Performance analysis of dual-hop amplify-and-forward relay system in mixed Nakagami-*m* and Rician fading channels

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The performance of a dual-hop amplify-and-forward relay system, which experiences mixed Nakagami-*m* and Rician fading channels, is investigated. Based on the closed-form cumulative probability density function of the equivalent end-to-end signal-to-noise ratio, the exact expressions for outage probability and average symbol error probability are derived. The theoretical observations are verified by the Monte Carlo simulation results.

Introduction: Amplify-and-forward (AF) relay has attracted much interest since it is practical and easily implemented [1]. In many practical application scenarios, different links of the relay system experience different types of fading channels. For instance, for an indoor relay system [2] there is no line-of-sight (NLoS) component in the base station-relay link, while there is always a line-of-sight (LoS) component in the relay-mobile link, which should be modelled as Rician fading. Furthermore, a lot of practical channel measurement results [3] have validated such asymmetric nature of the relay fading channels, termed as mixed type fading channels [4]. In contrast to the symmetric fading assumptions [1, 5, 6], there are few performance analysis results for mixed type fading channels. Recently, in [4], Suraweera *et al.* presented the performance analysis of mixed Rayleigh and Rician fading channels for the AF relay system. It is well known that the Nakagami-*m* fading channel often gives the best fit to land-mobile and indoor-mobile multipath propagation and Rayleigh fading is just a special case of it [7]. However, to the best of our knowledge, the performance analysis of relay systems in mixed Nakagami-*m* and Rician fading channels has not been studied. In this Letter, we propose the mixed Nakagami-*m* and Rician fading channel model to capture the asymmetric nature of relay channels and the exact expressions for outage probability and average symbol error probability (SEP) are also investigated.

System and channel model: Consider a two-hop AF relay system [1, Fig. 1] where source S communicates to destination D with the help of relay R. The relay system is assumed to operate in the half-duplex model. Therefore, the equivalent end-to-end signal-to-noise ratio (SNR) is given by [1]. It should be noted that there is another equivalent end-to-end SNR form [1] as $\gamma = \gamma_1\gamma_2/\gamma_1 + \gamma_2 + 1$. However, the equivalent SNR form in (1) has the advantage of mathematical tractability and its corresponding system performance can serve as a benchmark of all practical relays [1]. So we only focus on the end-to-end SNR form in (1).

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} \tag{1}$$

where γ_1 is the instantaneous SNR of the S-R link, and γ_2 is the instantaneous SNR of the R-D link.

As discussed before, the fading amplitude of the S-R link is subjected to Nakagami-*m* fading, hence the cumulative probability density function (CDF) of γ_1 is expressed as [7]

$$F_{\gamma_1}(\gamma) = \left[1 - \frac{1}{\Gamma(m)}\Gamma\left(m, \frac{m\gamma}{\overline{\gamma_1}}\right)\right]U(\gamma) \tag{2}$$

where m is the Nakagami-*m* factor, $\overline{\gamma_1}$ is the mean of γ_1, $\Gamma(n)$ is the gamma function [8, equation (8.310.1)], $\Gamma(n, x)$ is the upper incomplete gamma function [8, equation (8.350.2)], and $U(x)$ is the unit-step function.

For the R-D link, since the fading amplitude is subjected to Rician fading, the probability distribution function (PDF) of γ_2 is given as follows [7]

$$f_{\gamma_2}(\gamma) = \frac{1}{2\sigma^2}\exp\left(-\frac{\gamma + A^2}{2\sigma^2}\right)I_0\left(\frac{A\sqrt{\gamma}}{\sigma^2}\right)U(\gamma) \tag{3}$$

where the average SNR $\overline{\gamma}_2 = A^2 + 2\sigma^2$, Rician K factor is $K = A^2/2\sigma^2$, and $I_0(x)$ is the zeroth modified Bessel function of the first kind [8, equation (8.406.1)].

CDF of end-to-end SNR: According to the definition of CDF, we can calculate it as [4, equation 6]

$$F_\gamma(r) = P\left(\frac{\gamma_1\gamma_2}{\gamma_1 + \gamma_2} < r\right) = 1 - \int_r^{+\infty}\left[1 - F_{\gamma_1}\left(\frac{\gamma_2 r}{\gamma_2 - r}\right)\right]f_{\gamma_2}(\gamma_2)d\gamma_2 \tag{4}$$

Substituting (3) and (4) into (5), we can get

$$\begin{aligned} F_\gamma(r) = 1 - \int_r^{+\infty} &\frac{1}{2\sigma^2\Gamma(m)}\Gamma\left(m, \frac{m\gamma_2 r}{\overline{\gamma_2}(\gamma_2 - r)}\right) \\ &\times \exp\left(-\frac{\gamma_2 + A^2}{2\sigma^2}\right)I_0\left(\frac{A\sqrt{\gamma_2}}{\sigma^2}\right)d_{\gamma_2} \end{aligned} \tag{5}$$

With the help of the series representations of $\Gamma(n, x)$ [8, equation (8.352.7)] and $I_0(x)$ [8, equation (8.445)], and using the binominal theorem [8, equation (1.111)], (6) can be finally formulated as the integral problem in [8, equation (3.471.9)]. After some manipulations, we can get

$$\begin{aligned} F_\gamma(r) = 1 - &\sum_{k=0}^{m-1}\sum_{n=0}^{k}\sum_{i=0}^{+\infty}\sum_{j=0}^{i} T(k, n, i, j) \\ &\times \frac{r^{k+i+1}}{\exp[r(m/\overline{\gamma}_1 + 1/2\sigma^2)]}K_{j-n+1}\left(r\sqrt{\frac{2m}{\overline{\gamma}_1\sigma^2}}\right) \end{aligned} \tag{6}$$

where $T(k, n, i, j) = \frac{1}{\sigma^2}\frac{1}{k!}C_k^n C_i^j \frac{1}{(i!)^2}\left(\frac{A}{2\sigma^2}\right)^{2i}\left(\frac{m}{\overline{\gamma}_1}\right)^k\left(\frac{2m\sigma^2}{\overline{\gamma}_1}\right)^{j-n+1/2}$ $\exp\left(-\frac{A^2}{2\sigma^2}\right)$, $K_n(x)$ is the nth-order modified Bessel function of the second kind defined in [8, equation (8.407)], $\binom{i}{j} = \frac{i!}{j!(i-j)!}$.

Outage probability: Outage probability is defined as the probability that the instantaneous SNR falls below the threshold SNR γ_{th}. Thus the outage probability P_{out} is given as

$$P_{out} = P(\gamma > \gamma_{th}) = F_v(\gamma_{th}) \tag{7}$$

Note that for the case of $m = 1$, i.e., mixed Rayleigh and Rician fading channels, (7) is exactly the same as (8) in [4].

SEP: SEP is an important system performance measure in wireless communications. It can be calculated as [7]

$$P_s = E[Q\sqrt{a\gamma}] = \frac{1}{\sqrt{2\pi}}\int_0^{+\infty} e^{-x^2/2}F_\gamma\left(\frac{x^2}{a}\right)dx \tag{8}$$

where $E[\cdot]$ is the mathematical expectation operator, $Q(x)$ is the Gaussian Q-function [7, equation (4.1)], and a is a constant related to the modulation scheme. For instance, $a = 4$ for BPSK, and $a = 1$ for QPSK. Substituting (7) into (8) and simplifying, (8) can be expressed as the integral problem in [8, equation 6.621.3)]. After some mathematical manipulation, it can be obtained that

$$\begin{aligned} P_s = \frac{1}{2} - &\sum_{k=0}^{m-1}\sum_{n=0}^{k}\sum_{i=0}^{+\infty}\sum_{j=0}^{i}\frac{T(k, n, i, j)}{2\sqrt{2}a^{k+i+1}} \\ &\times \frac{(2/a\sigma\sqrt{2m/\overline{\gamma}_1})^{j-n+1}}{(1/2 + m/a\overline{\gamma}_1 + 1/2a\sigma^2 + 1/a\sigma\sqrt{2m/\overline{\gamma}_1})^{k+i+j-n+5/2}} \\ &\times \frac{\Gamma(k + i + j - n + 5/2)}{\Gamma(k + i + 2)} \times \Gamma\left(k + i - j + n + \frac{1}{2}\right) \\ &\times F\left(k + i + j - n + \frac{5}{2}, j - n + \frac{3}{2}; k + i + 2;\right. \\ &\left.\frac{1/2 + m/a\overline{\gamma}_1 + 1/2a\sigma^2 - 1/a\sigma\sqrt{2m/\overline{\gamma}_1}}{1/2 + m/a\overline{\gamma}_1 + 1/2a\sigma^2 + 1/a\sigma\sqrt{2m/\overline{\gamma}_1}}\right) \end{aligned} \tag{9}$$

where $F(a, b; c; d)$ is the Gaussian hyper-geometric function defined in [8, equation (9.100)]. Note that for the mixed Rayleigh and Rician fading channels, we have $m = 1$ and (9) reduces to the previously published result presented in [4, equation 14].

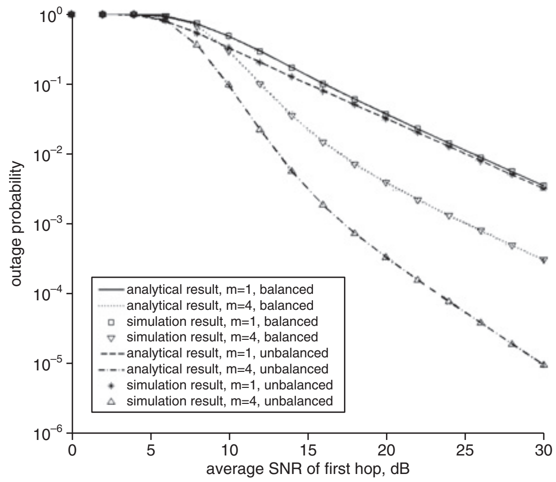

Fig. 1 *Outage probability for different m factors with $K = 6$ dB and $\gamma_{th} = 5$ dB under both balanced and unbalanced link scenarios*

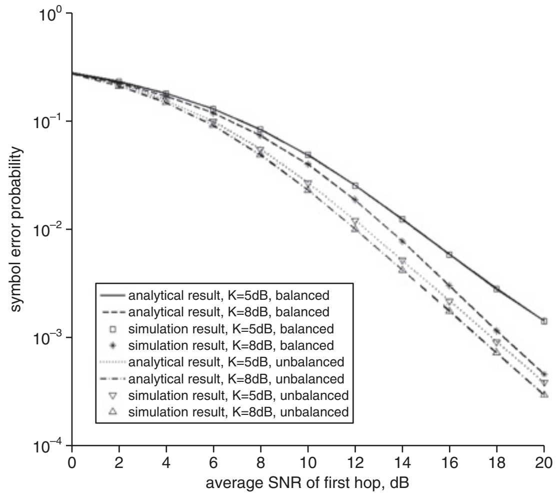

Fig. 2 *SEP for different K factors with QPSK modulation and $m = 3$ under both balanced and unbalanced link scenarios*

Simulation results: We propose some simulation results to verify the above analytical results. The theoretical and simulation results are presented for different Nakagami-*m* and Rician *K* factors under both balanced link ($\bar{\gamma}_1 = \bar{\gamma}_2$) and unbalanced link ($\bar{\gamma}_1 = 1.5\bar{\gamma}_2$) scenarios. As found in the numerical simulation, the infinite series of (7) and (9) keep constant when i is larger than 100. Therefore, the analytical results of (7) and (9) are calculated using 100 summation terms over i. Fig. 1 illustrates the outage probability with different m factors, while Fig. 2 shows the SEP for the QPSK system with different K factors. It is clear that all the simulation results match the analytical results exactly.

Conclusion: We have derived the closed-form CDF of the equivalent end-to-end SNR for a dual-hop AF relay system in mixed Nakagami-*m* and Rician fading channels, based on which the exact outage probability and SEP were investigated. Monte Carlo simulation results are presented to validate the theoretical analysis. Since the mixed Nakagami-*m* and Rician fading model generalised the channel models published before, such as the symmetric Rayleigh channel model and the mixed Rayleigh and Rician channel model, the analytical results derived in this work can be used to evaluate a wider range of relay application scenarios. To the best of our knowledge, such exact results have not been published in the literature.

Acknowledgments: This research was supported by the Natural Science Foundation of China (no. 60772113), the China Important Natural Science and Technology Specific projects (no. 2009ZX03003-004-01) and the National Basic Research Program of China (no. 2009CB320406).

8 June 2010
doi: 10.1049/el.2010.1555

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