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Joint optimization of time and frequency synchronization based on polynomial sequences for OFDM systems

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Abstract

In orthogonal frequency division multiplexing (OFDM) systems, time and frequency synchronization are two critical elements for guaranteeing the orthogonality of OFDM subcarriers. Conventionally, with the employment of pseudo-noise (PN) sequences in preamble design, the preamble information is not fully utilized in both symbol timing offset acquisition and carrier frequency offset estimation. In this article, a new synchronization algorithm is proposed for jointly optimizing the time and frequency synchronization. This algorithm uses polynomial sequences as synchronization preamble instead of PN sequences. Theoretical analysis and simulation results indicate that the proposed algorithm is much more accurate and reliable than other existing methods.

Keywords frequency synchronization, joint optimization, OFDM, polynomial sequences, timing synchronization

1 Introduction

OFDM has attracted increasing interest in recent years. It is favorable in that it splits a high-rate data stream into a parallel of low-rate data streams, and thus reinforcing resistance against frequency-selective multipath fading. OFDM has been proposed and standardized for high-speed broadcastings in Europe for digital audio broadcasting [1] and terrestrial digital video broadcasting [2]. It has also been proposed for high data-rate packet communications, i.e., wireless local area network (WLAN) and wireless metropolitan area networks (WMAN), as specified in IEEE 802.11 [3] and 802.16 [4].

To attain spectral efficiency, the spectrum of OFDM signal overlaps. As a result, OFDM is more sensitive to timing offset and carrier frequency offset (CFO) than single carrier systems. In general, the synchronization of OFDM system is carried out in three steps: frame timing synchronization (detection of OFDM frame timing offset (FTO)), frequency synchronization (estimation of the CFO), and symbol timing synchronization (finding the exact start of FFT window, i.e., symbol timing offset (STO)). In the existing related studies, several OFDM

synchronization schemes have been introduced to estimate the former parameters [5–11]. These estimators associated with OFDM systems are divided into two classes, i.e., data-aided schemes and blind methods. Data-aided schemes were proposed in Refs. [5–9], where periodically inserted known symbols were explicitly used. In Ref. [5], Schmidl and Cox proposed a training symbol based timing/frequency synchronization, which used an OFDM symbol with identical halves. The drawback of data-aided synchronization methods is the overhead caused by the training OFDM symbols. Blind estimators are bandwidth efficient since they do not require extra overhead [10–11]. These methods usually extract synchronization information by correlating the last samples of OFDM symbols with the cycle prefix (CP). However, estimation error of these methods is generally higher than 1% of the subcarrier spacing at moderate signal-to-noise ratio (SNR) (e.g., 10 dB). This is because CP is affected by the frequency selective fading. For the robustness of OFDM system, only data-aided methods are investigated in this article.

As described in Ref. [5] and other related studies, the CFO and STO estimators are always illustrated separately without joint optimization. Therefore, the information of the transmitted preamble sequences is not used efficiently in STO estimator, which lowers the correct acquisition probability

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(CAP). To solve the above problem, a novel synchronization algorithm using the polynomial sequence as synchronization sequence is proposed in this article. Based on the novel preamble pattern design and simple signal processing, the preamble information can be completely utilized both in timing and frequency synchronization, which can improve the STO acquisition performance both in additive white Gaussian noise (AWGN) and multipath Rayleigh channels. Furthermore, as shown in the simulation results, compared with conventional methods, the proposed algorithm can also enhance the CFO estimator accuracy at low SNR, whereas they both reach the Cramér-Rao lower bound at high SNR.

The remainder of this article is organized as follows. Sect. 2 proposes the signal model and problem formulation. In Sect. 3, the proposed synchronization scheme based on polynomial sequences is developed. The synchronization performance by computer simulations with comparison to Schmidl's method is evaluated in Sect. 4. Finally, conclusions are given in Sect. 5.

2 Problem formulation

2.1 OFDM signal model

In an OFDM system, the complex baseband samples of an OFDM symbol can be represented as:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N}; \quad -N_g \leq n \leq N-1 \quad (1)$$

where $X(k)$ is the frequency domain data before inverse fast Fourier transform (IFFT), N is the IFFT length, and N_g denotes the length of the CP, which is used to eliminate the intersymbol interference (ISI).

The channels between the transmitter and the receiver can be modeled as a wide-sense stationary, uncorrelated scattering, Rayleigh fading channel. Therefore, the complex baseband samples received at the receiver can be expressed as:

$$r(n) = e^{j2\pi\epsilon n/N} \sum_{l=0}^{L-1} x(n-l-\tau) h_l + w(n); \quad -N_g \leq n \leq N-1 \quad (2)$$

where h_l , τ , and ϵ denote the channel impulse response at the l th multipath, the integer-valued unknown timing offset, and the fractional carrier frequency offset, respectively. $w(n)$ is the time domain complex AWGN with zero mean and variance of σ_w^2 .

2.2 Review of conventional synchronization methods and problem description

In this section, the conventional synchronization methods utilizing repetitive synchronization pattern (SP) will be briefly introduced. In these methods, the preamble at the beginning

of a frame is composed of two repetitions of a shorter SP with length $N_{sp} = N/2$. Generally, the CFO $\hat{\epsilon}$ is estimated by calculating the angle of the autocorrelation of the received signal. Furthermore, the STO $\hat{\tau}_{STO}$ is found by searching the peak of cross-correlation function between the received signal and reference synchronization pattern. The above-mentioned process can be written as:

$$\hat{\epsilon} = \angle \left(\sum_{n=0}^{N_{sp}-1} |r(n + \hat{\tau}_{FTO}) r^*(n + \hat{\tau}_{FTO} + N_{sp})| \right) \quad (3)$$

$$\hat{\tau}_{STO} = \arg \max_d \left(\sum_{n=1}^{N_{cc}-1} |r_{fine}^*(n+d) c(n)| \right) \quad (4)$$

where $\hat{\tau}_{FTO}$ denotes the estimated FTO for frame synchronization, $r(n)$ is the received data, $r_{fine}(n) = r(n) e^{-j2\pi\hat{\epsilon}n/N}$, and N_{cc} is the cross-correlation length, which is generally equal to the half length of synchronization symbol in conventional methods.

As shown in Refs. [5,8], making an approximation at high SNR, the normalized variance of the estimator can be shown as:

$$\text{var}[\hat{\epsilon} - \epsilon] = \frac{1}{\pi^2 N_{sp} \text{SNR}} \quad (5)$$

It is clear from Eq. (5) that the mean squared error (MSE) decreases with the synchronization sequence length N_{sp} . Furthermore, it can be concluded from Ref. [6] that the correct acquisition probability of $\hat{\tau}_{STO}$ increases with the cross correlation length N_{cc} .

As a result, it can be seen that the longer N_{sp} and N_{cc} are, the better the estimator is. According to the existing research, the CFO estimator completely uses two halves of the preamble, whereas STO estimator only uses one half, which would directly decrease the STO acquisition probability.

3 Proposed synchronization algorithm

From the above description, it is inferred that the conventional theory cannot fully use the information of transmitted synchronization symbols. In this section, a new synchronization algorithm using polynomial synchronization sequences is proposed. By using this algorithm, the preamble information can be fully used both in CFO and STO estimation. Therefore, both the STO acquisition performance and CFO estimator accuracy can be enhanced, especially at low SNR.

Polynomial sequences represent the sequences expressed with a polynomial, which are chosen as constant amplitude zero auto correlation (CAZAC) [12] sequences in this article. A set of CAZAC sequences can be expressed as

$$c^u(n) = \begin{cases} e^{-j\frac{\pi n^2 u}{N_{SP}}}; & N_{SP} \text{ even} \\ e^{-j\frac{\pi u n(n+1)}{N_{SP}}}; & N_{SP} \text{ odd} \end{cases} \quad (6)$$

$$n = 0, 1, \dots, N_{SP} - 1, u = 0, 1, \dots, N_{SP} - 1$$

As mentioned earlier, if one wants to achieve the optimal STO acquisition performance, the cross-correlation length N_{CC} should be equal to its maximum value N . Consequently, the length of the CAZAC sequence used in this article is $N_{SP} = N_{CC} = N$, and there is no repetition in the preamble design. At the receiver, the STO can be directly acquired by using the N -length received signal to cross-correlate with the transmitted preamble. And then the repetition architecture can be reconstructed through its polynomial configuration to estimate the CFO. The CFO estimation principle is described as follows.

As depicted in Eq. (6), the N -length CAZAC synchronization sequence can be rewritten:

$$\left. \begin{aligned} c_1(n) &= c^u(n) = e^{j\pi u n^2 / N} \\ c_2(n) &= c^u\left(n + \frac{N}{2}\right) = c_1(n) e^{j\pi u (n+N/4)^2} \\ n &= 0 : \frac{N}{2} - 1 \end{aligned} \right\} \quad (7)$$

As a result, the received signal can be represented as:

$$\left. \begin{aligned} r_1(n) &= e^{j2\pi\epsilon n/N} \sum_{l=0}^L c_1(n-l) h_l(n) + w_1(n) \\ r_2(n) &= e^{j2\pi\epsilon n/N} e^{j(2\pi\epsilon(N/2)/N)} \sum_{l=0}^L c_1(n-l) e^{j\pi u ((n-l)+N/4)^2} \\ &\quad h_l(n) + w_2(n) = e^{j2\pi\epsilon n/N} e^{j\pi\epsilon} \sum_{l=0}^L c_1(n-l) e^{j\pi u ((n-l)+N/4)^2} \\ &\quad h_l(n) + w_2(n) \end{aligned} \right\} \quad (8)$$

With the assumption of high SNR, r_2 can be reconstructed to be a quasi-repetition of r_1 :

$$\tilde{r}_2(n) = r_2(n) e^{-j\pi u (n+N/4)^2} = e^{j2\pi\epsilon n/N} e^{j\pi\epsilon} \sum_{l=0}^L c_1(n-l) e^{-j\pi u l^2} h_l(n) \quad (9)$$

Then

$$\left. \begin{aligned} \sum_{n=0}^{N/2-1} \tilde{r}_2^*(n) r_1(n) &= e^{-j\pi\epsilon} \sum_{n=0}^{N/2-1} \sum_{l=0}^L |c_1(n-l)|^2 |h_l(n)|^2 e^{j\pi u l^2} \\ \tilde{r}_2(n) &= r_2(n) e^{-j\pi u (n+N/4)^2} = e^{j2\pi\epsilon n/N} e^{j\pi\epsilon} \sum_{l=0}^L c_1(n-l) e^{-j\pi u l^2} h_l(n) \end{aligned} \right\} \quad (10)$$

Therefore,

$$\hat{\epsilon} = \frac{1}{\pi} \angle \left[\sum_{n=0}^{N/2-1} \tilde{r}_2^*(n) r_1(n) \right] \quad (11)$$

Based on the above description, the proposed synchronization algorithm can be depicted as:

1) Frame timing synchronization

$$\hat{\tau}_{FTO} = \arg \max_d [\tilde{\Lambda}(d)] \quad (12)$$

where

$$\tilde{\Lambda}(d) = \sum_{n=0}^{N/2-1} \left| r(n+d) \left[r\left(n+d+\frac{N}{2}\right) e^{-j\pi u (n+N/4)^2} \right]^* \right| \quad (13)$$

2) Carrier frequency synchronization

$$\hat{\epsilon} = \angle [\tilde{\Lambda}(\hat{\tau}_{FTO})] \quad (14)$$

3) Symbol timing synchronization

$$\hat{\tau}_{fine} = \arg \max_d [\tilde{\Pi}(d)] \quad (15)$$

where

$$\tilde{\Pi}(d) = \sum_{n=1}^{N-1} |\tilde{r}_{fine}^*(n+d) c(n)| \quad (16)$$

and $\tilde{r}_{fine}(n) = e^{j2\pi\hat{\epsilon}n/N} r(n)$. From Fig. 1, it can be observed that the normalized cross-correlation amplitude of the proposed algorithm is approximately two times higher than the Schmidl's method, which might be concluded that this difference would bring the enhancement of STO acquisition probability. Furthermore, it can also be seen that the two autocorrelation patterns are similar. As a result, the FTO and CFO estimation performance would not be decreased.

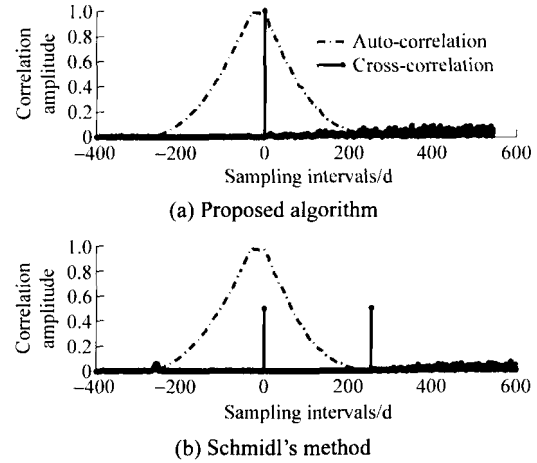


Fig. 1 Correlation pattern comparison between proposed algorithm and Schmidl's method in OFDM systems for AWGN channel, SNR = 20 dB

4 Simulation results

The performance of the proposed synchronization algorithm will be demonstrated in this section and it would be compared with Schmidl's method. The simulation parameters are listed in Table 1. The delay and power coefficients of chosen multipath channel are $[0, 0.31, 0.61, 1.09, 1.83, 2.61]$ μs and $[0, -1, -9, -10, -15, -20]$ dB, and the vehicle velocity is 3 km/h. The CFO are pseudorandom values drawn from a uniform distribution on the unit interval $[-0.5, 0.5]$ in the simulations.

Table 1 Simulation parameters

| Parameters | Values |
|-------------------------|---------------|
| Bandwidth | 1.25 MHz |
| Sampling frequency | 1.92 MHz |
| Number of subcarriers | 128 |
| Subcarrier spacing | 15 kHz |
| OFDM symbol length | 76.12 μ s |
| Length of cyclic prefix | 4.69 μ s |

Fig. 2 illustrates the STO acquisition performance in the AWGN channel, where it is readily apparent in the graph that the proposed symbol timing offset estimation algorithm can obtain obvious CAP enhancement compared with Schmidl's method at low SNR range (-10 dB to 0 dB). Similarly, Fig. 3 demonstrates the same simulation in multipath channel. From the figure, the STO CAP is about 15% to 21% higher than Schmidl's method, which would result in prominent enhancement in the system BER performance.

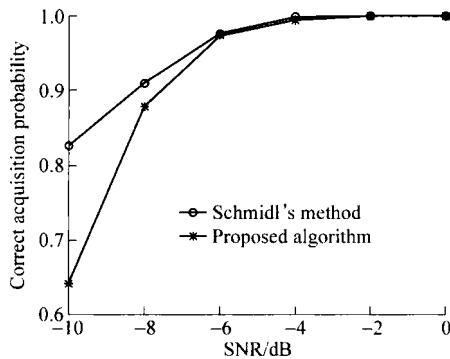


Fig. 2 Comparison between proposed and Schmidl's STO acquisition in OFDM systems for AWGN channel

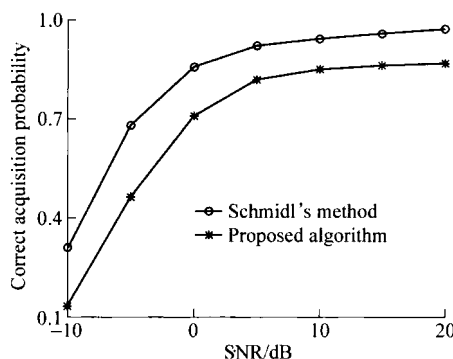


Fig. 3 Comparison between the proposed and Schmidl's STO acquisition in OFDM systems for multipath channel

The simulation results for the MSE as function of the SNR for proposed and Schmidl's carrier frequency offset estimation algorithms are depicted in Figs. 4 and 5. The simulation curves are compared with Cramér-Rao lower bound [5,7] both in AWGN and multipath channels. It is clear from these figures that the theoretical value, which equals the Cramér-Rao lower bound, is a good estimate of the variance for high SNR values, and the curves corresponding to different methods are aligned together. However, when the

SNR is low, although the theoretical value underestimates the variance than simulation results in both estimations, the proposed algorithm has obvious lower MSE curve than the Schmidl's.

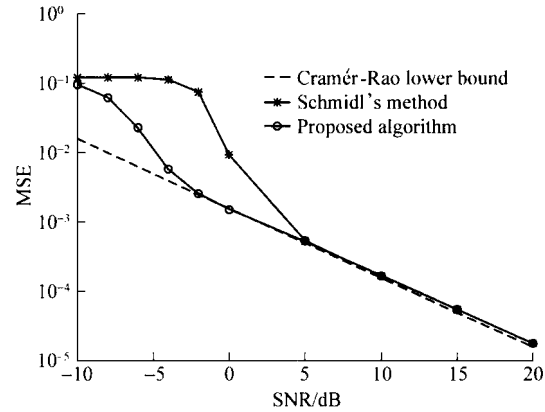


Fig. 4 Comparison between proposed and Schmidl's carrier frequency offset estimation to Cramér-Rao lower bound in OFDM systems for AWGN channel

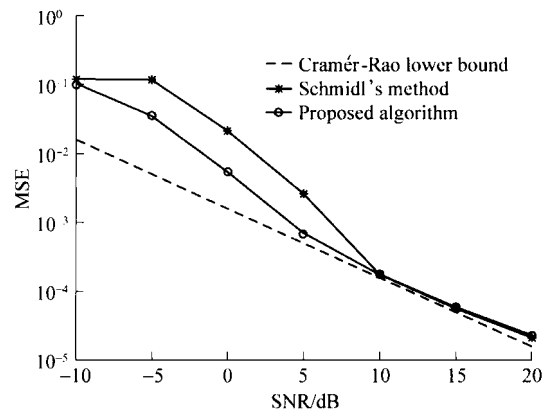


Fig. 5 Comparison between proposed and Schmidl's carrier frequency offset estimation to Cramér-Rao lower bound in OFDM systems for multipath channel

5 Conclusions

A novel algorithm for joint optimization of timing and frequency synchronization is proposed in this article. The algorithm uses one polynomial sequence instead of two repeated PN sequences to be transmitted as the preamble. The corresponding synchronization scheme is proposed at the receiver. Theoretical analysis and simulation results show that this proposed algorithm can significantly enhance performance in both timing and frequency synchronization in OFDM systems.

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