|_2} \right]^T, \tag{37}$$

and the overall optimum is obtained as

$$\hat{\mathbf{f}}_i = \min_p \min_s \left\| \mathbf{f}_i - e^{j\theta_p} \hat{\mathbf{f}}_{i,p,s} \right\|_2. \tag{38}$$

Notice that if QPSK or 8PSK are used as $\mathbf{\Xi}_M$, α_q in (36) can be dropped to reduce complexity. The Pseudo-code for the proposed differential quantization is summarized in Table II.

IV. NUMERICAL RESULTS

In this section, Monte-Carlo simulation is performed to evaluate the performance of the proposed algorithm. Both FOGM channel model and geometry-based standard channel model are adopted, and as a alternative in massive MIMO channel quantization, NTCQ in [8], [9] are compared. For each model, the block interval T_B is assumed as $T_B = T_f \lceil N_T / N_{\text{pilot}} \rceil$, where $\lceil a \rceil$ denotes the minimal integer not smaller than a. $N_{\text{pilot}} = 4$ is the maximum number of antenna that can be estimated in a subframe, and $T_f = 5$ [ms] is the subframe interval. The carrier frequency is fixed at $f_c = 2.4$ [GHz]. For simplicity, feedback delay is not considered. In massive MIMO, where $N_T = 100$, performance metric is set as the average beamforming gain in dB scale, defined as

$$J_{\text{avg}} = 10 \log_{10} \left(\mathbb{E} \left[\left| \mathbf{h}_i^H \hat{\mathbf{h}}_i \right|^2 \right] \right). \tag{39}$$

For FOGM channel, NTCQ with QPSK, NTCQ with 8PSK and differential NTCQ with QPSK are compared to the proposed differential quantization. In (2), η is given by Jakes

TABLE III
STANDARD MODEL PARAMETERS

Parameters	Value
Scenario	Urban micro-cell (UMi)
Propagation Condition	non-line-of-sight (NLOS)
Number of paths	19 [11]
Elevation AS of departure (EASD) [degree]	4 [10]
Azimuth AS of departure (AASD) [degree]	25.7 [11]
Power angular spectrum (PAS)	Wrapped Gaussian [11]
Azimuth Radiation direction [degree]	$[-60, 60]$
Elevation Radiation direction [degree]	$[-45, 45]$
Antenna spacing [m]	0.0625

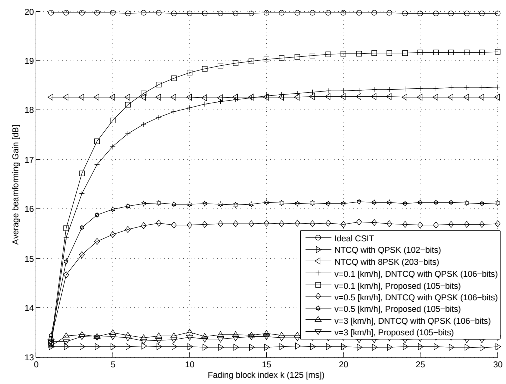

Fig. 2. J_{avg} as a function of fading block index. Proposed scheme with $M = 4, V = 4$ is compared to NTCQ and DNTCQ under different temporal correlation.

model, given as $\eta = J_0 (2\pi f_D T_B)$, where $J_0(\cdot)$ is the 0th order Bessel function, and f_D denotes the maximum Doppler frequency. $\mathbf{R}$ is assumed to be modeled as

$$\mathbf{R} = \begin{bmatrix} 1 & r & \cdots & r^{N_T-1} \\ r^\dagger & 1 & & \\ \vdots & & \ddots & \\ r^{(N_T-1)\dagger} & r & & 1 \end{bmatrix}, \tag{40}$$

where $|r| = 0.5$, and the phase of r is uniformly generated for 0 to 2π. We plot the average beamforming gain for one UT when $v_{\text{UT}} \in \{0.1, 0.5, 3\}$ [km/h], and $\eta \in \{0.9924, 0.8185, -0.0979\}$ consequently. In Fig. 2, basic NTCQ with QPSK and 8PSK, or differential NTCQ (DNTCQ) with QPSK are compared to the proposed quantization with QPSK. It is shown that the proposed scheme outperforms DNTCQ with almost the same overhead. After a few feedback intervals for "initialization", they can also outperform basic NTCQ with 8PSK in quasi-static channel as $v_{\text{UT}} = 0.1$ [km/h], which shows the effectiveness of exploiting channel correlation. Moreover, even when temporal correlation is low, as $v_{\text{UT}} = 3$ [km/h], their performance is not worse than basic NTCQ with the same overhead.

For standard models, we adopt ITU-R channel model [11] and WINNER+ channel model [10] to simulate the realistic transmission environment, where limited angular spread (AS) is often observed in channel measurements. 10×10 uniform planar array (UPA) is used, and parameters are summarized

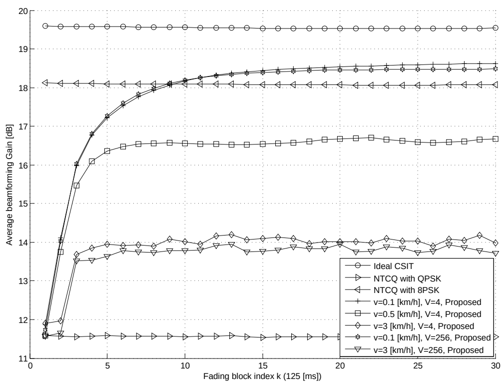

Fig. 3. J_{avg} as a function of fading block index. Proposed scheme with $M = 4$ is compared to NTCQ under realistic channel correlation.

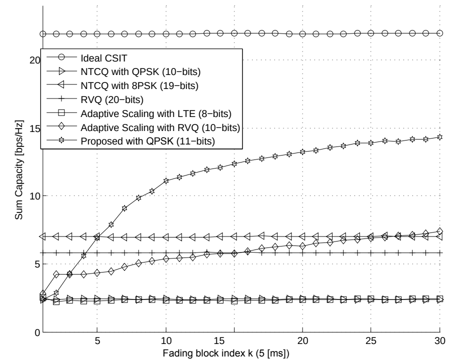

Fig. 4. Capacity of ZFP when element number of Tx array $N_T = K = 8$, UT moving speed $v = 0.1$ km/h and SNR = 25 dB.

in Table III. In Fig. 3, basic NTCQ with QPSK and 8PSK are compared to the proposed quantization with QPSK. For DNTCQ, η is not explicitly given here, and methods for estimation on η are missing in [9], so it is not compared as a alternative here. It is shown that in realistic environment, the proposed quantization method can effectively exploit the spatial correlation. It can be also observed that increasing V is not beneficial to the performance in the proposed scheme.

If small-scale array is considered, and MIMO zero-forcing precoding (ZFP) is applied, sum rate capacity is used as a performance metric, written as

$$C_{\text{ZFP}} = \sum_{k=1}^{K} \log_2 \left(1 + \frac{\frac{\rho}{K} \left| \bar{\mathbf{h}}_{k,i}^H \mathbf{v}_{k,i} \right|^2}{\frac{\rho}{K} \sum_{j=1, j \neq k}^{K} \left| \bar{\mathbf{h}}_{k,i}^H \mathbf{v}_{j,i} \right|^2 + 1} \right), \quad (41)$$

where K is the number of UTs, and $\mathbf{v}_{k,i}, k = 1, ..., K$ is a the ZFP beam vector selected as a unit vector orthogonal to the row vector subspace $\mathcal{S}_{k,i} = \text{span}\left\{ \hat{\mathbf{h}}_{j,i}^H : j \neq k \right\}$. Here suffixes k and j denote the UT indexes. In Fig. 4, a downlink transmission in realistic environment with $K = 8$ is simulated, where 4×2 PLA is used. The proposed method with QPSK is compared with RVQ [6], basic NTCQ [8] and adaptive skewing [5]. For the preceding techniques, it is shown that under pedestrian velocity, superiority of basic NTCQ with 8PSK is obvious against RVQ based method due to the minimum-ED maximization property. We can also observe that this superiority is even enlarged in the proposed quantization, as the performance reaches about 2/3 of maximal capacity in ideal CSIT. These demonstrate that the proposed scheme can also be applied in small-scale arrays.

V. Conclusions

In this paper, a practical differential quantization scheme for massive MIMO systems is designed. The complexity of this scheme scales linearly to the number of antennas, while adaptive skewing that needs no prior knowledge on channel correlation statistics can be performed on a designed DCB. Simulation results confirm that the proposed scheme outperforms preceding approach on differential NTCQ, and is effective in realistic propagation. It is a good candidate for future massive MIMO applications.

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