Joint Timing Synchronization and Channel Estimation for OFDM Systems via MMSE Criterion

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***Abstract*— A joint timing synchronization and channel estimation algorithm is proposed for orthogonal frequency division multiplexing (OFDM) system. In the proposed scheme, the shift delay characteristic of synchronization sequence is revealed in channel estimation process. Through utilizing this characteristic, the correct symbol timing offset (STO) is jointly optimized with channel estimation via the minimum mean square error (MMSE) criterion. Simulation results demonstrate that the proposed scheme could bring almost ideal performance improvement for both channel and timing offset estimation.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) has received intense interest during the past few decades. Its robustness to frequency-selective channels has made it one of the main candidates for high data rate transmission for wireless applications. All OFDM-based transmission systems suffer from several sources of impairment such as synchronization [1] and channel estimation. Accurate timing and frequency synchronization is required to maintain orthogonality among sub-carriers and to avoid inter-symbol interference (ISI). Additionally, the channel impulse response (CIR) must be known to coherently detect the data per sub-carrier. When we consider the massive amount of research devoted to these problems (see, e.g., [1]–[10] and references therein), it becomes clear that synchronization and channel estimation are critical issues. For timing synchronization, conventional techniques are either data-aided (i.e., exploiting training symbols in the time- or frequency-domain) [1]–[7] or blind (e.g., exploiting the presence of the cyclic prefix) [8]–[10]. Furthermore, some literatures, i.e. [11], have attempted to estimate synchronization parameters jointly with channel estimation, which generally maximize a likelihood function of the above parameters. Unfortunately, as known from the aforementioned references, the probability of correct STO estimation is unsatisfactory in multi-path Rayleigh fading channel with large rms delay spread. In this condition, the channel coefficient for the first path sometimes has a smaller value compared to that for other delayed paths. As a consequence, the correlation peak will fall at the position corresponding to the path with the largest value.

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In this article, we propose an algorithm based on minimum mean square error (MMSE) criterion to jointly optimize the timing synchronization and channel estimation. In the proposed algorithm, a set of possible symbol timing offsets (STOs) are obtained by coarse synchronization firstly. Then the preceded STOs are eliminated through utilizing the shift delay characteristic of synchronization sequence exhibiting in channel estimation process. In addition, the correct STO is acquired via the MMSE criterion, where the estimated CIRs corresponding to the delayed STOs are processed in advance with the above mentioned shift delay characteristic. Through employing the proposed algorithm, the influence caused by multi-path Rayleigh channel could be conquered efficiently. Meanwhile, the CIR could be obtained combined with the STO optimizing.

The organization of this paper is as follows. Section II introduces the signal model and problem formulation. Section III presents the proposed joint timing synchronization and channel estimation scheme. The simulation results are shown in Section IV and finally Section V concludes the paper.

II. Signal Model and Problem Formulation

A. Signal Model

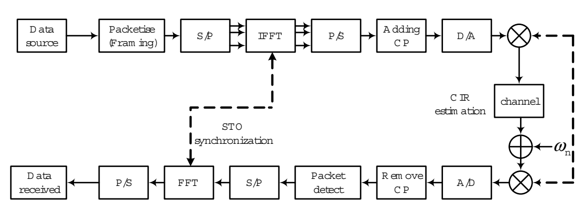

Fig. 1. Block diagram of OFDM system

An OFDM system could be shown as Fig.1, which also depicts the position of STO synchronization and CIR estimation. In an OFDM system, the transmitter broadcasts identical synchronization symbols periodically and simultaneously. An OFDM preamble symbol x_n is created by applying an Inverse

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Fast Fourier Transform (IFFT) operator to a N-length synchronization sequence X_k in frequency domain. This sequence should exhibit ideal correlation properties and have a constant envelope in both time and frequency domains. Furthermore, each OFDM symbol is preceded by a cyclic prefix (CP), an exact replica of the N_{CP} last samples of the OFDM symbol.

Assume that the effect of the propagation channel can be described by a finite impulse response (FIR) filter with an effective length of $L \leq N_{CP}$, the received frequency complex baseband signal can be written as

$$Y_k(\theta) = \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h_l x(n-l-\theta) \cdot e^{j\frac{-2\pi kn}{N}} + W_k \tag{1}$$

where h_l denotes the slowly time-varying discrete time complex CIR with $\sum_{l=0}^{L-1} E(|h_l|^2) = 1$, $W(k)$ indicates the frequency domain complex Additive White Gaussian Noise (AWGN) with variance σ_ω^2 and θ stands for the arrival time of the first multi-path component.

B. Problem Formulation

As shown in Fig. 2, the timing synchronization error could be classified as preceded STO and delayed STO. The effect of each error type is different, which would bring challenges to the decision of correct STO estimation.

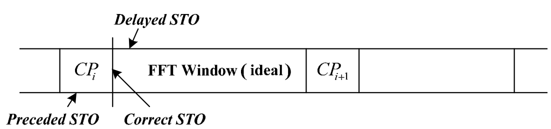

Fig. 2. The influence of STO estimation error

Our task in this paper is to identify the correct arriving time of the first multi-path component θ and to estimate the discrete CIR of the synchronization symbol, so that ISI and ICI can be removed or significantly reduced during OFDM demodulation.

III. Proposed Joint Timing Synchronization and Channel Estimation Algorithm

A. Review of Timing Synchronization Methods

In this section, the fine timing synchronization methods mentioned in [2] and [11] will be briefly introduced, which are named as Yip's and Lim's.

The Yip's method is based on the cross-correlation characteristic of the synchronization sequence. In this algorithm, the timing synchronization can be performed by correlating the received samples and the transmitted preamble symbol which is known on the receiver. The metric of the estimator can be given by

$$\hat{\theta}_{\mathrm{Yip}} = \arg\max_{d} \left(\Phi\left(d\right)\right) \tag{2}$$

and

$$\Phi\left(d\right) = \left|\sum_{n=0}^{N-1} x\left(n\right) r^*\left(n+d\right)\right|^2 \tag{3}$$

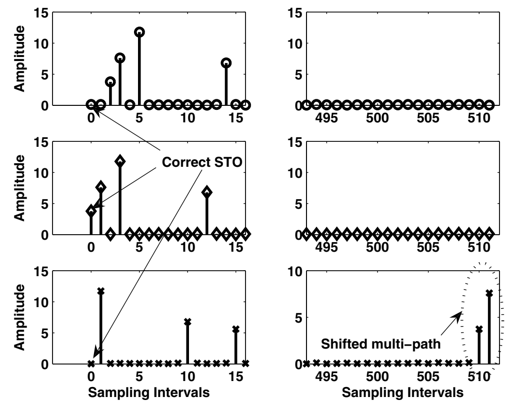

Fig. 3. Estimated time domain channel amplitude with different STO, TU channel. Top: 2 intervals preceded; Middle: correct STO; Bottom: 2 intervals delayed

From formula (2), it could be known that the Yip's scheme would find the multi-path with the strongest power to be the estimated STO.

An existing joint synchronization and channel estimation algorithm has been presented by Lim [11]. The strategy derived a maximum likelihood estimation solution for channel coefficients which turns out to be the correlator as below:

$$\hat{\theta}_{\mathrm{Lim}} = \arg\max_{m} \left(\Psi(m)\right) \tag{4}$$

where

$$\Psi(m) = \sum_{k=m}^{m+N_{Cp}-1} \left|\mathbf{x}^H \mathbf{r}(m)\right| \tag{5}$$

and $\mathbf{r}(m) = [r(m), r(m+1), \cdots, r(m+N-1)]^T$. As shown in (12), $\Psi(m)$ implements the summation of the correlation amplitudes with length of N_Cp.

B. Proposed timing synchronization method

In this paper, it is assumed that coarse timing recovery based on auto correlation has been performed firstly, so that the desired STO θ is searched only in a finite set $\mathbf{\Lambda} = [\theta_0, \theta_1, \cdots, \theta_{M-1}]$, where M is the cardinal number of $\mathbf{\Lambda}$. The time domain least square (LS) estimated CIR corresponding to possible STO θ_m could be expressed with element notation as

$$\hat{h}(\theta_m, n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(\theta_m, k) \cdot e^{j\frac{2\pi kn}{N}} / S(k) \tag{6}$$

Fig. 3 is the illustration of the estimated CIR during one symbol in time domain, which corresponds to variable types of STOs, as shown in formula (2). The figure only shows the head and end part of the whole symbol ($N = 512$) for the purpose of clear expression. On the one hand, it could be seen from the figure that when the STO is preceded, the CIR would be shifted rightward. Therefore, the amplitude corresponding to the position of correct STO only owns noise component.

However, when the STO is correct, there is not only noise, but also multi-path channel component on the same position. This characteristic could help us to eliminate the preceded STOs. On the other hand, when the STO estimated is delayed, the direct and some other multi-path components would be shifted leftward to the end of the symbol. Consequently, if the amplitudes of the sampling intervals in the right window are set to be zero, the channel estimation error would increase tremendously. Based on the above analysis, the proposed scheme could be described as follows:

Algorithm 1 Joint Optimization of Timing Synchronization and Channel Estimation Algorithm

1) Assume $\mathbf{\Lambda} = [\theta_0, \theta_1, \cdots, \theta_{M-1}]$ stands for the set of all possible STOs and $\mathbf{R} = [\mathbf{Y}(\theta_0), \mathbf{Y}(\theta_1), \cdots, \mathbf{Y}(\theta_{M-1})]^T$ corresponding received preamble symbol set, where $\mathbf{Y}(\theta_m) = [Y(\theta_m, 0), Y(\theta_m, 1), \cdots, Y(\theta_m, N-1)]$.

2) Calculate $\hat{h}(\theta_m, n)$, $n = 0 : N-1$ according to formula (2), which could result in a matrix as $\hat{\mathbf{h}}_\Lambda = [\hat{\mathbf{h}}(\theta_0), \hat{\mathbf{h}}(\theta_1), \cdots, \hat{\mathbf{h}}(\theta_{M-1})]^T$, $\hat{\mathbf{h}}_\Lambda \in \mathbb{C}^{N \times M}$.

3) Eliminate the preceded STOs

$$\tau_i = \left\{ \theta_m \,\middle|\, |h(\theta_m, 0)| > \beta \cdot \max_{\theta_m}(|h(\theta_m, 0)|) \right\}, \quad (7)$$

$m = 0 : M-1$, $i = 0 : M'-1$, β is a threshold, $M' \leq M$. Therefore, $\hat{\mathbf{h}}_\Lambda$ would be converted to $\hat{\mathbf{h}}_\Gamma = [\hat{\mathbf{h}}(\tau_0), \hat{\mathbf{h}}(\tau_1), \hat{\mathbf{h}}(\tau_{M'-1})]^T$, where $\mathbf{\Gamma} = [\tau_0, \tau_1, \cdots, \tau_{M'-1}]$, $\hat{\mathbf{h}}_\Gamma \in \mathbb{C}^{N \times M'}$.

4) Acquire the correct STO with MMSE criterion

$$\hat{\theta} = \arg\min_{\tau_i}(E\{\left\|\hat{\mathbf{Y}}(\tau_i) - \mathbf{Y}\right\|^2\}), \quad (8)$$

where

$$\hat{Y}(\tau_i, k) = \sum_{n=0}^{N-1} \hat{h}_0(\tau_i, n) \cdot e^{\frac{2\pi k n}{N}} \cdot X(k) \quad (9)$$

and

$$\hat{h}_0(\tau_i, n) = \begin{cases} \hat{h}(\tau_i, n) & n = 0 : N_{CP} - 1 \\ 0 & n = N_{CP} : N-1 \end{cases},$$

$$i = 0 : M' - 1. \quad (10)$$

5) Estimate the frequency domain CIR corresponding to the true STO.

$$\hat{H}(\hat{\theta}, k) = \sum_{n=0}^{N-1} \hat{h}_0(\hat{\theta}, n) \cdot e^{\frac{2\pi k n}{N}}. \quad (11)$$

IV. Simulation Results

The performance of the proposed joint timing synchronization and channel estimation method will be demonstrated in this section. And the proposed algorithm would be compared with Lim's and Yip's methods. The simulation parameters, recommended by 3GPP (3rd Generation Partnership Project) LTE (Long Term Evolution), are listed in TABLE 1. As shown in the table, the chosen channel type is COST207 typical urban (TU) channel, which owns the nature that the direct path is not the strongest for most of the time. In consequence, the requisition of combating multi-path fading interference is rigorous.

TABLE I
Simulation Parameter

Parameter	Values
Band width	5MHz
Sampling frequency	7.68MHz
Number of sub-carriers	512
OFDM symbol length	76.12s
Length of cyclic prefix	4.69s
Efficient path number	6
Time delay	[0, 0.2, 0.5, 1.6, 2.3, 5] us
Power	[-3, 0, -2, -6, -8, -10] dB
Vehicle velocity	3Km/h
Number of total simulated frame	10000

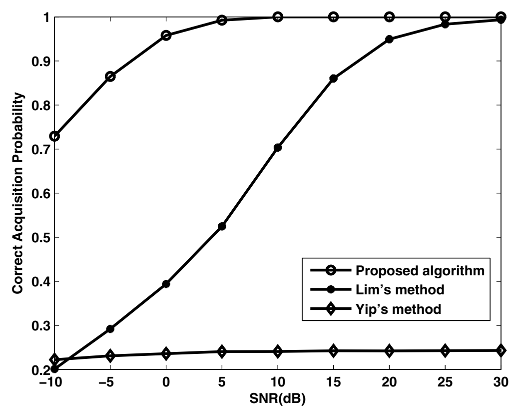

Fig. 4. Comparison of the of correct synchronization acquisition probability for proposed algorithm with Lim's and Yip's, TU channel

The probability of correct timing synchronization versus signal-to-noise ratio (SNR) is evaluated in Fig.4. It can be observed from the figure that upon utilizing the proposed joint optimization method, the probability of the correct STO estimation is improved to be much higher than the other two. In most cases, the proposed scheme can give perfect acquisition probability (equals to 1) while the other two could not. The simulation results also show that, compared with Lim's method, the proposed scheme can earn about 20dB or more performance improvement when the probability is 0.9 and 1. Meanwhile, the Yip's method is almost out of work even at high SNR when TU channel is utilized. Therefore, it could be concluded that the proposed algorithm could efficiently combat the multi-path Rayleigh fading channel with large rms delay spread and obtain excellent acquisition performance.

Fig.5 denotes the comparison among mean square errors (MSE) of channel estimation, which are based on the STOs obtained from the above three schemes. From this figure, it could also be seen that because of the low acquisition proba-

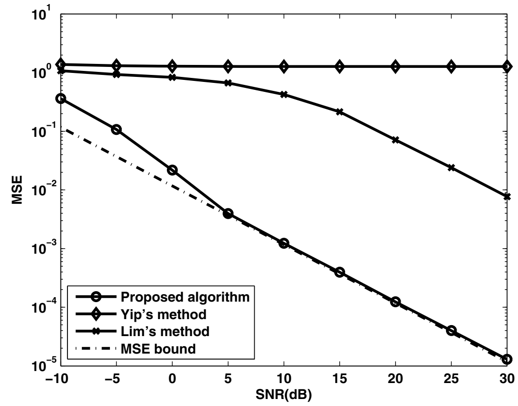

Fig. 5. Comparison of channel estimation MSE with non-ideal timing synchronization for proposed, Lim's, Yip's and the ideal MSE bound, TU channel

bility in timing synchronization, the Yip's method performs the worst as well. Nevertheless, compared with the other two, the performance of our proposed method is the best, whose MSE could achieve 10^{-3} at SNR=10dB and 10^{-4} at SNR=20dB, which is very close to the ideal bound of the time domain LS channel estimation algorithm which can be depicted as:

$$MSE_{LS} = \frac{L}{10^{SNR/10} \cdot N} \tag{12}$$

It could also be observed from Fig. 4 and Fig. 5 that both at low and high SNR, the proposed method performs best. And the two figures illustrate the robustness of our proposed joint optimization of timing synchronization and channel estimation approach.

V. Conclusion

In this article, a novel joint optimization of timing synchronization and channel estimation method is proposed for OFDM systems. Based on the shift delay characteristic of the synchronization sequence in channel estimation process, the algorithm could jointly enhance the correct acquisition probability performance of fine timing synchronization and mean channel estimation error. Simulation results testify that, in multi-path Rayleigh fading channel, the proposed method can afford nearly ideal STO acquisition accuracy (equal to 1 when SNR=3) and CIR MSE (close to ideal bound) performance.

References

[1] H. Steendam and M. Moeneclaey, "Synchronization sensitivity of multicarrier systems," *Eur. Trans. Commun., (ETT Special Issue on Multi-Carrier Spread Spectrum)*, vol. 52, p. 834C844, May 2004.

[2] K.-W. Yip, Y.-C. Wu, and T.-S. Ng, "Timing-synchronization analysis for ieee 802.11 a wireless lans on frequency- nonselective rician fading channels," in *Wireless Communications and Networking, 2003. WCNC 2003. 2003 IEEE*, vol. 1, 16-20 March 2003, pp. 78–82vol.1.

[3] M. Morelli, "Timing and frequency synchronization for the uplink of an ofdma system," *Trans. Commun.*, vol. 52, p. 296C306, Feb. 2004.

[4] T. M. Schmidl and D. C. Cox, "Robust frequency and timing synchronization for ofdm," vol. vol. 45, p. pp. 1613C1621, Dec 1997.

[5] M. Morelli and U. Mengali, "A comparison of pilot-aided channel estimation methods for ofdm systems," *IEEE Trans. Signal Processing*, vol. 49, no. 12, p. 3065C3073, Dec. 2001.

[6] G. F. M. Speth, S. A. Fechtel and H. Meyr, "Optimum receiver design for wireless broadband systems using ofdm. i," *IEEE Trans. Commun.*, vol. 47, no. 11, p. 1668C1677, Nov. 1999.

[7] J. L. E. G. Larsson, G. Liu and G. B. Giannakis, "Joint symbol timing and channel estimation for ofdm based wlans," *IEEE Commun. Lett.*, vol. 5, no. 8, p. 325C327, Aug. 2001.

[8] Y.Yao and G. B. Giannakis, "Blind carrier frequency offset estimation in siso, mimo, and multiuser ofdm systems," *IEEE Trans. Commun.*, vol. 53, no. 1, p. 173C183, Jan. 2005.

[9] M. Tanda, "Blind symbol-timing and frequency offset estimation in ofdm systems with real data symbols," *IEEE Trans. Commun.*, vol. 52, no. 10, p. 1609C1612, Oct. 2004.

[10] M. C. Necker and G. L. Stuber, "Totally blind channel estimation for ofdm on fast varying mobile radio channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, p. 1514C1525, Sep. 2004.

[11] W. C. Lim, B. Kannan, and T. Tjhung, "Joint channel estimation and ofdm synchronization in multipath fading," in *Communications, 2004 IEEE International Conference on*, vol. 2, 20-24 June 2004, pp. 983–987Vol.2.