Power Control for Limited Feedback Precoding: Achievable SINRs and Optimal Capacity Analysis

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***Abstract*—This paper analyzes the performance of MIMO downlink broadcast channel employing precoding techniques, with imperfect channel state information at the transmitter (CSIT) due to limited feedback. We focus on two optimization issues that related to power control (PC) in precoding. Firstly we employ a distributed and competitive PC scheme for the interference-limited network, and derive two sufficient conditions of Nash Equilibrium, which are the upper bounds for achievable SINRs. Secondly, maximum sum capacity (SC) is investigated to show the maximal ability of PC in precoding. From Monte-Carlo simulation, it reveals that the capacity boost of non-linear Tomlinson-Harashima precoding (THP) with PC is significant as compared to its alternatives. Specially, the more users are served, the higher capacity gain is achieved. Maximally about 0.9 bps/Hz capacity gain can be achieved for THP, which is about 10.8% better than equal power allocation (EP).**

I. Introduction

Multiuser multiple-input multiple-output (MU-MIMO) has been regarded as a good potential to increase the overall downlink capacity of next-generation cellular network [1]. It has been proved that the maximal capacity of MIMO broadcast channel can be achieved by dirty paper coding (DPC) if the channel state information (CSI) is perfectly known both at the transmitter and user terminal (UT) [1], [2]. However, the requirement that the transmitter has accurate CSI is difficult to meet. In such limited feedback system, random vector quantization (RVQ) codebook has been proven to be a suboptimal and computational-affordable solution [3]. Still, the imperfect CSI at the transmitter (CSIT) will mislead the precoding, then cause co-channel interference (CCI) and make the network interference-limited. Increasing feedback bits will surely suppress CCI, but will cost more uplink resources. This dilemma has limited the implementation of MIMO precoding in a realistic cell.

If we accept the fact that the network is unavoidably interference-limited, the objective becomes finding a power control (PC) scheme that can balance CCI among UTs to achieve desired performance. Most of preceding researches simply used equal power setting (EP) [3], [4]. However, it is desirable to allocate varied power to different UTs based on their Quality-of-Service (QoS) requirements in practical systems. For this purpose, a distributed and fast converging PC scheme called Foschini-Miljanic (FM) algorithm is devised for interference-limited network [5], while all UTs competing for their required SINRs end up in a Nash Equilibrium [6]. However, the required SINRs of all UTs might not be satisfied because of the strength of CCI. Therefore, the conditions guaranteeing SINR achievable is of interest. Moreover, waterfilling [7] PC that aims to maximize the overall capacity, is not feasible when CCI exists. Therefore, as far as the authors know, no preceding research has been conducted to analyze the performance of limited feedback system when PC schemes are employed.

This paper investigates two issues of PC schemes for limited feedback precoding. The sufficient conditions of Nash Equilibrium are derived, providing achievable SINRs for UTs. As another goal in this paper, sum capacity (SC) maximized by PC is analyzed via a numerical method. Linear zero-forcing precoding (ZFP) and non-linear Tomlinson-Harashima precoding (THP) are studied comparatively based on numerical simulations, and results show that for the case of THP incorporating PC under limited feedback, capacity boost is up to 10.8% superior than EP scheme with 8 UTs.

The remainder of this paper is organized as follows. The system model is introduced in Section II before the sufficient conditions of Nash Equilibrium are presented in Section III. Section IV discusses the ability of PC about maximizing SC. Section V analyzes simulation followed by conclusion in Section VI.

Notation: Transposition and conjugate transposition are denoted by $(\cdot)^T$, $(\cdot)^H$, respectively. $\mathrm{diag}(\cdot)$ is diagonal matrix formed by a vector or a series of elements, while $\mathrm{diag}\{\cdot\}$ is the main diagonal of a matrix. $|\cdot|$ replaces the absolute value of a number. Expectation is $\mathbb{E}\{\cdot\}$, and $\|\cdot\|$ is used for Frobenius norm. $\Re(\cdot)$ denotes real part. Uppercase boldface letters stand for matrices and lowercase boldface ones represent vectors. Italic denotes scalar.

II. System Description and Precoding schemes

A. MU-MIMO Signal Model

Considering a MU-MIMO downlink system in an urban microcell (UMi) illustrated in Fig.1, where one base station (BS) equipped with N_T Tx antennas communicates with $K = N_T$ active UTs, each equipped with one antenna. The received signal for the kth UT can be modeled as

$$r_k = \sqrt{\rho_k}\mathbf{h_k}\mathbf{x} + n_k, \tag{1}$$

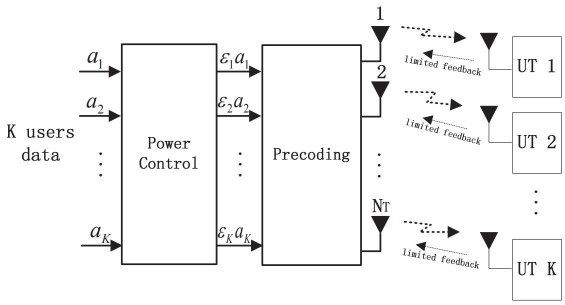

Fig. 1. MU-MIMO downlink broadcast network

where $\sqrt{\rho_k}$ denotes the path loss. $\mathbf{h}_k \in \mathbb{C}^{1\times N_T}$ is the flat Rayleigh fading channel vector from the BS to the kth UT. $\mathbf{x} = [x_1, ..., x_K]^T \in \mathbb{C}^{N_T\times 1}$ represents the transmitted vector for all UTs, where x_k is the kth transmitted symbol at the kth antenna. $n_k \sim \mathcal{CN}(0,1)$ is the additive white Gaussian noise at the kth UT.

B. Limited Feedback and Random Vector Quantization

Throughout this paper we assume perfect CSI at the receiver (CSIR) and imperfect CSIT. Following the studies of quantized CSI feedback in [3] and [8], channel direction vector is quantized at each UT, and the corresponding index is fed back to the BS. Define the quantization codebook $\mathbb{W} = \{\mathbf{w}_1, ..., \mathbf{w}_{N_c}\}$ with size $N_c = 2^{l_c}$, l_c is quantization length. $\mathbb{W}$ is known for both BS and all the UTs, and each UT chooses the feedback direction vector as

$$\hat{\mathbf{h}}_k = \arg\max_{\mathbf{w}_i \in \mathbb{W}} \left\{ \left|\bar{\mathbf{h}}_k \mathbf{w}_i\right|^2 \right\}, \tag{2}$$

where $\bar{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ is the channel direction vector of the kth UT. Using results in [3], the $\bar{\mathbf{h}}_k$ containing quantization error is modeled by

$$\bar{\mathbf{h}}_k = \hat{\mathbf{h}}_k \cos\theta_k + \tilde{\mathbf{h}}_k \sin\theta_k, \tag{3}$$

where $\cos\theta_k = \left|\bar{\mathbf{h}}_k \hat{\mathbf{h}}_k^H\right|$, and $\sin\theta_k = \left|\bar{\mathbf{h}}_k \tilde{\mathbf{h}}_k^H\right|$. $\tilde{\mathbf{h}}_k$ is a unit norm vector which is orthogonal to $\hat{\mathbf{h}}_k$ and independent of $\sin\theta_k$. $\sin^2\theta_k = 1 - \cos^2\theta_k$ is the quantization error with $\tilde{\mathbf{h}}_k$ as its direction vector.

C. PC Schemes for Precoding

In Fig.1, $\mathbf{a} = [a_1, ..., a_K]^T \in \mathbb{C}^{K\times 1}$ is the symbol vector for all UTs. Each a_k uses the same M-ary square constellation (M is squared number) and has average power (i.e.$\sigma_{a_k}^2 = 1$). $\mathbf{s} = [\varepsilon_1 a_1, ..., \varepsilon_K a_K]^T$ is the power loaded symbol vector where ε_k denotes the kth PC coefficient. Thus the power of each component in $\mathbf{s}$ is $\sigma_{s_k}^2 = \varepsilon_k^2$. Precoding with PC can be viewed as the mapping from $\mathbf{s}$ to $\mathbf{x}$, written as $\mathbf{x} = f(\mathbf{s})$. The vector of PC coefficients $\boldsymbol{\varepsilon} = [\varepsilon_1, ..., \varepsilon_k]^T$ is the key factor to realize PC. Details of ZFP and THP are shown below.

1) Zero-Forcing Precoding (ZFP): f is a linear operator in ZFP, written as $\mathbf{x} = \mathbf{V}\mathbf{s}$, where $\mathbf{V} \in \mathbb{C}^{N_T\times K}$ is a given precoding matrix comprising $\mathbf{v}_k \in \mathbb{C}^{N_T\times 1}$ for each UT. The beamvector $\mathbf{v}_k$ is selected as a unit vector orthogonal to the row vector subspace $\mathcal{S}_k = \mathrm{span}\{\hat{\mathbf{h}}_j : j \neq k\}$, where $\hat{\mathbf{h}}_j$ is the quantized feedback of the channel of the jth UT. Then the received signal is provided for the kth UT as

$$\begin{aligned} r_{k,ZFP} &= \sqrt{\rho_k}\mathbf{h}_k\mathbf{V}\mathbf{s} + n_k \\ &= \sqrt{\rho_k}\left\|\mathbf{h}_k\right\| \sum_{j=1}^{K} (\hat{\mathbf{h}}_k \cos\theta_k + \tilde{\mathbf{h}}_k \sin\theta_k)\mathbf{v}_j s_j + n_k \\ &= \sqrt{\rho_k}\left\|\mathbf{h}_k\right\| \varepsilon_k (\cos\theta_k \left|\hat{\mathbf{h}}_k\mathbf{v}_k\right| + \sin\theta_k\left|\tilde{\mathbf{h}}_k\mathbf{v}_k\right|) a_k \\ &\quad + \sqrt{\rho_k}\left\|\mathbf{h}_k\right\| \sin\theta_k \sum_{j\neq k} \left[\varepsilon_j \left|\tilde{\mathbf{h}}_k\mathbf{v}_j\right|\right] a_j + n_k. \end{aligned} \tag{4}$$

Let γ_k stand for the received SINR for the kth UT. Similar to [3], it can be expressed as

$$\gamma_{k,ZFP} = \frac{\rho_k \left\|\mathbf{h}_k\right\|^2 \varepsilon_k^2 (\cos^2\theta_k \left|\hat{\mathbf{h}}_k\mathbf{v}_k\right|^2 + \sin^2\theta_k \left|\tilde{\mathbf{h}}_k\mathbf{v}_k\right|^2)}{\rho_k \left\|\mathbf{h}_k\right\|^2 \sin^2\theta_k \sum\limits_{j\neq k} \left[\varepsilon_j^2 \left|\tilde{\mathbf{h}}_k\mathbf{v}_j\right|^2\right] + 1} \tag{5}$$

where $i, j \in \{1, ..., K\}$.

2) Tomlinson-Harashima Precoding (THP): The major characteristics of THP are pre-cancellation of interference and modulus operation in this pre-cancellation. The main process of this scheme can be summarized as follows:

Step 1: assuming $\hat{\mathbf{H}} = \left[\hat{\mathbf{h}}_1^T, ..., \hat{\mathbf{h}}_K^T\right]^T$ is the CSIT obtained via feedbacks. QR decomposed $\hat{\mathbf{H}}^T$ results in $\hat{\mathbf{H}} = \hat{\mathbf{R}}\hat{\mathbf{Q}}$ where $\hat{\mathbf{R}} = [R_{i,j}] \in \mathbb{C}^{K\times N_T}$ is a lower triangle matrix, $\hat{\mathbf{Q}} \in \mathbb{C}^{N_T\times N_T}$ is a unitary matrix. The feed-forward precoding matrix is set as $\mathbf{F} = \hat{\mathbf{Q}}^H$.

Step 2: The constellation is bounded by the square region of width 2τ, where $\tau = \sqrt{\frac{3M}{2(M-1)}}$. Let $\mathbf{y} = [y_1, ..., y_K]^T \in \mathbb{C}^{N_T\times 1}$ be the result of modulus operation performed in pre-cancellation. Define $\mathbf{d} = [d_1, ..., d_K]^T \in \mathbb{C}^{N_T\times 1}$ where $d_k \in \{2\varepsilon_k\tau\,(p_I + jp_Q)\,|p_I, p_Q \in \mathbb{Z}\}$ is properly selected to ensure $y_k \in \{\alpha + j\beta\,|\alpha, \beta \in (-\varepsilon_k\tau, \varepsilon_k\tau)\}$, then

$$\mathbf{y} = \hat{\mathbf{R}}^{-1}\boldsymbol{\Delta}^{-1}\mathbf{z}, \tag{6}$$

where $\boldsymbol{\Delta} = \mathrm{diag}(R_{1,1}^{-1}, ..., R_{k,k}^{-1})$, $\mathbf{z} = \mathbf{s} + \mathbf{d}$.

Step 3: $\mathbf{x} = \mathbf{F}\mathbf{y}$ is achieved.

Rewritten (1), the received signal is

$$r_{k,THP} = \sqrt{\rho_k}\mathbf{h_k}\mathbf{F}\mathbf{y} + n_k, \tag{7}$$

where the power of $\mathbf{y}$ can be accurately approximated as $\frac{M}{M-1}\mathrm{diag}\{\mathbb{E}\{\mathbf{s}\mathbf{s}^H\}\}$ due to the shaping loss in modulus operation. The received signal for the kth UT can be calculated as

$$\begin{aligned} r_{k,THP} &= \sqrt{\rho_k}\left\|\mathbf{h}_k\right\| \sqrt{\frac{M-1}{M}} R_{k,k}\cos\theta_k(\varepsilon_k a_k + d_k) \\ &\quad + \sqrt{\rho_k}\left\|\mathbf{h}_k\right\| \sin\theta_k \tilde{\mathbf{h}}_k \hat{\mathbf{Q}}^H \mathbf{y} + n_k. \end{aligned} \tag{8}$$

Similar to [4], the received SINR for the kth UT is

$$\gamma_{k,THP} = \frac{\frac{M-1}{M}\rho_k \left\|\mathbf{h}_k\right\|^2 \cos^2\theta_k \varepsilon_k^2 \left|R_{k,k}\right|^2}{\rho_k \left\|\mathbf{h}_k\right\|^2 \sin^2\theta_k \sum\limits_{j=1}^{K} \left[\varepsilon_j^2 \left|\tilde{\mathbf{h}}_k\mathbf{q}_j\right|^2\right] + 1}, \tag{9}$$

where $\mathbf{q}_j$ is the jth column of $\hat{\mathbf{Q}}^H$.

III. Upper Bounds For Achievable SINRs

In order to obtain the optimal balance between achievable SINRs and CCI, two sufficient conditions for Nash Equilibrium are derived in this section, which are the upper bounds of achievable SINRs. We start by defining the notations:

- *Power propagation channel matrix:* $\mathbf{A}$. $\mathbf{A}$ is a random constant $K \times K$ matrix having only nonnegative entries that are strictly positive along the diagonal. For clarity, we take $\mathbf{A} = \mathbf{T} + \mathbf{U}$, where the matrices $\mathbf{T} = \mathrm{diag}(t_{1,1}, t_{2,2}, ..., t_{K,K})$ and $\mathbf{U} = [u_{k,j}] \in \mathbb{R}^{K \times K}$ denote the signal power propagation and the interference power propagation, respectively.
- *Signal power of the kth UT:* ${\varepsilon_k}^2 t_{k,k}$.
- *Interference power of the kth UT from jth UT:* ${\varepsilon_j}^2 u_{k,j}$.
- *Received SINR of the kth UT :* $\gamma_k = \frac{{\varepsilon_k}^2 t_{k,k}}{\sum\limits_{j \neq k} {\varepsilon_j}^2 u_{k,j} + 1}$.
- *Required SINRs:* $\bar{\boldsymbol{\gamma}}$. The required SINRs from different UTs is $\bar{\boldsymbol{\gamma}} = [\bar{\gamma}_1, ..., \bar{\gamma}_K]^T$. $\bar{\gamma}_k$ inherently represents the QoS or data-rate requirement for the kth UT.
- *Normalized CCI level from the jth UT to the kth UT:* $\iota_{k,j} = \frac{u_{k,j}}{t_{k,k}}$.

As background, a useful Lemma guaranteeing stability of a matrix is needed:

Lemma 1 Let $\mathbf{M}$ be a square matrix and $\sigma(\mathbf{M})$ be its spectrum. The stability modulus of $\mathbf{M}$ is [9]

$$s(\mathbf{M}) = \max\{\Re(\lambda) : \lambda \in \sigma(\mathbf{M})\}, \tag{10}$$

and the logarithmic norm of $\mathbf{M}$ is defined by

$$\mu(\mathbf{M}) = \lim_{h \to 0^+} \frac{\|\mathbf{I} + h\mathbf{M}\| - 1}{h}, \tag{11}$$

then $s(M) \leq \mu(M)$.

Now upper bounds can be proved with this Lemma. In either theorem given as follows, the required SINR is achievable:

Theorem 1 (Equal power SINR constraint) For every UT in the pool, if the target SINR $\bar{\gamma}_k$ satisfies

$$\bar{\gamma}_k < \frac{1}{\sum\limits_j \iota_{kj}}, k \in \{1, ..., K\}, \tag{12}$$

then $\bar{\boldsymbol{\gamma}}$ is achievable. EP has been demonstrated to satisfy this case.

Theorem 2 (Maximum interference constraint) For every UT in the pool, if the normalized interference it caused to other UTs satisfies

$$\max_{k \in \{1,...,K\}} \{\bar{\boldsymbol{\gamma}}^T \boldsymbol{\iota}_k\} < 1, \tag{13}$$

where $\boldsymbol{\iota}_k = [\iota_{1,k} ..., \iota_{K,k}]^T$, then $\bar{\boldsymbol{\gamma}}$ is achievable.

Proof: Assuming $\bar{\boldsymbol{\gamma}}$ is achievable when the vector of PC coefficients $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^*$. It has been proven by FM algorithm that $\boldsymbol{\varepsilon}^*$ can be generated from

$$\boldsymbol{\varepsilon}^* = \mathbf{B}^{-1}\boldsymbol{\eta}. \tag{14}$$

where $\mathbf{B} = \mathbf{I} - \mathbf{T}^{-1}\mathrm{diag}\{\bar{\boldsymbol{\gamma}}\}\mathbf{U}$, $\boldsymbol{\eta} = \mathbf{T}^{-1}\bar{\boldsymbol{\gamma}}$. The step factor matrix $-\mathbf{B} = [b_{i,k}] \in \mathbb{R}^{K \times K}$ is generated as

$$b_{i,k} = \begin{cases} \frac{\bar{\gamma}_i u_{i,k}}{t_{i,k}} - 1 & \text{if } i = k \\ \frac{\bar{\gamma}_i u_{i,k}}{t_{i,k}} & \text{if } i \neq k \end{cases}. \tag{15}$$

To guarantee the SINR requirements $\bar{\boldsymbol{\gamma}}$ achievable, $-\mathbf{B}$ must be *stable*, which means the eigenvalues of $-\mathbf{B}$ must satisfy [9]

$$s(-\mathbf{B}) < 0. \tag{16}$$

Applying Lemma 1 with $M = -B$, the following expression is obtained

$$s(-\mathbf{B}) \leq \mu(-\mathbf{B}). \tag{17}$$

If the $\mu(-\mathbf{B})$ with respect to the two common logarithmic norms $|x|_\infty = \sup_i |x_i|$, $|x|_1 = \sum_i |x_i|$ satisfies

$$\begin{aligned} \mu_\infty(-\mathbf{B}) &= \sup_i \left(\Re(b_{ii}) + \sum_{k,k \neq i} |b_{ik}| \right) \\ &= \sup_i \left(\frac{\bar{\gamma}_i u_{i,i}}{t_{i,i}} - 1 + \sum_{k,k \neq i} \frac{\bar{\gamma}_i u_{i,k}}{t_{i,k}} \right) \\ &= \bar{\gamma}_i \sum_{k=1}^{K} \frac{u_{i,k}}{t_{i,k}} - 1 < 0, \forall i, \end{aligned} \tag{18}$$

i.e.Theorem 1 or

$$\begin{aligned} \mu_1(-\mathbf{B}) &= \sup_k \left(\Re(b_{kk}) + \sum_{i,i \neq k} |b_{ik}| \right) \\ &= \sup_k \left(\frac{\bar{\gamma}_k u_{k,k}}{t_{k,k}} - 1 + \sum_{i,i \neq k} \frac{\bar{\gamma}_i u_{i,k}}{t_{i,k}} \right) \\ &= \max_{k \in \{1,...,K\}} \sum_{i=1}^{K} \frac{\bar{\gamma}_i u_{i,k}}{t_{i,k}} - 1 < 0 \end{aligned} \tag{19}$$

i.e.Theorem 2, then $s(-\mathbf{B}) < 0$ can be ensured. (18) and (19) yield (12) and (13), respectively. This completes the proof. ■

It can be concluded from (12) and (13) that achievable SINR will only be affected by the quantization error and the collinearity of quantized MIMO channel vectors, but not by the Rayleigh fading or the distance between UTs and BS. Therefore, for the purpose of higher required SINRs, it is more efficient to decrease the CCI caused by quantization error rather than increasing the Tx power. Thus CCI is worthy of further observation. Based on (4) and (8), $t_{k,j}$ and $u_{k,j}$ are expressed in detail as

$$\left.\begin{aligned} t_{k,k} &= \rho_k \|\mathbf{h}_k\|^2 (\cos^2\theta_k \left|\hat{\mathbf{h}}_k \mathbf{v}_k\right|^2 + \\ &\qquad \sin^2\theta_k \left|\tilde{\mathbf{h}}_k \mathbf{v}_k\right|^2), \\ u_{k,j} &= \rho_k \|\mathbf{h}_k\|^2 \sin^2\theta_k \left|\tilde{\mathbf{h}}_k \mathbf{v}_j\right|^2, j \neq k, \\ u_{k,k} &= 0 \end{aligned}\right\} \text{for ZFP}, \tag{20}$$

and

$$\left.\begin{aligned} t_{k,k}&=\frac{M-1}{M}\rho_k\left\|\mathbf{h}_k\right\|^2\cos^2\theta_k\left|R_{k,k}\right|^2,\\ u_{k,j}&=\rho_k\left\|\mathbf{h}_k\right\|^2\sin^2\theta_k\left|\tilde{\mathbf{h}}_k\mathbf{q}_j\right|^2\end{aligned}\right\}\text{for THP.} \quad (21)$$

The normalized CCI levels can be obtained as

$$\iota_{ij,ZFP}=\frac{\sin^2\theta_i\left|\tilde{\mathbf{h}}_i\mathbf{v}_j\right|^2}{\cos^2\theta_i\left|\hat{\mathbf{h}}_i\mathbf{v}_i\right|^2+\sin^2\theta_i\left|\tilde{\mathbf{h}}_i\mathbf{v}_i\right|^2}, \quad (22)$$

where $i\neq j$ for ZFP, and

$$\iota_{ij,THP}=\frac{M}{M-1}\tan^2\theta_i\frac{\left|\tilde{\mathbf{h}}_i\mathbf{q}_j\right|^2}{\left|R_{i,i}\right|^2} \quad (23)$$

for THP.

IV. Power Control for sum capacity optimization

In order to evaluate the maximal ability of PC schemes in limited feedback system, SC is investigated in this section. The expression of SC is written as

$$C_{sum}=\sum_{k=1}^{K}\log_2(1+\gamma_k), \quad (24)$$

which is regarded as the function of γ_k in this paper. Notice that the Tx power is normalized here for comparative study. As mentioned before, waterfilling algorithm is no more adoptable as a result of CCI. Meanwhile, it is very difficult to get the analytical expression of the optimal PC scheme mathematically. Therefore, numerical method is used in searching the optimal PC coefficients, and most results are shown in Section V.

Since searching can be prohibitively large, particle swarm optimizer (PSO) is employed to reduce complexity. The PSO used here is based on the study of [10]. PSO is based on a social-psychological model of social influence and social learning. Improvements of PSO, such as velocity clamping, constriction coefficient, increasing neighbours, and combination with other intelligent methods, are proposed by preceding researches. These methods quite improve the speed of convergence and quality of solutions, and make PSO a mature technique in the field of convex optimization.

V. Simulation Analysis

It is concluded that higher required SINRs can be achieved with less CCI from (12) and (13). Thus the distribution of CCI is presented under assumptions of Rayleigh flat fading. In Fig.2, the cumulative distribution function (CDF) of the normalized CCI is plotted separately by offering $N_T=K=4$ and $N_T=K=8$ for both ZFP and THP schemes. Notice that for THP when $i\geq j$, we have $\lim_{\iota_0\to\infty}P(\iota_{i,j}<\iota_0)>0$ due to $\iota_{11,THP}=0$. Specially, the results of 0 dB for all schemes are worth being noticed because the CCI caused by the kth UT is almost as strong as its beamforming signal at that point. It can be viewed that ZFP is worse than THP with $i\geq j$ when

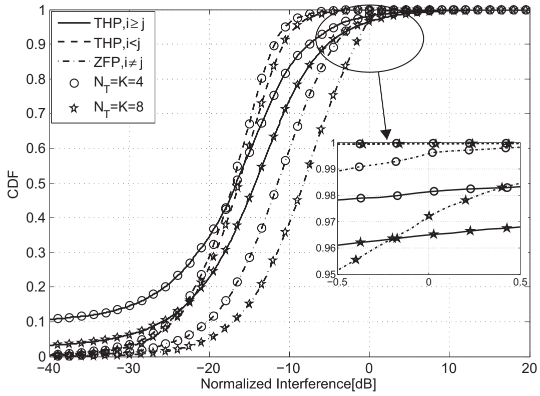

Fig. 2. CDF of the CCI when Tx antenna number is 4 ($l_c=16$) and 8 ($l_c=20$).

CCI is less than 0 dB. However, as to THP with $i\geq j$, the probability of CCI larger than 0 dB is higher over ZFP. In the case of THP with $i<j$, over 90% of CCI is less than -10 dB, implying that it is more beneficial to allocate more power to the UTs who have low ranking in the THP pre-cancellation. Still, there is a certain amount for CCI larger than 0 dB. This is caused by high occurrence of large collinearity between channel vectors of different UTs in high-order multiplexing. For the rest of our simulations, some user pairing procedure is introduced to eliminate this impact, so that the maximal channel vector collinearity between UTs satisfies

$$\max_{k\in 1,\ldots,K}\left[1-\left|\hat{\mathbf{h}}_k\mathbf{v}_k\right|^2\right]<\xi, \quad (25)$$

where ξ is a pre-given threshold given by 0.96.

Now we start to evaluate the SC performance when PC schemes are applied in limited feedback precoding. Alternative schemes are selected for comparative analysis: THP with EP (THP-EP), THP with PSO (THP-PC), ZFP with EP (ZFP-EP), ZFP with PSO (ZFP-PC). Monte-Carlo simulations are performed to evaluate the mean SC of all schemes on the DL bandwidth of each UT with 6 PRBs. Simulation parameters are listed in Table I [1] following LTE specification (Rel-10) [11]. Considering only single cell, hexagonal cell layout of Non-Line-of-Sight (NLOS) path loss model for scenario Urban Micro (UMi) is used [12]. In Fig.3, UT number is fixed to $K=8$. All UTs have the same distance to BS, and they are randomly distributed on a circle with BS as their center, whose radius is varying from 50 m to 150 m. Obviously, when PC scheme is applied, higher SC can be observed for both ZFP and THP, which proves the effectiveness of PC schemes. Maximally THP-PC is 1bps/Hz higher than THP-EP which is almost the same value holded by ZFP. The gap between EP and PC is maximized when distance (d_{BU}) is around 100 m, which infers that the SC gain achieved from PC may decrease at both high and low SINR regions.

[1] Details of Table I are provided as: Noise Power $=-174[dBm]+10\lg(180000\times 6)+9=-105[dBm]$; Path Loss $=36.7\lg(d_{BU}[m])+22.7+26\lg(2.4[GHz])=36.7\lg(d_{BU}[m])+32.6\ [dB]$; $10\lg\varepsilon_k-$ Path Loss $-$ Noise Power $=\mathrm{SNR}_k[dB]$.

TABLE I
SIMULATION PARAMETERS

Parameters	Setting
Antenna number (N_T)	8
Bandwidth of PRB	180 KHz
Number of PRBs	6
Noise Figure at UE	9 dB
Tx power	10 dBm
DL Path loss	$36.7 \lg (d_{BU}[m]) + 32.6$ dB
Quantization length (l_c)	20 bits
Fading	Rayleigh

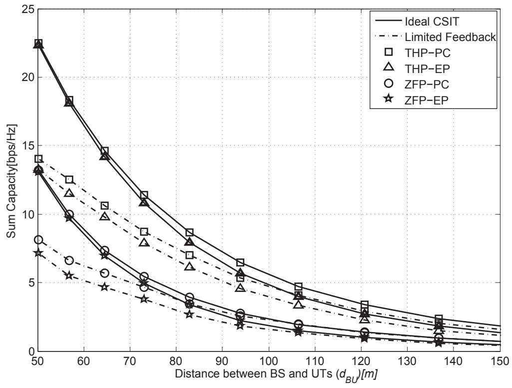

Fig. 3. Theoretical SC comparing among THP with EP (THP-EP), THP with PSO (THP-PC), ZFP with EP (ZFP-EP), ZFP with PSO (ZFP-PC) under ideal CSIT or limited feedback.

Fig.4 shows the improvement of SC when number of UTs increases from 2 to 8. The distance from each UT to BS is distributed uniformly on the interval of [50 m, 150 m] in this part. Similar improvement can be achieved when PC is applied. Wider gap can be achieved if more UTs are conducted, indicating that PC scheme is able to effectively deal with CCI and create an optimized balance in the interference-limited network. Furthermore, the superiority of THP is obvious, and combining with PC schemes can further increase the SC overall performance. Maximally about 0.9 bps/Hz capacity gain can be achieved by THP, which is about 10.8% better than EP.

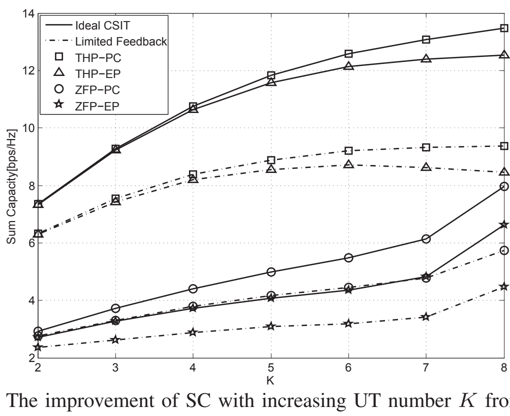

Fig. 4. The improvement of SC with increasing UT number K from 2 to 8.

VI. CONCLUSION

In this paper, we have investigated the advantages achieved by applying PC to precoding in limited feedback system. Two cases are investigated. Firstly, employing a competitive PC scheme, the upper bounds of achievable SINRs are obtained, while the assumption that power can be ultimately large is taken. Secondly, by employing numerical method into PC and normalizing total power, the maximal ability of PC in improving SC is analyzed. Based on Monte-Carlo simulations, it is concluded from CCI results that UT ranking lower in the THP pre-cancellation should be assigned more Tx power. As to the SC performance, expected results can be observed that PC outperforms EP significantly. Specifically, it can be seen that THP-PC is superior than others by capacity gain up to 10.8%.

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