# Multiuser Access in Distributed Multichannel Cognitive Radio Systems

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***Abstract*—In this paper, we investigate a novel slotted ALOHA-based distributed cognitive network in which a secondary user (SU) selects a *random subset* of channels for sensing, detects an idle (unused by licensed users) subset therein, and transmits in any one of those detected idle channels. First, we derive a range for the number of channels to be sensed per SU. Based on that, the analytical average system throughput is derived in both saturation and non-saturation networks. Second, the relationship between the average system throughput and the number of sensing channels is attained. We show that the optimal number of sensed channel in a given number of SUs is dependent on the number of licensed channels, the number of idle channels, and the transmission probability of each SU. Finally, the analytical results are validated by substantial simulations.**

## I. Introduction

The surge in demand for high bandwidth applications on mobile devices is driving the current imperative for either additional spectrum allocations and/or more efficient use of existing ones. Cognitive radios (CR) [1]–[3] performing dynamic spectrum access seems to be a natural pathway for realizing improved spectrum efficiencies by allowing secondary users on hitherto licensed spectrum. Utilization of licensed band imposes the constraint of sensing channel availability for opportunistic access by secondary users (SUs), without imposing inadmissible interference to primary users (PUs).

Coexistence of SUs and PUs may be achieved using either a scheduled (centralized) mechanism or a distributed scheme. For a scheduled mechanism, a central control channel is necessary to schedule SUs on spectrum sensing and packet accessing. Among the former, Cordeiro *et al.* [4] presented a Cognitive Media Access Control (MAC) protocol over the vacant TV broadcasting spectrum based on instructions from base station and analyzed efficiency improvement of the SU in terms of throughput. Hamdaoui *et al.* [5] proposed an opportunistic spectrum MAC protocol through periodically listening to a control channel. However, the works presented above need central control channels among the SUs and PUs, this will be difficult to be applied in distributed systems. For a distributed access, existing serial or random sensing mechanisms are "*classic sensing strategy* (CSS)", where each SU serially or randomly searches the channels within the spectrum until it detects an idle channel. In [6] and [7], the authors proposed a cross-layer based opportunistic multi-channel MAC protocol with CSS in SUs coordination and analyzed the throughput. Liang *et al.* [8] studied the problem of designing the sensing slot duration to maximize the achievable throughput for the multiple distributed SUs with cooperative sensing. In terms of coordination among SUs, an extra control channel is necessary for above two works. Choe *et al.* [9] and [10] provided analytical results for throughput of a slotted ALOHA-based multi-channel CR system using CSS without the control channel, but under the assumption of the infinite SUs in the network.

We generalize these efforts by allowing each of the finite number of SUs to select a fixed number of licensed channels for sensing and undertake packet transmission in any one of the detected idle channels among that subset. This sensing method is termed here as "*extended sensing strategy* (ESS)". Based on ESS, two significant problems become open for further investigation. One is the range of the number of channels for sensing by each SU. The other concern is the tradeoff between the number of channels to be sensed and the system throughput. In other words, how many channels are needed to be sensed in order to achieve an optimal system throughput? Thus, in this paper, we consider the ESS applied in a slotted ALOHA-based multi-channel random access CR system without the central controller. The major contributions of this paper are:

- *Obtain the range of choosing the channels for sensing and the analytical system throughput expression*;
- *Derive a relationship between the average system throughput and the number of sensing channels,* $N_s$, and attain the optimal $N_s$ for throughput maximization.

The rest of the paper is organized as follows. Section II briefly discusses the system model of multi-user multi-channel cognitive access. In Section III, an analytical derivation of the average throughput based on the ESS is provided. Section

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IV deals with the optimization problem with respect to the number of sensed channels $N_s$. Section V discusses the average throughput achieved by comparing the simulation results with those obtained from our theoretical analysis. Section VI concludes the paper.

## II. System Model

As shown in Fig. 1, we consider a licensed spectrum of $N$ channels, out of which $M$ random channels are idle or unoccupied by the PUs. Further, there are $K$ SUs synchronized with the PUs and are opportunistically accessing the idle channels without imposing interference to the PUs. Multiple users are accessing the spectrum using access frames (AFs) of fixed durations. In one AF, the time, $T_{\text{AF}}$, for an SU is divided into two parts namely, sensing time slot, $T_S$, and packet transmission time slot, $T_P$, depicted in Fig. 2.

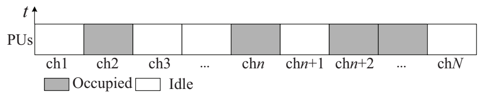


Fig. 1. Channel states of PUs in one Slot

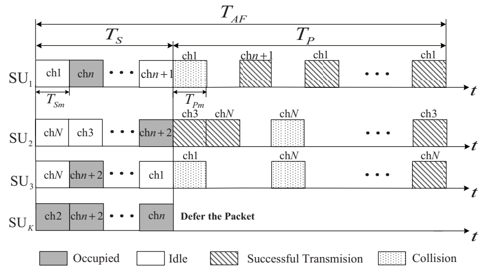


Fig. 2. Multiple access frame for SUs based on the channel states in Fig. 1

### A. ESS in one AF

In the sensing slot, we assume that each SU accomplishes sensing one licensed channel in one unit time slot, $T_{Sm}$, within $T_S$. $N_s$ channels will be randomly selected from the $N$ channels in sequence and further detected by each SU. Thus, the total time for sensing will be $T_S = T_{Sm} N_s$. Due to the absence of the central control channel, one SU will not know which channels are sensed by the other SUs. The detection of each individual channel status as busy/idle is subject to (occasional) error, determined by the probability of (correct) detection of an idle channel $P_d$ and probability of false alarm $P_f$ (probability of falsely declaring a busy channel as idle). In this work, we assume *ideal* detection, *i.e.,* $P_d = 1$ and $P_f = 0$ for simplicity, and defer the sensitivity assessment to future work. This allows us to focus for now on the impact of parameter selection for the core features of *ESS*.

### B. MAC for Packet Transmission in one AF

In the packet transmission slot, slotted ALOHA is applied for packets transmission to multiple idle channels as shown in Fig. 2. Assume each SU transmits a packet independently with the probability $p_{\text{tra}}$ on any idle channel which is included in the $N_s$ sensed channels, at the beginning of each unit time slot, $T_{Pm}$, within $T_P$. Thus, the number of transmitted packets in one slot, $U$, is a random variable. Since there is no coordination among SUs when the packets access the selected idle channels , a collision will happen if more than one packet is transmitted to the same idle channel in one time slot (*e.g.*, both $\text{SU}_1$ and $\text{SU}_3$ transmitting packets in ch1 in the first time slot in Fig. 2). Moreover, an SU (*e.g.,* $\text{SU}_K$ in Fig. 2) will 'hold' its packet if there is no idle channel detected within the $N_s$ sensed channels. Both of the above situations are viewed as the *failed packet transmission*. Correspondingly, the *successful packet transmission* of an SU in one time slot occurs when two conditions are satisfied simultaneously:

- The SU has a packet transmitted on one idle channel which is randomly selected from the detected idle channels in $T_S$;
- No packet from other SUs accesses the selected idle channel in this time slot.

Further, each SU will know about the success and failure after its transmission. The collided packet will be retransmitted in the further time slot within $T_P$ and the hold packet will be transmitted until the SU detects the idle channels in the following $T_{\text{AF}}$. A new packet will be generated after a successful packet transmission at an SU, which means an SU always has at least one packet in its buffer. Simultaneously, we assume that the states of all the PUs' channels will be stable within one $T_{\text{AF}}$.

Consequently, we have,

$$T_{\text{AF}} = T_S + T_P = N_s T_{Sm} + N_p T_{Pm}, \tag{1}$$

where we let $T_{Pm} = \eta T_{Sm}, \eta > 1$. For higher throughput systems where the transmission time far exceeds the sensing time, $N_p >> N_s$ is typical.

## III. Throughput Analysis based on "Extended Sensing Strategy"

According to *Section II*, one packet can only be transmitted to a subset of these $M$ idle channels in one $T_{\text{AF}}$ and this subset is different for different SUs. Thus, the range of $N_s$ will affect the probability of one given idle channel selected by one packet for transmission and the number of packets in successful transmission.

### A. Range of Choosing $N_s$ Channels for Sensing

*Lemma 1:* The probability, $p_{\text{sac}}^{(i)}$, of the $i^{\text{th}}$ idle channel (any a given idle channel) selected for access by one packet will be

$$p_{\text{sac}}^{(i)} = \begin{cases} \frac{1}{M}\left(1 - \frac{(N-N_s)^{[M]}}{N^{[M]}}\right), & \text{if } N_s \le N - M, \\ \frac{1}{M}, & \text{if } N_s \ge N - M + 1, \end{cases} \tag{2}$$

and $p_{\text{sac}}^{(i)} \in [1/N, 1/M]$.

*Proof:* Please refer to *Appendix* for the derivation of $p_{\text{sac}}^{(i)}$ in order to arrive at (2) and the proof for monotonic property. ■

From *Lemma 1*, $p_{\text{sac}}^{(i)}$ is not related to $i$ according to (2), which implies that $p_{\text{sac}}^{(1)} = p_{\text{sac}}^{(2)} = \ldots = p_{\text{sac}}^{(M)}$. we can simplify $p_{\text{sac}}^{(i)}$ to $p_{\text{sac}}$. Moreover, $p_{\text{sac}}$ increases with $N_s$ if $N_s \le N - M$ and be constant and independent with $N_s$ if $N_s \ge N - M + 1$.

### *B. Throughput Analysis in Saturation and Non-saturation Networks*

Based on the result in *Lemma 1*, we will derive the average number of packets in successful transmission, when there are $U$ packets transmitted from the SUs. Here, we firstly treat $U = K$ as a constant, which means each SU is in saturation.

*Theorem 1:* For $U$ packets transmitted in order to access the $M$ idle channels which is out of $N$ licensed channels, the average number of packets in successful transmission of the system is given by:

$$S_{\text{suc}}(U) = MUp_{\text{sac}}(1 - p_{\text{sac}})^{U-1}. \tag{3}$$

*Proof:* Define $X_i = \sum_{j=1}^{U} X_{ij}$ as the number of packets transmitted on the $i^{th}$ idle channel in one time, where $X_{ij}$ is an indicator random variable with $i \in \{1, 2, \ldots, M\}$ and $j \in \{1, 2, \ldots, U\}$. $X_{ij} = 1$ if $j^{\text{th}}$ the packet selects the $i^{\text{th}}$ idle channel for access and $p(X_{ij} = 1) = p_{\text{sac}}$, otherwise $X_{ij} = 0$.

We introduce another random variable $Z_i$ defined as $Z_i = 1$, if the $i^{th}$ idle channel is only selected by one packet, otherwise, $Z_i = 0$ (*i.e.,* no packet or more than one packet selects the $i^{th}$ idle channel). That means $p(Z_i = 1) = p(X_i = 1)$.

In fact, $S_{\text{suc}}(U)$ equals to the average number of the idle channels selected only one packet. So, it can be expressed as

$$\begin{aligned} S_{\text{suc}}(U) &= E[Z|U] = E[\sum_{i=1}^{M} Z_i|U] = \sum_{i=1}^{M} E[Z_i|U] \\ &= \sum_{i=1}^{M} p(Z_i = 1|U) = \sum_{i=1}^{M} p(X_i = 1|U), \end{aligned} \tag{4}$$

where $Z = \sum_{i=1}^{M} Z_i$ is the number of idle channels selected by only one packet . Since $p(X_1 = 1|U) = p(X_2 = 1|U) = \ldots = p(X_M = 1|U)$ and each $X_{ij}$ represents an independent Bernoulli trial, we can reformulate (4) as,

$$\begin{aligned} S_{\text{suc}}(U) &= \sum_{i=1}^{M} p(X_i = 1|U) = Mp(\sum_{j=1}^{U} X_{ij} = 1|U) \\ &= M \binom{U}{1} p_{\text{sac}}(1 - p_{\text{sac}})^{U-1}. \end{aligned} \tag{5}$$

Thus, we arrive at the expression in (3). ∎

From (3), we deduce that $N_s$ is independent of $S_{\text{suc}}$ when $N_s > N - M + 1$. *It implies that the number of successful packet transmissions will not always increase with rising $N_s$. While there is an initial benefit in sensing more channels, it is not necessary to choose more than $N - M + 1$ channels for sensing.* We will verify this fact with simulation results in *Section V*.

Following *Theorem 1*, we will derive the expression of the average system throughput, when each SU is not in saturation, *i.e.,* $p_{\text{tra}} < 1$ and $U$ is a random variable. Since the packets transmission for each SU follows the identical independent distribution, the average number of packets in transmission from all SUs is $E[U] = Kp_{\text{tra}}$.

*Theorem 2:* The average system throughput can be approximated as,

$$S_{\text{ave}} = MKp_{\text{sac}}p_{\text{tra}}(1 - p_{\text{tra}}p_{\text{sac}})^{K-1}. \tag{6}$$

*Proof:* From *Theorem 1*, the average system throughput can be expressed as:

$$\begin{aligned} S_{\text{sys}} &= E[\frac{N_p T_{Pm} S_{\text{suc}}(U)}{N_s T_{Sm} + N_p T_{Pm}}] = \frac{N_p T_{Pm} E[S_{\text{suc}}(U)]}{N_s T_{Sm} + N_p T_{Pm}} \\ &= \frac{\eta N_p}{N_s + \eta N_p} E[S_{\text{suc}}(U)]. \end{aligned} \tag{7}$$

We define $S_{\text{ave}} = E[S_{\text{suc}}(U)]$ as the average number of the packets in successful transmission. Since $\eta > 1$ and $N_p >> N_s$, we can approximate (7) as,

$$S_{\text{sys}} \approx S_{\text{ave}} = E[S_{\text{suc}}(U)], \tag{8}$$

which implies we ignore the time consumption for sensing compared with the time for packet transmission. According to the result in *Theorem 1*, $S_{\text{ave}}$ can be presented as,

$$\begin{aligned} S_{\text{ave}} &= E[MUp_{\text{sac}}(1 - p_{\text{sac}})^{(U-1)}] = Mp_{\text{sac}}E[U(1 - p_{\text{sac}})^{(U-1)}] \\ &= Mp_{\text{sac}} \sum_{U=1}^{K} U(1 - p_{\text{sac}})^{U-1} p_{\text{num}}(U), \end{aligned} \tag{9}$$

where $p_{\text{num}}(U)$ is the probability that there are $U$ packets transmitted from $K$ SUs. Since the packet transmission of each SU is the independent Binomial distribution, $p_{\text{num}}(U)$ can be obtained as

$$p_{\text{num}}(U) = \binom{K}{U} (p_{\text{tra}})^U (1 - p_{\text{tra}})^{K-U}. \tag{10}$$

Substituting (10) into (9) simplifies to

$$\begin{aligned} S_{\text{ave}} &= MKp_{\text{sac}}p_{\text{tra}}(p_{\text{tra}}(1 - p_{\text{sac}}) + 1 - p_{\text{tra}})^{K-1} \\ &= MKp_{\text{sac}}p_{\text{tra}}(1 - p_{\text{tra}}p_{\text{sac}})^{K-1}. \end{aligned} \tag{11}$$

yielding (6). ∎

From (6), it is observed that the average system throughput will be affected by both $N_s$ and $p_{\text{tra}}$. Since $N_s$ is a discrete and finite variable, we can in principle obtain the optimal $N_s$ via exhaustive search in (6) and selecting the $N_s$ corresponding to the maximal $S_{\text{ave}}$. However, this is not effective for increasing or variable $p_{\text{tra}}$.

## IV. Relationship between $N_s$ and Average System Throughput

In this section, we focus on the throughput maximization w.r.t. $N_s$. According to (6), the problem can be presented as

$$\arg\max_{(N_s)} S_{\text{ave}} = MKp_{\text{sac}}p_{\text{tra}}(1 - p_{\text{tra}}p_{\text{sac}})^{K-1} \tag{12a}$$

$$\text{s.t. } N_s \in \{1, 2, \ldots, N - M + 1\}. \tag{12b}$$

Since the above is neither *convex* nor *concave* on the domain of $N_s$, the well-known Karush-Kuhn-Tucker conditions will only provide the local optimal solution. In the following, we analyze the optimal $N_s$ according to the relationship among $N$, $M$ and the average number of packets in transmission $E[U]$.

*Proposition 1:* For the maximal average system throughput, $N_s$ and $S_{\text{ave}}$ for a given $p_{\text{tra}}$ are related via

$$N_s^* = \begin{cases} 1, & Kp_{\text{tra}} > N; \\ \max(S_{\text{ave}}(\lfloor \hat{N}_s \rfloor), S_{\text{ave}}(\lceil \hat{N}_s \rceil)), & M \le Kp_{\text{tra}} \le N; \\ N - M + 1, & Kp_{\text{tra}} < M, \end{cases} \tag{13}$$

where $\hat{N}_s$ satisfies $(N-\hat{N}_s)^{[M]} = (1-\frac{M}{Kp_{\text{tra}}})N^{[M]}$.

*Proof:* We relax $N_s$ as $\bar{N}_s \in [0, N-M+1]$ and differentiate $S_{\text{ave}}$ in (6) with respect to $\bar{N}_s$.

i) When $Kp_{\text{tra}} > N$, which means $\frac{1}{Kp_{\text{tra}}} < \frac{1}{N} \le p_{\text{sac}}$, then $1 - Kp_{\text{tra}}p_{\text{sac}} < 0$ and $\frac{\partial S_{\text{ave}}}{\partial N_s} < 0$. That indicates $S_{\text{ave}}$ decreases with $p_{\text{sac}}$. Further, $p_{\text{sac}}$ is an increasing function of $N_s$ through differentiation with respect to $N_s$ Therefore, $S_{\text{ave}}$ achieves the maximal value at $N_s^* = 1$.

ii) $M \le Kp_{\text{tra}} \le N$: then $\frac{1}{N} \le \frac{1}{Kp_{\text{tra}}} \le \frac{1}{M}$ and $p_{\text{sac}} = \frac{1}{Kp_{\text{tra}}}$ leads to $\frac{\partial S_{\text{ave}}}{\partial N_s} = 0$ and $\frac{\partial^2 S_{\text{ave}}}{\partial N_s^2} < 0$. That implies $p_{\text{sac}} = \frac{1}{Kp_{\text{tra}}}$ is the only maximal point on the domain and can be reduced to

$$\frac{1}{M}\left(1 - \frac{(N-N_s)^{[M]}}{N^{[M]}}\right) = \frac{1}{Kp_{\text{tra}}} \tag{14}$$

Thus, $N_s^* = \max(S_{\text{ave}}(\lfloor \hat{N}_s \rfloor), S_{\text{ave}}(\lceil \hat{N}_s \rceil))$, where $\hat{N}_s$ satisfies $(N-\hat{N}_s)^{[M]} = (1-\frac{M}{Kp_{\text{tra}}})N^{[M]}$.

iii) $Kp_{\text{tra}} < M$, *i.e.* $p_{\text{sac}} \le \frac{1}{M} \le \frac{1}{Kp_{\text{tra}}}$: then $1 - Kp_{\text{tra}}p_{\text{sac}} \ge 0$ and $\frac{\partial S_{\text{ave}}}{\partial N_s} \ge 0$. Hence, (6) is an increasing function of $p_{\text{sac}}$ and $p_{\text{sac}}^* = \frac{1}{M}$ when $N_s \ge N - M + 1$ according to *Theorem 1*. However, sensing more channels means more resource consumption, *e.g.* time and energy, but with no throughput enhancement. Thus, the optimal $N_s$ will be $N_s^* = N - M + 1$ and the theorem is proved. ■

From *Proposition 1* and the fact that $E[U] = Kp_{\text{tra}}$, we conclude that if the average number of transmitted packets is larger than $N$, each SU does not need to sense the channels for more than one $T_{Sm}$ - longer sensing will only introduce more collisions and thereby reduce the system throughput. For $E[U] = Kp_{\text{tra}} < N$, it is worth investing more time on sensing the idle channel for each SU.

## V. Simulation Results

In this section, we present the results of the average system throughput via Monte-Carlo simulation. In our simulation, we have considered $N = 10$ licensed channels and the number of packet transmission slots $N_p = 40$ and $\eta = 5$. The $M$ idle channels are randomly located within the $N$ channels. Within an access frame duration $T_{\text{AF}}$, each SU randomly selects $N_s$ channels for sensing in $T_S$ and transmits the packet on any one of the idle channels identified as idle among the $N_s$ channels selected. The average throughput is obtained by counting the number of the idle channels which have only one packet access in each time slot within $N_p$ packet transmissions slots.

Fig. 3 and Fig. 4 present the average number of packets in successful transmissions when each SU is in saturation in the two cases, $U = K = 4$ and $U = K = 12$, respectively. First, we see that the analytical results in (3) fit well with the simulation results in Fig. 3 and Fig. 4, respectively. The average number of successful transmitted packets is constant when $N_s > N-M$ as indicated by the circled points in Fig. 3 and Fig. 4, which verifies the results in *Lemma 1*. As $M$ increases, it is intuitive that the average number of successful transmitted packets increases with $M$. Further, when $U < M$, the average number of packets in successful transmission increases with $N_s$ until $N_s = N-M+1$. Conversely, when $U > N$, the average number of successful transmitted packets declines as $N_s$ increases until $N_s = N-M+1$. For $M \le U \le N$, it is noted that the number of successful transmitted packets first increases with $N_s$ growing, then decreases after the maximum point. This occurs because increasing $N_s$ ultimately brings more collisions and reduces successful transmissions.

In Fig. 5 and Fig. 6, the average system throughput are plotted when the probability of packet transmission is given. It can been observed that if $Kp_{\text{tra}} > N$, *e.g.* $K = 12$ in Fig. 5, the throughput is decreasing with $N_s$ raising, thus the optimal $N_s^* = 1$. For $M \le Kp_{\text{tra}} \le N$, the throughput achieves the maximal point according to *Proposition 1*. Further, the throughput in $K = 3$ is larger than that in both $K = 6$ and $K = 12$ when $N_s > 4$ as shown in Fig. 5, since more collisions result if the average number of transmitted packets $Kp_{\text{tra}} > M$. Correspondingly, if the probability of packet transmission is reduced to 0.4 as shown in Fig. 6, the throughput for $K = 12$ is larger than that for the other two cases and grows as $K$ increases.

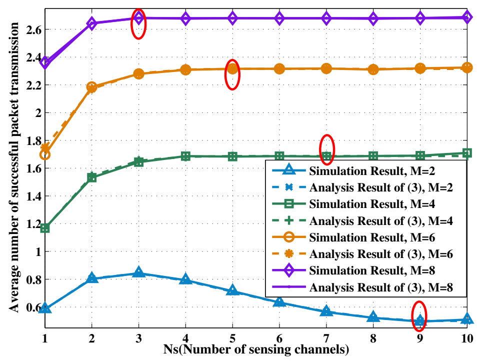


Fig. 3. Average number of successful packet transmission in $N = 10$ and $U = K = 4$

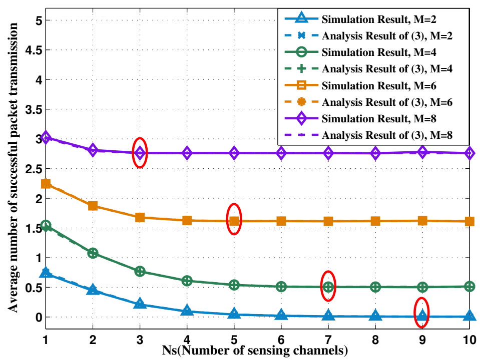


Fig. 4. Average number of successful packet transmission in $N = 10$ and $U = K = 12$

## VI. Conclusion

In this paper, we have introduced the "extended sensing strategy" and analyzed its throughput in a distributed, multi-channel cognitive radio setting. From the analysis, we have shown that the effective number of sensing channels $N_s$ is dependent on the total number of licensed channels $N$ and the

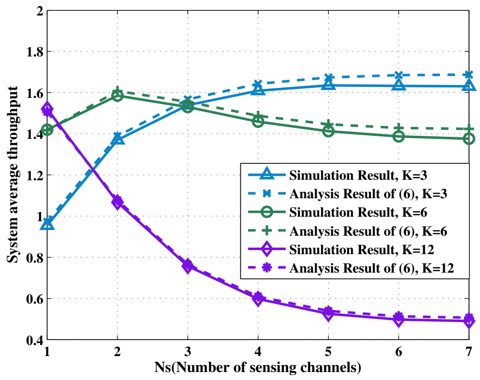


Fig. 5. System average throughput when $N = 10$, $M = 4$ and $p_{\rm tra} = 1$.

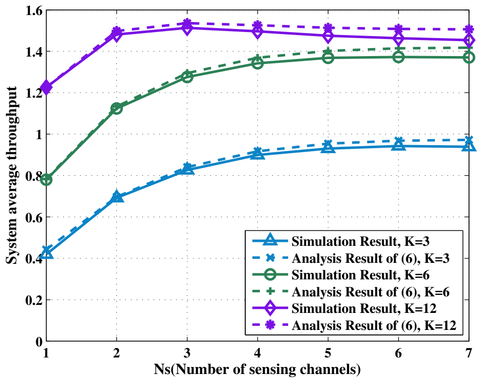


Fig. 6. System average throughput when $N = 10$, $M = 4$ and $p_{\rm tra} = 0.4$.

number of the idle channels $M$. The optimal $N_s$ is derived, which reveals the relationship between $N_s$ and the average system throughput. Comparing the simulation and analytical results, it is concluded that the system throughput from our analysis almost arrives at the simulation results.

## Appendix

The conditional probability, $p^{(i)}_{\text{sac}|M_s}$, that the $i^{\text{th}}$ idle channel is selected by one packet in the condition that $M_s$ idle channels are detected in $T_S$, can be presented as,

$$p^{(i)}_{\text{sac}|M_s} = p_{\text{idl}}(M_s) \cdot \binom{M-1}{M_s-1} \Big/ \binom{M}{M_s} \cdot \frac{1}{M_s} \tag{15a}$$

$$= p_{\text{idl}}(M_s) \cdot \frac{1}{M}, \tag{15b}$$

where $p_{\text{idl}}(M_s)$ is the probability of $M_s$ idle channels sensed by one SU and the second term in (15a) is the probability of the $i^{\text{th}}$ idle channel included in the $M_s$ idle channels. The probability of the packet transmitted on the $i^{\text{th}}$ idle channel out of the $M_s$ idle channels is $1/M_s$ as the third term in (15a), since the probability of selecting any one idle channel within $M_s$ idle channels is equal to each other. Hence, the probability of the $i^{\text{th}}$ channel selected for access by one packet, $p^{(i)}_{\text{sac}}$, is

$$p^{(i)}_{\text{sac}} = \sum_{M_s=1}^{\min\{M,N_s\}} p^{(i)}_{\text{sac}|M_s} = \sum_{M_s=1}^{\min\{M,N_s\}} \frac{p_{\text{idl}}(M_s)}{M} \tag{16a}$$

$$= \frac{1}{M} p_{\text{idl}}(M_s \geq 1) = \frac{1}{M}(1 - p_{\text{idl}}(M_s = 0)). \tag{16b}$$

If $N_s \geq N - M + 1$, it will always contains at least one idle channel within the $N_s$ detected channels which means $p_{\text{idl}}(M_s = 0) = 0$. While, if $N_s \leq N - M$, the packet will fail to transmit when no idle channel is detected within $N_s$ channels, so

$$p_{\text{idl}}(M_s = 0) = \begin{cases} \binom{N-M}{N_s} \Big/ \binom{N}{N_s}, & \text{if } N_s \leq N - M; \\ 0, & \text{else} . \end{cases} \tag{17}$$

Consequently, according to (16b), $p^{(i)}_{\text{sac}}$ is presented as

$$p^{(i)}_{\text{sac}} = \begin{cases} \frac{1}{M}\left(1 - \frac{(N-N_s)^{[M]}}{N^{[M]}}\right), & \text{if } N - M \geq N_s, \\ \frac{1}{M}, & \text{else.} \end{cases} \tag{18}$$

where $L^{[k]} = L(L-1)\cdots(L-(k-1))$. To prove the monotonic property of $p^{(i)}_{\text{sac}}$, we firstly relax $N_s$ as $0 < \bar{N}_s \leq N - M$. Then we can differentiate $p^{(i)}_{\text{sac}}$ as followings,

$$\frac{\partial p^{(i)}_{\text{sac}}}{\partial \bar{N}_s} = \Upsilon \sum_{j=0}^{M-1} \frac{1}{(N - \bar{N}_s - j)} > 0, \tag{19a}$$

$$\frac{\partial^2 p^{(i)}_{\text{sac}}}{\partial \bar{N}_s^2} = \Upsilon \sum_{j=0}^{M-1} \frac{1}{(N - \bar{N}_s - j)^2} - \frac{(\frac{\partial p^{(i)}_{\text{sac}}}{\partial \bar{N}_s})^2}{\Upsilon} < 0, \tag{19b}$$

where $\Upsilon = \frac{(N-\bar{N}_s)^{[M]}}{M \cdot N^{[M]}}$. Through differentiating (18) with respect $\bar{N}_s$, we can know the probability $p_{\text{sac}}$ is an increasing function versus $N_s$.

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