Low-Complexity Energy-Efficient Power and Subcarrier Allocation in Cooperative Networks

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***Abstract*—This letter addresses the joint subcarrier and power allocation problem in downlink multiuser OFDM cooperative networks. The sum of the logarithmic per user time-averaged bits-per-Joule is maximized. The optimization problem is decomposed into two subproblems. First, the solution for the subcarrier allocation considering the proportional fairness is obtained. Second, an alternating power allocation (PA) scheme with separate transmit power constraints at the source and relays is derived under frequency selective fading channels. Then the complexity of the proposed scheme is analyzed. Finally, numerical results present the convergence, energy efficiency and outage probability performance of the proposed scheme.**

***Index Terms*—Cooperative network, energy-efficiency, subcarrier allocation, power allocation.**

I. INTRODUCTION

GREEN radios have drawn much attention in both academia and industry. The energy efficient resource allocation in single-hop cellular networks has been extensively studied in [1]-[4]. The schemes for optimizing the overall bits transmitted per Joule of energy have been discussed under flat and frequency selective fading channels.

Relaying transmission is viewed as one promising candidate for green radios. Relays bring mobile users closer to the network. It is deployed with flexibility and low cost. However, limited research about resource allocation in relay systems has been conducted for maximizing throughput per Joule. The maximum bits-per-Joule is investigated in [5] by optimizing the transmission time and the transmit power at each node for one-way relay transmission and two-way relay transmission with the direct transmission as the benchmark. But the system model is limited to the basic three-node relay network. A single relay selection and PA scheme to minimize the total transmit power is presented in [6]. But the tradeoff between the throughput and the transmit power consumption is not taken into account. The joint subcarrier, PA and relay selection scheme for maximizing energy efficiency per user in uplink OFDMA decode-and-forward (DF) relay networks is proposed in [7]. The fairness is further considered in [8]. But both of them have mainly focused on uplink scenarios or mobile terminal sides. The energy efficient resource allocation of relay systems in the downlink or base station sides is yet to be exploited for the green design target.

This letter addresses the subcarrier and power allocation problem under frequency selective fading channels in downlink multiuser OFDM cooperative networks. The optimization problem targeting at maximizing the time-averaged bits-per-Joule in the overall cooperative network with proportional fairness is decomposed into two subproblems. The analytical solution for each subproblem is obtained. The complexity of the joint subcarrier and power allocation scheme is also analyzed. Numerical results are presented to validate the convergence and performance of the proposed scheme.

II. SYSTEM MODEL

Consider a relay-enhanced OFDM downlink system, where one source S transmits signals to M destinations $(D_1, \cdots, D_M)$ with the help of N half-duplex amplify-and-forward (AF) relays $(R_1, \cdots, R_N)$. The serving user set of the n_{th} relay at time t is defined as $\Psi_n(t)$, which satisfies

$$\begin{array}{l}\overset{N}{\underset{n=1}{\cup}}\Psi_n(t)=\{1,2,\ldots,M\}\\ \Psi_i(t)\cap\Psi_j(t)=\emptyset,\forall i,j\in\{1,2,\ldots,N\}\ \text{and}\ i\neq j\end{array}. \tag{1}$$

The transmission from S→D covers two equal time slots. In the S→R link, a total number of K subcarriers are allocated to N relays orthogonally. The index set of all subcarriers is denoted as $\xi=\{1,2,\cdots,K\}$. In the R→D link, all destinations share the K subcarriers among relays in OFDMA mode. Each destination connects to one best relay at a time according to a predefined criterion. The signal is transmitted on the same subcarrier for the two hops. We assume independent fading among links and perfect synchronization among nodes. Then the instantaneous data rate on the k_{th} subcarrier at the m_{th} destination D_m at time t is given as

$$r_{m,k}(t) = \frac{1}{2}B\log_2\left(1+\frac{\eta_{s,k}(t)\eta_{m,k}(t)}{1+\eta_{s,k}(t)+\eta_{m,k}(t)}\right), \tag{2}$$

$$\eta_{s,k}(t) = \frac{p_{s,k}(t)|h_{s,k}(t)|^2}{N_0B}, \tag{3}$$

$$\eta_{m,k}(t) = \frac{p_{r,k}(t)|h_{m,k}(t)|^2}{N_0B}, \tag{4}$$

where we denote by $\eta_{s,k}(t)$ and $\eta_{m,k}(t)$ the signal-to-noise ratio (SNR) of S-R and R-D links at time t, respectively. The transmit power at S to its connected relay on the k_{th} subcarrier at time t is represented by $p_{s,k}(t)$. The transmit power of the relay on the k_{th} subcarrier at time t is represented by $p_{r,k}(t)$. We denote by $h_{s,k}(t)$ and $h_{m,k}(t)$ the channel gain on the k_{th} subcarrier from S to its connected relay and from this relay to D_m at time t accordingly. The noise power spectral density is denoted by N_0. The subcarrier bandwidth is denoted by B.

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III. PROBLEM FORMULATION

This letter focuses on maximizing the time-averaged bits-per-Joule to target at energy-efficiency. The time-averaged bits-per-Joule for D_m is defined as the ratio of the average data rate to the average power consumption[1], which is mathematically represented as

$$u_m\left(t\right)=\frac{R_m\left(t\right)}{P_m\left(t\right)}=\frac{\left(1-\frac{1}{T}\right)R_m\left(t-1\right)+\frac{1}{T}s_m\left(t\right)}{\left(1-\frac{1}{T}\right)P_m\left(t-1\right)+\frac{1}{T}p_m\left(t\right)}, \tag{5}$$

where the window length is denoted by T and $R(0)=0$. We denote by $s_m\left(t\right)$ the instantaneous data rate of D_m at time t, which is formulated as

$$s_m\left(t\right)=\sum_{k=1}^{K}\rho_{m,k}(t)r_{m,k}\left(t\right), \tag{6}$$

where we denote by $\rho_{m,k}(t)$ the subcarrier allocation indicator, which satisfies $\rho_{m,k}(t)=1$ if subcarrier k is allocated to D_m at time t and $\rho_{m,k}(t)=0$, otherwise.

The instantaneous power consumption at time t is represented as $p_m\left(t\right)$. It is formulated as

$$p_m\left(t\right)=\sum_{k=1}^{K}\rho_{m,k}(t)\left(p_{s,k}(t)+p_{r,k}(t)\right)+p_{cm}(t), \tag{7}$$

where the circuit power consumed by the device electronics of D_m is denoted by $p_{cm}(t)$. It is user and time dependent.

On the basis of the constraints in (2)-(7), the power and subcarrier allocation problem aiming at the energy efficiency in the overall cooperative network considering the proportional fairness is formulated as

$$\max_{p_{s,k}(t),p_{r,k}(t),\rho_{m,k}(t)} U\left(t\right)=\sum_{m=1}^{M}\log u_m\left(t\right), \tag{8}$$

$$\text{s.t.}\quad \sum_{m=1}^{M}\rho_{m,k}(t)=1,\forall k\in\{1,2,\cdots,K\}, \tag{9}$$

$$\sum_{k=1}^{K}p_{s,k}(t)\leq P_s, \tag{10}$$

$$\sum_{k=1}^{K}\sum_{m\in\Psi_n(t)}\rho_{m,k}(t)p_{r,k}(t)\leq P_{r,n},\forall n\in\{1,2,\cdots,N\}. \tag{11}$$

The constraint (9) implies that each subcarrier can only be assigned to one destination in the two-time-slot period. Then constraints (10) and (11) limit the maximum transmit power at the source and the n_{th} relay, respectively. The sum of the logarithmic per user time-averaged bits-per-Joule indicates that the optimization solution is proportionally fair.

IV. THE PROPOSED SUBCARRIER AND POWER ALLOCATION SCHEME

It is observed from (8)-(11) that the optimization problem is a mixed-integer non-linear programming. Considering that the computational complexity of finding the global optimum by joint optimization is prohibitive, the optimization is decomposed into two subproblems.

A. Subcarrier Allocation

Due to the high computational complexity of exhaustive search, a low-complexity subcarrier allocation scheme will be demonsrated in this subsection. $U\left(t-1\right)$ represents the previous overall energy efficiency at time $t-1$. It has been a fixed constant at time t. Thus maximizing $U\left(t\right)$ is equivalent to maximizing $U\left(t\right)+U\left(t-1\right)$, which is expressed as

$$\begin{aligned}&U\left(t\right)+U\left(t-1\right)\\&=\sum_{m=1}^{M}\log u_m\left(t\right)+\sum_{m=1}^{M}\log u_m\left(t-1\right)\\&=\sum_{m=1}^{M}\log\left(\frac{R_m(t)}{P_m(t)}\frac{R_m(t-1)}{P_m(t-1)}\right)\end{aligned}. \tag{12}$$

Then substituting (5) into (12), $U\left(t\right)+U\left(t-1\right)$ is reformulated as

$$\begin{aligned}&U\left(t\right)+U\left(t-1\right)\\&=\sum_{m=1}^{M}\log\left[\frac{\left(1-\frac{1}{T}\right)R_m^2(t-1)+\frac{1}{T}R_m(t-1)\sum_{k=1}^{K}\rho_{m,k}(t)r_{m,k}(t)}{P_m(t)P_m(t-1)}\right],\\&\approx\sum_{m=1}^{M}\log\left[\frac{(1-\frac{1}{T})R_m^2(t-1)}{P_m^2(t-1)}+\frac{\frac{1}{T}R_m(t-1)}{P_m^2(t-1)}\sum_{k=1}^{K}\rho_{m,k}(t)r_{m,k}(t)\right]\end{aligned} \tag{13}$$

where the approximation is tight for $T\gg 1$.

It is observed that the first part of (13) is a constant at time t. Thus the maximization of $U\left(t\right)$ can be reduced to the allocation of $\rho_{m,k}(t)$ and $r_{m,k}\left(t\right)$. Since the logarithmic function is monotonically increasing, $U\left(t\right)$ is maximized by assigning each subcarrier to the destination achieving the largest $r_{m,k}\left(t\right)$, which is mathematically represented as

$$\rho_{m,k}^{*}(t)=\arg\max_{\rho_{m,k}(t)} r_{m,k}\left(t\right). \tag{14}$$

The allocation of $\rho_{m,k}(t)$ should also obey the constraints in (9) and (11). Specifically, each subcarrier should be allocated to only one destination during the two-time-slot transmission period. No more subcarriers should be assigned to a relay if its total transmit power reaches the upper limit. Then the index set of subcarriers assigned to D_m is derived as

$$\Omega_m^{*}=\left\{k\left|r_{m,k}\left(t\right)>r_{n,k}\left(t\right),\forall m\neq n\right.\right\},\forall m. \tag{15}$$

B. Power allocation

Suppose that the channel information is known for each optimizing node. First, we investigate the optimal transmit PA over subcarriers at the source assuming a given (e.g., uniform) transmit PA at relays. Second, we investigate the optimal transmit PA over subcarriers at relays assuming a given transmit PA at the source. Third, an alternating separate optimization of relays and source is proposed. Before demonstrating the algorithm, the definitions of superlevel set and quasi-concave function are given:

Definition 1: The α-superlevel set of a function $f:\mathbf{R}^n\rightarrow\mathbf{R}$ is defined as

$$S_\alpha=\left\{x\in\operatorname{dom}f\left|f(x)\geq\alpha\right.\right\}. \tag{16}$$

Definition 2: A function $f:\mathbf{R}^n\rightarrow\mathbf{R}$ is called quasi-concave if its domain and all its superlevel sets S_α for $\alpha\in R$ are convex.

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[1] The average data rate $R_m\left(t\right)$ and the average power consumption $P_m\left(t\right)$ at time t are obtained using the exponentially weighted low-pass filter. The value of $R_m\left(t-1\right)$ and $P_m\left(t-1\right)$ can be obtained at the end of time $t-1$. They are updated after each transmission time [2].

Based on the above definitions, authors in [2] prove that:

Theorem: $u_m(t)$ is a function of $r_{m,k}(t)$. The superlevel sets $S_\alpha = \{\mathbf{r}_m(t)\,|u_m(t) \geq \alpha, \alpha \in \forall R\}$ are strictly convex, where $\mathbf{r}_m(t) = [r_{m,k_1}(t), r_{m,k_2}(t), \cdots, r_{m,k_c}(t)]$. $c = |\Omega_m(t)|$ denotes the number of elements in the subcarrier set allocated to D_m and $\Omega_m(t) = \{k_i\,|k_1 < k_2 < \cdots < k_c\} \subseteq \xi$. ξ is the index set of all subcarriers. Then $u_m(t)$ is a strictly quasi-concave function optimized on a convex set $\mathbf{r}_m(t)$ and a unique globally optimal rate vector $\mathbf{r}^*(t)$ exists and if $r_{m,k}(t) > 0$, every element in $\mathbf{r}^*(t)$ satisfies

$$\frac{\partial u_m(t)}{\partial r_{m,k}(t)} = 0. \tag{17}$$

1) Optimization of Source PA: Since we assume that the transmit power at relays are given, only the transmit power at the source will be optimized such that the time-averaged bits-per-Joule for each destination is maximized. According to the theorem and substituting (5) into (17), the optimal rate condition is derived as

$$\frac{\partial p_{s,k}(t)}{\partial r_{m,k}(t)} = \frac{1}{u_m(t)}. \tag{18}$$

When $T \geq 1$, $P_m(t) \approx P_m(t-1)$ and $R_m(t) \approx R_m(t-1)$. Then (18) can be approximated as

$$\frac{\partial p_{s,k}(t)}{\partial r_{m,k}(t)} \approx \frac{P_m(t-1)}{R_m(t-1)} = \frac{1}{u_m(t-1)}. \tag{19}$$

In order to decrease the computation complexity, $r_{m,k}(t)$ in (2) is approximated by the harmonic mean of the SNRs in the S-R and R-D links as [9]

$$r_{m,k}(t) \approx \frac{1}{2} B \log_2\left(1 + \frac{\eta_{s,k}(t)\eta_{m,k}(t)}{\eta_{s,k}(t) + \eta_{m,k}(t)}\right). \tag{20}$$

Then combining (20) and (3), the relationship between $r_{m,k}(t)$ and $p_{s,k}(t)$ is derived as

$$p_{s,k}(t) = \frac{N_0 B}{|h_{s,k}(t)|^2} \cdot \frac{\left(2^{\frac{2r_{m,k}(t)}{B}} - 1\right)\eta_{m,k}(t)}{\eta_{m,k}(t) - 2^{\frac{2r_{m,k}(t)}{B}} + 1}. \tag{21}$$

Then taking partial derivative of $p_{s,k}(t)$ with respect to $r_{m,k}(t)$ and substituting the result into (19), we obtain

$$A_1 x^2 - [2A_1(1 + \eta_{m,k}(t)) + 1]\,x + A_1(1 + \eta_{m,k}(t))^2 = 0, \tag{22}$$

where $x \triangleq 2^{\frac{2r_{m,k}(t)}{B}}$ and $A_1 \triangleq \frac{|h_{s,k}(t)|^2}{2 \cdot \ln 2 \cdot N_0 \cdot (\eta_{m,k}(t))^2 \cdot u_m(t-1)}$. From (21), it is seen that x is constrained within $[1, 1 + \eta_{m,k}]$. Then only one root of this quadratic function is satisfied, i.e.,

$$x = 1 + \eta_{m,k}(t) + \frac{1}{2A_1} - \frac{\sqrt{1 + 4A_1 + 4A_1\eta_{m,k}(t)}}{2A_1}. \tag{23}$$

Substituting (23) into (21), the optimal transmit power at the source on the k_{th} subcarrier is derived as

$$p^*_{s,k}(t) = \left[\frac{N_0 B \eta_{m,k}(t)}{|h_{s,k}(t)|^2} \cdot \frac{1 + 2A_1\eta_{m,k}(t) - \sqrt{1 + 4A_1 + 4A_1\eta_{m,k}(t)}}{\sqrt{1 + 4A_1 + 4A_1\eta_{m,k}(t)} - 1}\right]^+, \tag{24}$$

where $[x]^+ = \max(0, x)$.

2) Optimization of Relay PA: Now it is assumed that the transmit power at the source is given, the transmit power at relays will be optimized such that the time-averaged bits-per-Joule for each destination is maximized. The optimization objective is symmetric with respect to the transmit power at the source and relays. Thus similar to the derivation steps of the optimal transmit power at the source, the optimal transmit power at relays is derived as

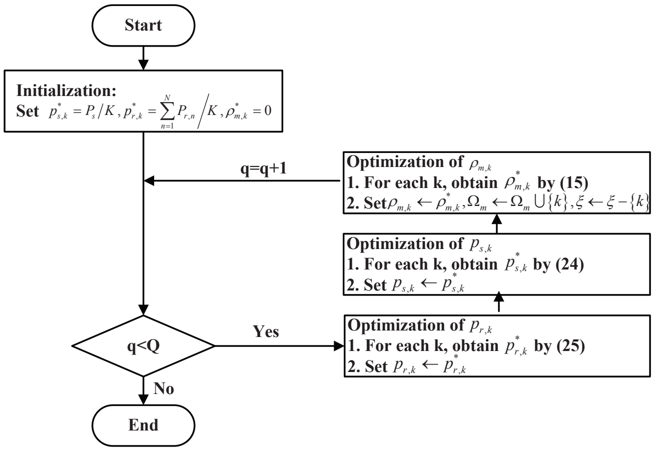

Fig. 1. Flowchart of the proposed scheme.

$$p^*_{r,k}(t) = \left[\frac{N_0 B \eta_{s,k}(t)}{|h_{m,k}(t)|^2} \cdot \frac{1 + 2A_2\eta_{s,k}(t) - \sqrt{1 + 4A_2 + 4A_2\eta_{s,k}(t)}}{\sqrt{1 + 4A_2 + 4A_2\eta_{s,k}(t)} - 1}\right]^+, \tag{25}$$

where $A_2 \triangleq \frac{|h_{m,k}(t)|^2}{2 \cdot \ln 2 \cdot N_0 \cdot (\eta_{s,k}(t))^2 \cdot u_m(t-1)}$.

3) Alternate Optimization of Source and Relay PA: At the beginning of this alternate optimization, the transmit power on each subcarrier at the source and relays is usually uniformly distributed considering the convergence requirement. Then the optimal solutions in (24) and (25) can be repeated alternately such that the output of the previous optimal solution is the input of the next one.

The final solution can be obtained after Q times iterations between these subproblems as demonstrated in Fig. 1. The complexity of finding the optimal solution by exhaustive search is more than $O(2M^K K \log K)$, which increases exponentially with the number of subcarriers and users. But the complexity of the proposed scheme is $O[Q(2MK + K \log K + \sum_{i=1}^{N} K_i \log K_i)]$, which only increases linearly. K_i is the number of subcarriers allocated to the i_{th} relay.

V. NUMERICAL RESULTS

In this section, the convergence and the performance of the proposed scheme is investigated. In the simulations, a single cell with radius $600m$ is considered. Relays are deployed with equal angle spacing on a circle with $300m$ radius centered at the source. The destinations are randomly distributed within the cell. The nearest relay is selected for each destination at the beginning of each simulation. Each transmitted signal suffers from large-scale path loss proportional to d^{-2}, where d denotes the distance between two nodes. A system with 64 subcarriers is considered. The subcarriers experience independent identically distributed Rayleigh fading with unit average power gain. The noise power spectral density is

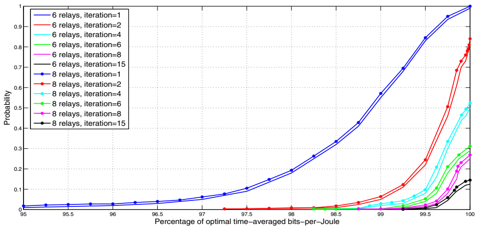

Fig. 2. Convergence of the proposed scheme ($M = 8$).

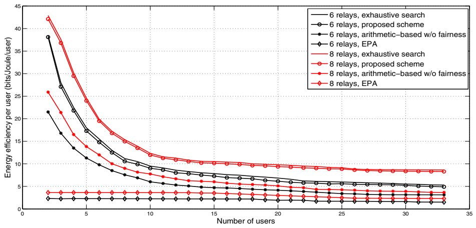

Fig. 3. Energy efficiency performance ($Q = 15$).

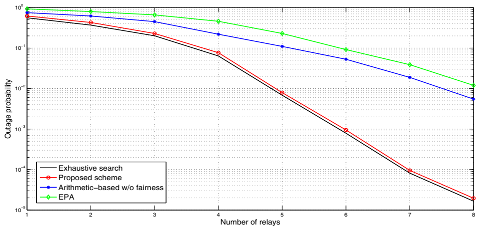

Fig. 4. Outage probability performance ($M = 8$, $R_t = 2$kbps and $Q = 15$).

$N_0 = 1.4 \times 10^{-5}$. The system bandwidth is 960kHz. The window length is $T = 100$.

Fig. 2 shows the cumulative distribution function (CDF) of the iterative-based time-averaged bits-per-Joule normalized by the optimal one under 6 and 8 relays. The optimal value is obtained by numerical optimization. One iteration is defined as the procedure of one optimization in (24) and (25) accordingly. It reveals that as the number of iterations increases, the relative error between the optimal value and the iterative-based one decreases. When the number of iteration is 4, almost 50% of the optimal value is achieved. When the number of iterations increases to 15, nearly 90% of the optimal value is achieved. The lower bound of all CDFs exists, which increases with the number of iterations. Therefore, it is concluded that the proposed scheme is convergent. Moreover, the proposed scheme converges faster when the number of relays is smaller.

Fig. 3 compares the energy efficiency performance of the proposed scheme with three other schemes over different number of users under 6 and 8 relays. The global optimum is obtained by exhaustive search. The arithmetic-based scheme maximizes the sum of the per user time-averaged bits-per-Joule without fairness consideration. The equal power allocation (EPA) scheme allocates equal transmit power to each subcarrier and assigns each subcarrier to the destination with the maximum data rate. It is observed that the proposed scheme achieves similar performance to the exhaustive search but with low-complexity. Thus the approximations in our derivation are reasonable. It can be seen that the proposed scheme outperforms the arithmetic-based one by 40% approximately when the number of users is less than 10. The performance of EPA is the worst. Moreover, the energy efficiency of the network under 8 relays is better than that under 6 relays. The increasing number of relays provides higher diversity gain. By deploying more relays in the network, destinations have more chances to choose a link with better channel conditions. But the cost is a relatively slower convergence as shown in Fig. 2.

Fig. 4 depicts the outage probability of the above four schemes over different number of relays. The outage probability is defined as the number of destinations whose achievable data rate is below the target rate R_t. The outage probability of the proposed scheme is smaller than EPA and arithmetic-based scheme. It approaches to exhaustive search closely and decreases rapidly as the number of relays increases.

VI. CONCLUSION

In this letter, the energy efficient subcarrier and power allocation problem to maximize the time-averaged bits-per-Joule in downlink OFDM cooperative systems was formulated. The problem was decomposed into subcarrier allocation and alternative optimization of PA at source and relays. The analytical solution for each subproblem was derived. The simulations have demonstrated that the proposed scheme is convergent with low complexity. It has superior energy efficiency and low outage probability.

REFERENCES

[1] Y. Wang, W. Xu, K. Yang, and J. Lin, "Optimal energy-efficient power allocation for OFDM-based cognitive radio networks," *IEEE Commun. Lett.*, vol. 16, no. 9, pp. 1420–1423, Sept. 2012.

[2] G. Miao, N. Himayat, G. Y. Li, and S. Talwa, "Low-complexity energy-efficient OFDMA," in *Proc. 2009 IEEE ICC,* pp. 1–5.

[3] X. Chen, X. Xu, and X. Tao, "Energy efficient power allocation in generalized distributed antenna system," *IEEE Commun. Lett.*, vol. 16, no. 7, pp. 1022–1025, July 2012.

[4] Q. Shi, W. Xu, D. Li, Y. Wang, X. Gu, and W. Li, "On the energy efficiency optimality of OFDMA for SISO-OFDM downlink system," *IEEE Commun. Lett.*, vol. 17, no. 3, pp. 541–544, Mar. 2013.

[5] C. Sun and C. Yang, "Energy efficiency analysis of one-way and two-way relay systems," *EURASIP J. Wireless Commun. Networking,* vol. 2012, no. 46, pp. 1–18, Feb. 2012.

[6] M. Zhou, Q. Cui, R. Jantti, and X. Tao, "Energy-efficient relay selection and power allocation for two-way relay channel with analog network coding," *IEEE Commun. Lett.*, vol. 16, no. 6, pp. 816–819, June 2012.

[7] C. Y. Ho and C.-Y. Huang, "Energy efficient subcarrier-power allocation and relay selection scheme for OFDMA-based cooperative relay networks," in *Proc. 2011 IEEE ICC,* pp. 1–6.

[8] Y. Jiang, J. Zhang, X. Li, and W. Xu, "Energy-efficient resource optimization for relay-aided uplink OFDMA systems," in *Proc. 2012 IEEE VTC – Spring,* pp. 1–5.

[9] M. O. Hasna and M. S. Alouini, "Performance analysis of two-hop relayed transmissions over Rayleigh fading channels," in *Proc. 2002 IEEE VTC – Spring,* vol. 4, pp. 1992–1996.