

Transmit Beamforming for MIMO-OFDM Systems with Limited Feedback

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Abstract—The performance of a multi-input multi-output orthogonal frequency division multiplexing (MIMO-OFDM) system can be improved when the channel state information (CSI) is available for beamforming at the transmitter. However, even with the quantization of the CSI, the feedback requirement is still prohibitively unaffordable in a frequency division duplexing based MIMO-OFDM system with a large number of subcarriers. In this paper, two schemes with limited feedback, i.e., a nonlinear interpolation and a modified clustering based transmit beamforming schemes, are proposed. In the first method, the beamforming vectors for the non-pilot subcarriers in each cluster are constructed via that of the pilot subcarriers through nonlinear interpolation. In the second method, the beamforming vector for each cluster is selected to optimize the performance of a group of subcarriers which are chosen from that cluster based on equal spacing or a predefined random matrix. Simulations demonstrate that the proposed nonlinear interpolation and the modified clustering based transmit beamforming schemes outperform the existing limited feedback methods with the same feedback.

I. INTRODUCTION

The coupling of multi-input multi-output (MIMO) and orthogonal frequency division multiplexing (OFDM) techniques can inarguably improve spectral efficiency and reduce bit error rate (BER) [1], [2]. When the channel state information (CSI) is available at the transmitter, one popular approach of leveraging this gain is to make use of beamforming in which the transmitted waveform can be customized to provide higher link capacity and lower error rate in a MIMO-OFDM system [3].

Many researchers have demonstrated that a MIMO-OFDM system with better performance could be found even if only quantized CSI is available at the transmitter [4]–[10]. The quantized CSI may be sent to the transmitter from the receiver through a low-rate feedback channel in frequency division duplexing (FDD) systems. Unfortunately, even with the quantization of the CSI, the feedback requirement still grows in proportion to the number of the active subcarriers. There have existed some literatures on addressing the problem of how to best represent the set of beamforming vectors for the entire subcarriers in a MIMO-OFDM system with as few feedback bits as possible [8]–[10]. Beamforming vectors for the non-pilot subcarriers are determined by using linear interpolation and geodesic interpolation in [8] and [9], respectively.

This work was supported by TD-tech.

Although the Karcher mean clustering [9] may outperform the conventional clustering [8], it suffers high computational complexity. In [10], beamforming vectors for the non-pilot subcarriers are obtained by solving a weighted least squares problem on the Grassmann manifold. However, the above schemes can not fully exploit the correlation between beamforming vectors.

In this paper, two different schemes with limited feedback, i.e., the nonlinear interpolation and the modified clustering based transmit beamforming schemes, are presented. In the first approach, the quantized information of both the beamforming vectors and phase parameters are feedback to the transmitter. Then, beamforming vectors for the non-pilot subcarriers in each cluster are interpolated via that of the two neighboring pilot subcarriers using nonlinear interpolation. This method has better performance than the existing linear interpolation and geodesic interpolation approaches. In the modified clustering scheme, beamforming vector for each cluster is chosen to optimize the performance of a group of selected subcarriers in that cluster and the quantized information is also provided to the transmitter. Compared to the Karcher mean clustering and the concurrent clustering, this scheme shows a tradeoff between the computational complexity and the performance.

The rest of this paper is organized as follows. System model is presented in section II. The proposed nonlinear interpolation based transmit beamforming is explained in section III followed by the modified clustering based transmit beamforming in section IV. Simulation results are given in section V, while the conclusions are made in section VI.

II. SYSTEM MODEL

Block diagram of a MIMO-OFDM system using M_t transmit antennas and M_r receive antennas with limited feedback is illustrated in Fig. 1. At the transmitter, an independent data stream is dealt with a signal processing module followed by beamforming vector selection, which is based on the CSI feedback from the receiver. After that the conventional OFDM modulation is applied on each data stream of M_t transmit antennas, which is finally transmitted via individual antenna. The inverse operations are performed to recover the original data at the receiver. Assuming the size of inverse fast Fourier

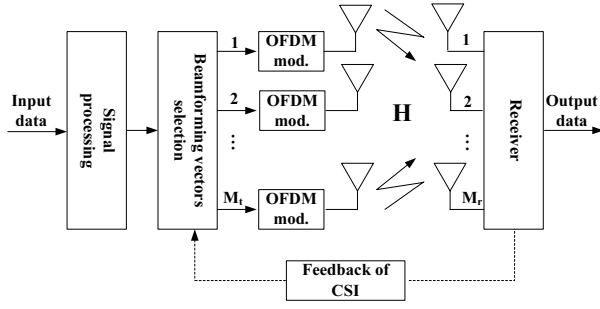


Fig. 1. Block diagram of a MIMO-OFDM system with limited feedback using M_t transmit antennas and M_r receive antennas.

transformation (IFFT) is N , the received signal at the k -th subcarrier yields a M_r -dimensional vector $\mathbf{y}(k)$, which is expressed as

$$\mathbf{y}(k) = \mathbf{H}(k) \mathbf{f}(k) s(k) + \mathbf{n}(k) \quad (1)$$

where $1 \leq k \leq N$, $s(k)$ is a symbol transmitted over the k -th subcarrier, which is multiplied by a $M_t \times 1$ beamforming vector $\mathbf{f}(k)$. The channel $\mathbf{H}(k)$ has independent and identically distributed (i.i.d.) entries according to $\mathcal{CN}(0, 1)$, which is perfectly known at the receiver but unknown to the transmitter. The M_r -dimensional noise vector $\mathbf{n}(k)$ has i.i.d. entries with zero mean and N_0 variance. Additionally, the transmitted signal energy is constrained to $E[s(k)s^H(k)] = \mathcal{E}_s$, where \mathcal{E}_s is the total transmitted power. For notational convenience, bold uppercase letters denote matrices, bold lowercase letters represent vectors, $E[\cdot]$ stands for the expectation operator, $(\cdot)^H$ and $(\cdot)^T$ denote conjugate transposition and transposition operators, respectively.

III. PROPOSED NONLINEAR INTERPOLATION BASED TRANSMIT BEAMFORMING

In broadband communication systems, there exists correlation between beamforming vectors illustrated in Fig. 2, in which pedestrian B (PB) channel model is utilized with 15 subcarriers per cluster, four transmit antennas and four receive antennas. It is observed that the correlation between beamforming vectors decreases nonlinearly with an increase in the distance between subcarriers. Furthermore, the beamformer correlation between two subcarriers with long distance is still large, e.g., a correlation of 0.8 with 10-subcarrier spacing. However, the linear interpolation coefficient [8], determined by (2), approaches zero with 14-subcarrier spacing as well as decreases linearly with an increase of subcarrier spacing

$$f_1(k) = 1 - (k-1)M/N + m \quad (2)$$

where $mN/M + 1 \leq k \leq (m+1)N/M$, $0 \leq m \leq M-1$, and M is the number of clusters per OFDM symbol.

In order to approximate the correlation between beamforming vectors more accurately, two functions of nonlinear

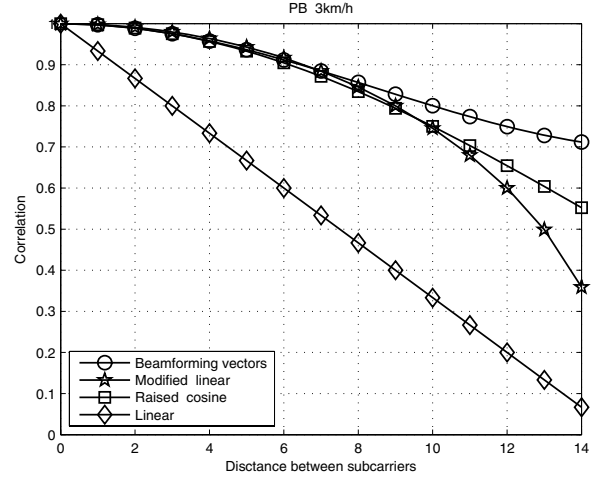


Fig. 2. Comparison among linear interpolator, two nonlinear interpolators, and the correlation between beamforming vectors with four transmit antennas and four receive antennas.

interpolation coefficient are proposed, i.e., the modified linear interpolation coefficient expressed by (3)

$$f_2(k) = \sqrt{1 - ((k-1)M/N - m)^2} \quad (3)$$

and the raised cosine based interpolation coefficient given by

$$f_3(k) = (\cos(\pi M (\text{mod}(k-1, N/M))/(2N)) + 1)/2 \quad (4)$$

where $\text{mod}(\cdot)$ represents modulo operator.

As in conventional linear interpolation [8], the beamforming vector for the pilot subcarrier in each cluster is first derived according to some criteria, such as minimize MSE (Mean Square Error) determined by (6)

$$\text{MSE}(\mathbf{f}(k)) = \mathcal{E}_s \left(1 + \frac{\mathcal{E}_s}{N_0} \mathbf{H}_{eq}^H(k) \mathbf{H}_{eq}(k) \right)^{-1} \quad (5)$$

$$\mathbf{f} \left(\frac{mN}{M} + 1 \right) = \arg \min_{\mathbf{f} \in \mathcal{F}} \text{MSE}(\mathbf{f}) \quad (6)$$

or maximize capacity given by (8)

$$C(\mathbf{H}_{eq}(k)) = \log_2 \det \left(1 + \frac{\mathcal{E}_s}{N_0} \mathbf{H}_{eq}^H(k) \mathbf{H}_{eq}(k) \right) \quad (7)$$

$$\mathbf{f} \left(\frac{mN}{M} + 1 \right) = \arg \max_{\mathbf{f} \in \mathcal{F}} C \left(\mathbf{H} \left(\frac{mN}{M} + 1 \right) \mathbf{f} \right) \quad (8)$$

where $\mathbf{H}_{eq}(k) = \mathbf{H}(k)\mathbf{f}(k)$ is the effective channel gain, \mathcal{F} is the predefined codebook consisting of quantized beamforming vectors.

Then the beamforming vectors for the non-pilot subcarriers in the m -th cluster are interpolated by that of the two adjacent pilot subcarriers using two nonlinear interpolators, i.e., the modified linear interpolator (modified LI), which is given by (9)

$$\mathbf{f}(k) = \sqrt{(1 - c_k^2)} \mathbf{f} \left(\frac{mN}{M} + 1 \right) + c_k \mathbf{f} \left(\frac{(m+1)N}{M} + 1 \right) e^{j\theta} \quad (9)$$

where $c_k = (k-1)M/N - m$, $mN/M + 1 < k \leq (m+1)N/M$, $\mathbf{f}(N+1) = \mathbf{f}(1)$, θ is a phase parameter for optimization which has a substantial influence on the performance of an interpolator, and the raised cosine based interpolator determined by (10)

$$\hat{\mathbf{f}}(k) = c_k \mathbf{f}\left(\frac{mN}{M} + 1\right) + s_k \mathbf{f}\left(\frac{(m+1)N}{M} + 1\right) e^{j\theta} \quad (10)$$

where $c_k = (\cos(\pi M(\text{mod}(k-1, N/M))/(2N)) + 1)/2$ and $s_k = (\sin(\pi M(\text{mod}(k-1, N/M))/(2N)) + 1)/2$.

It should be noticed that it is unnecessary to normalize the interpolated beamforming vectors given by (11) in the modified LI, which is required in the linear interpolation and raised cosine based interpolation. However, the beamforming vector of the modified LI combined with the phase parameter is subject to (12)

$$\mathbf{f}(k) = \hat{\mathbf{f}}(k) \times \left(\hat{\mathbf{f}}^H(k) \hat{\mathbf{f}}(k) \right)^{-1/2} \quad (11)$$

$$\mathbf{f}^H(k_{p1}) \mathbf{f}(k_{p2}) e^{j\theta} + e^{-j\theta} \mathbf{f}^H(k_{p2}) \mathbf{f}(k_{p1}) = 0 \quad (12)$$

where $k_{p1} = mN/M + 1$ and $k_{p2} = (m+1)N/M + 1$.

For the sake of maximizing the performance of the two proposed nonlinear interpolators, the receiver evaluates the optimal phase parameter θ , which follows the line of [8]. And the optimal way is to apply phase parameter optimization on all the subcarriers in each cluster based on the same metrics used to derive the beamforming vectors for the pilot subcarriers. However, the center subcarrier of each cluster is only considered for reducing the computational complexity and a phase parameter which has the minimum MSE given by (13)

$$\theta_m = \arg \min_{\theta \in \Theta} \text{MSE}(\mathbf{f}(\lceil k \rceil; \theta)) \quad (13)$$

or the maximum capacity determined by (14) is chosen as the phase parameter of that cluster

$$\theta_m = \arg \max_{\theta \in \Theta} C(\mathbf{H}(\lceil k \rceil) \mathbf{f}(\lceil k \rceil; \theta)) \quad (14)$$

where $\Theta = \{0, 2\pi/P, \dots, 2\pi(P-1)/P\}$ is a predefined phase parameter codebook containing quantized phases, P is the number of quantized levels, $k = (m+1/2)N/M$, and $\lceil A \rceil$ is a minimum integer that is larger than A .

Finally, the indices of the beamforming vectors are conveyed back to the transmitter in conjunction with phase parameters.

IV. MODIFIED CLUSTERING BASED TRANSMIT BEAMFORMING

In conventional clustering based approach [8], the beamforming vectors for the center subcarriers in each cluster are reused for the other subcarriers in that cluster. Although clustering is an efficient quantization strategy that exploits beamformer correlation to reduce feedback requirement, it suffers from mismatch at the edges of the boundaries, especially with a large number of subcarriers per cluster. Karcher mean clustering based approach developed in [9] avoids the

drawback of the current clustering; however, the beamformer reused for each cluster is derived through locally minimizing the objective function given by (15) and it is computationally expensive

$$\mathbf{f}_m = \arg \min_{\mathbf{f} \in \mathcal{F}} \sum_{l=1}^{N/M} d^2(\mathbf{f}, \mathbf{f}(mN/M + l)) \quad (15)$$

where $d(\cdot, \cdot)$ is a subspace distance.

In the modified clustering based scheme, each cluster is first partitioned into several subclusters according to a predefined threshold of beamformer correlation BC_0 . Then, one subcarrier of each subcluster is selected to form a group in that cluster and there are two styles of selecting this group of subcarriers, i.e., the first style is to select the center subcarrier of each subcluster (namely interleaved way), whereas the second one is based on a predefined random matrix shown in (16) to select one subcarrier of each subcluster (namely random matrix based way)

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1L} \\ a_{21} & a_{22} & \cdots & a_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ a_{V1} & a_{V2} & \cdots & a_{VL} \end{pmatrix} \quad (16)$$

where \mathcal{V} represents the number of candidate random combinations, $a_{vl} \in \{1, 2, \dots, N/(LM)\}$, $1 \leq v \leq \mathcal{V}$, $1 \leq l \leq L$, and L denotes the number of the selected subcarriers per cluster.

Finally, the beamforming vector for each cluster can be chosen according to the capacity criterion by using the interleaved and random matrix based approaches, which are expressed by (17) and (18), respectively

$$\mathbf{f}_m = \arg \max_{\mathbf{f} \in \mathcal{F}} \sum_{l=1}^L C\left(\mathbf{H}\left(\frac{mN}{M} + \left\lceil \left(l - \frac{1}{2}\right) \frac{N}{ML} \right\rceil\right) \mathbf{f}\right) \quad (17)$$

$$\mathbf{f}_m = \arg \max_{\mathbf{f} \in \mathcal{F}} \sum_{l=1}^L C\left(\mathbf{H}\left(\frac{mN}{M} + \left\lceil a_{vl} + (l-1) \frac{N}{ML} \right\rceil\right) \mathbf{f}\right) \quad (18)$$

Otherwise, the beamforming vector for each cluster is determined by the MSE criterion using the above two approaches, which are given by (19) and (20), respectively

$$\mathbf{f}_m = \arg \min_{\mathbf{f} \in \mathcal{F}} \max_{1 \leq l \leq L} \text{MSE}\left(\mathbf{H}\left(\frac{mN}{M} + \lceil k_{p3} \rceil\right) \mathbf{f}\right) \quad (19)$$

$$\mathbf{f}_m = \arg \min_{\mathbf{f} \in \mathcal{F}} \max_{1 \leq l \leq L} \text{MSE}\left(\mathbf{H}\left(\frac{mN}{M} + \lceil k_{p4} \rceil\right) \mathbf{f}\right) \quad (20)$$

where $k_{p3} = (l-1/2)N/(ML)$ and $k_{p4} = a_{vl} + (l-1)N/(ML)$.

V. SIMULATION RESULTS

In the simulations, In the simulation, a MIMO-OFDM system, which has $M_t = M_r = 4$, $N = 512$, and a cyclic prefix of 36, employs Pedestrian B (PB) Channel Model with a bandwidth of 5MHz, a sampling rate of 7.68MHz, and a carrier frequency of 2GHz. The system employs cyclic redundancy

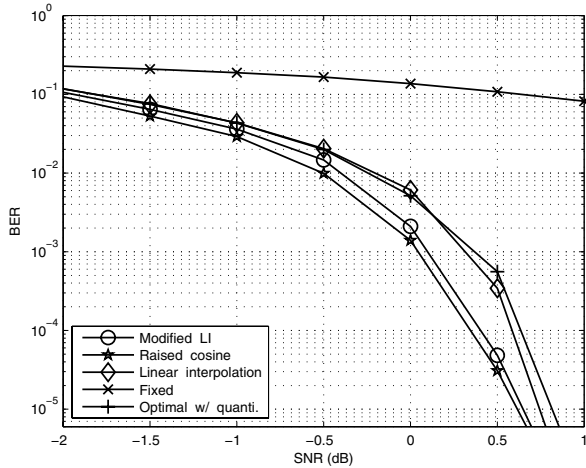


Fig. 3. Performance comparison among various limited feedback techniques with 15 subcarriers per cluster.

check (CRC) of length-16, Turbo code with rate one-half, a block interleaver, and 16QAM modulation. The total transmitted power is normalized to one, which is equally allocated over all subcarriers. Furthermore, the receiver exploits ideal channel estimation with linear MMSE (LMMSE) equalizer and every point of the simulation results was obtained by averaging over 2000 independent realizations of channel and noise. The beamformer codebook is obtained from [11]. Furthermore, the capacity criterion is used to select the beamforming vectors and the phase parameters throughout the following simulations.

Fig. 3 compares the BER performance among various limited feedback schemes with 15 subcarriers per cluster. For the two proposed nonlinear interpolators and linear interpolator, a codebook of size $|\mathcal{F}| = 4$ is utilized and θ is uniformly quantized using 2 bits. Therefore, a total of 80 bits of feedback information are required per OFDM symbol. A codebook of size $|\mathcal{F}| = 4$ is used in the optimal transmit beamforming with quantization requiring a total of 600 bits, whereas $|\mathcal{F}| = 1$ in the fixed transmit beamforming without any feedback. There are some conclusions that can be drawn from this figure. First, the fixed transmit beamforming has much worse performance than the other limited feedback strategies in the whole simulated SNR regime. Second, because of exploiting the correlation between beamformers sufficiently, the raised cosine based interpolator gains over Choi's linear interpolator by 0.3 dB at a BER of 10^{-3} . Moreover, this nonlinear interpolator has slightly better performance than the modified LI at the expense of higher computational complexity. Finally, it is of particular interest to note that the two proposed nonlinear interpolators provide better performance than the optimal transmit beamforming with quantization. The reason for this performance gain is that codebook size utilized in this simulation is small, which has significant effect on the BER performance. Furthermore, the two proposed interpolators has four extra phase parameters for optimizing the interpolated

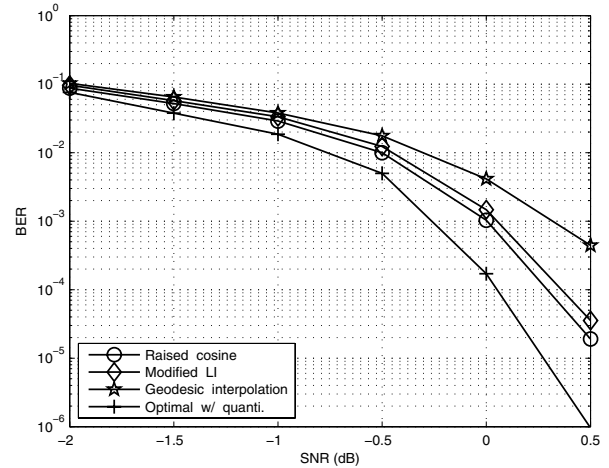


Fig. 4. Performance comparison among various limited feedback techniques with 15 subcarriers per cluster. Codebook partition technique is utilized in the two proposed nonlinear interpolators.

beamforming vectors.

Fig. 4 shows the BER performance among various limited feedback techniques with 15 subcarriers per cluster. In this experiment, for the two proposed nonlinear interpolators, a beamformer codebook of size $|\mathcal{F}| = 4$ and a phase codebook of size $|\Theta| = 2$ are used resulting in 60 bits per OFDM symbol. Both the geodesic interpolator and the optimal transmit beamforming with quantization use a beamformer codebook of size $|\mathcal{F}| = 8$. Thus, a total of 60 bits and 900 bits per OFDM symbol are required in the geodesic interpolator and the optimal beamforming with quantization, respectively. Since the codebook size has significant effect on the BER performance, a codebook partition technique is utilized in the two proposed nonlinear interpolators. The principle of this technique is to partition the beamformer codebook of size $|\mathcal{F}| = 8$ into two beamformer codebook subsets of size 4, and then these two subsets are utilized in each cluster under certain sequence, which is expressed by (21)

$$CB_m = \text{mod}(m, 2) + 1. \quad (21)$$

Additionally, the beamforming vectors for the non-pilot subcarriers in each cluster is determined by (??) in the geodesic interpolation. In the observed SNR range, the raised cosine based interpolation has slightly better performance than the modified LI at the expense of higher computational complexity. At a BER of 10^{-3} , this approach outperforms the geodesic interpolation by 0.3 dB and performs 0.25dB within the optimal beamforming with quantization.

Fig. 5 illustrates the BER performance comparison among different limited feedback schemes with 60 subcarriers per cluster. A codebook of size $|\mathcal{F}| = 16$ is utilized in all the simulated schemes. Thus, a total of 20 bits and 1200 bits are required in the modified clustering and the optimal transmit beamforming with quantization, respectively. Moreover, the feedback bits of the Karcher mean clustering, the geodesic

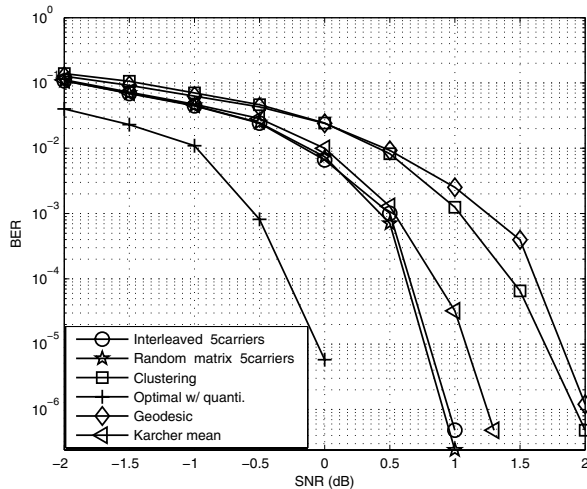


Fig. 5. Performance comparison among various limited feedback techniques when $|\mathcal{F}| = 16$ and 60 subcarriers per cluster.

interpolation, and the conventional clustering are the same as that of the two modified clustering schemes. For consideration of the computational complexity, $L = 5$ is used in the two modified clustering schemes. Furthermore, $V = 4$ is utilized in the random matrix based clustering. In addition, the beamforming vectors chosen for each cluster is given by (15) in the Karcher mean clustering with chordal distance metric. It is observed that the performance of the interleaved approach is identical with that of random matrix based approach. At the probability of BER of 10^{-4} , the interleaved approach provides around 0.25dB, 1dB, and 1.2dB over the Karcher mean clustering, the conventional clustering, and the geodesic interpolation, respectively. Moreover, the interleaved approach exhibits 1dB degradation from the case of the optimal beamforming with quantization.

VI. CONCLUSION

In this paper, a nonlinear interpolation and a modified clustering based transmit beamforming schemes are proposed to reduce the feedback information and improve performance for a MIMO-OFDM system. In the first approach, the indices of both beamforming vectors of the pilot subcarriers and phase parameters are fed back to the transmitter. Then the beamforming vectors for the non-pilot subcarriers in each cluster are constructed via that of the pilot subcarriers using nonlinear interpolation, which fully exploits the correlation between beamforming vectors and outperforms the existing linear interpolation and geodesic interpolation. In the second scheme, only the indice of the beamforming vector in each cluster is conveyed back to the transmitter and the beamforming vector is reused for the whole subcarriers in that cluster. This method, considered as a tradeoff between the computational complexity and the performance, provides better performance than the Karcher mean clustering, the conventional clustering, and the geodesic interpolation.

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