# Channel Estimation Error on Performance of Zero Force Beamforming based Multiuser SDMA in Downlink

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***Abstract*-In a MIMO OFDMA multiuser system, the multiuser diversity gain can be achieved both in spatial and frequency domain by joint spatial and frequency scheduling. Although the Channel Estimation Error (CEE) may degrade such multiuser diversity gain, little work has been performed on it. In this paper, we investigate the effect of the CEE on MIMO OFDMA downlink based on Multiuser Zero Force Beamforming (ZFB). The effect of channel estimation error on the Signal to Noise Ratio of the desired user is analyzed, and the system spectrum efficiency is evaluated for different CEE value. The simulation results show that ZFB is not sensitive to CEE when a reasonable CEE value is guaranteed in a cellular system. Compared to the multiuser diversity gain from joint spatial frequency scheduling, the performance degradation from CEE is acceptable.**

**Key words: ZFB, channel estimation error, MIMO**

## I. INTRODUCTION

In recent research on the multiuser Multiple Input and Multiple Output (MIMO) transmission and scheduling [1] has been investigated much, and it is proved that multiuser spatial multiplexing and diversity can improve the total system capacity of multiuser MIMO greatly. On the other hand, the dynamical subcarrier and power allocation [2] in multiuser Orthogonal Frequency Division Multiplexing Access (OFDMA) system can also achieve the multiuser diversity in frequency domain and obtain much higher system capacity than the single user system. Naturally, the joint spatial and frequency multiuser diversity gain can be achieved to improve the multiuser system capacity by dynamical resource allocation in MIMO OFDMA system.

However, in most work on the multiuser diversity, it is assumed that the Channel Status Information (CSI) is full available at the transmitter, which is too idealistic for a multiuser MIMO cellular system. First, the channel estimation will introduce some error to the channel status information, especially for MIMO OFDM channel estimation. Since the pilots for different users and antennas should be orthogonal to each other, and the heavy overhead for the pilot should be avoided, the pilots should be scattered in spatial, frequency and time domain

in discrete, and thus the interpolation in frequency and time domain is necessary to obtain the channel estimation for all the data symbols. Furthermore, the pilot is polluted by noise and the inter-cell interference. All these lead to error on the channel estimation. Second, different duplex mode supports different method to acquire CSI [4]. For a TDD system, the channel reciprocity can be utilized to obtain the CSI at the transmitter for downlink and downlink share the same frequency band and identified by different timeslots. For FDD system, CSI can only be feedback from receiver or transformed from the receiving link since downlink and downlink is separated in different frequency band and CSI for downlink and downlink are independent anytime. Generally, CSI can be utilized to perform cooperative processing in multiuser MIMO TDD system.

So the effect of channel estimation error (CEE) on the system performance should be considered in a multiuser MIMO OFDMA system. Recently, the optimization on MIMO scheduling is investigated with CEE in [5].

In this paper, the CEE of MIMO is summarized, and the impact of the CEE on performance of MIMO OFDMA downlink based on Zero Force Beamforming (ZFB) [3] is investigated, and the performance degradation is simulated. From the simulation results, the more users multiplexed on the same subcarrier group, the more reduction on spectrum efficiency with the same CEE. However, ZFB is little sensitive to CEE, and compared to the multiuser diversity gain from the joint and dynamic spatial and frequency scheduling, the performance degradation is acceptable when the reasonable CEE value can be guaranteed in a cellular system.

## II. MIMO CHANNEL ESTIMATION ERROR

In wireless system, the CSI is estimated from the pilot symbols distributed on different time and frequency resource units, which is known at both the transmitter and receiver. Even the power on the pilot symbol can be much higher than that on the data symbols, the channel estimation could introduce error to CSI for the limited samples both in time, frequency and spatial domain since orthogonal pilots should be transmitted for multiple users independently. The real CSI is obtained by the estimation on limited samples in spatial-time and frequency domain and approached by the interpolation in time and frequency domain. The orthogonality among different users' pilots can be implemented by Time Division Multiplexing (TDM), Code Division Multiplexing (CDM) or Frequency Division Multiplexing (FDM), while the density of the pilot should meet the requirements of the Niquist sampling theory in spatial, frequency and time domain to avoid the aliasing. However, to avoid heavy overhead for the system, the pilot density is expected as little as small as possible. Further the interference and the noise in a multi-cell scenario of cellular system will degrade the channel estimation accuracy, which

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leads to error on the CSI. The channel estimation error usually is expressed as its Mean squared Error (MSE), which usually is concerned to the Ratio of Signal to Interference and Noise (SINR) on the pilot as mentioned above.

The instantaneous CSI for user $k$ can be expressed as:

$$\hat{\mathbf{H}}_k = \mathbf{H}_k + \Delta\mathbf{H}_k \quad (1)$$

Where $\mathbf{H}_k$ and $\Delta\mathbf{H}_k$ are the ideal channel impulse response matrix and the CEE matrix of MIMO channel respectively, and then the MSE can be calculated as:

$$MSE = \sum_{i=}^{N_R}\sum_{j=1}^{N_T}\left|\Delta h_{i,j}\right|^2 \quad (2)$$

Where $\Delta h_{i,j}$ is the element $(i,j)$ of $\Delta\mathbf{H}_k$, which means the channel estimation error on the channel impulse response between transmitter antenna $j$ and receiver antenna $i$. MSE is concerned to the SINR on the pilot for transmitter antenna $j$, the higher the SINR, the less the MSE. To evaluate the effect of the channel estimation error on the multiuser MIMO performance, suitable model for the channel estimation error and MSE is necessary.

Usually, it is assumed that $\Delta\mathbf{H}_k$ and $\mathbf{H}_k$ are independent each other since $\mathbf{H}_k$ is decided by the physical environment where the UE and the Node B locate, and $\Delta\mathbf{H}_k$ is decided by the pilot pattern and the channel estimation method. The element of $\Delta\mathbf{H}_k$ satisfies a Zero Mean Cyclical Symmetrical Complex Gaussian (ZMCSCG) distribution, its variance is [6]:

$$\sigma_{\Delta\mathbf{H}}^2 = \mathrm{E}\left(\left|\left(\mathbf{H}_k\right)_{i,j}\right|^2\right) - \mathrm{E}\left(\left|\left(\hat{\mathbf{H}}_k\right)_{i,j}\right|^2\right) \quad (3)$$

The distribution of the element of $\hat{\mathbf{H}}_k$ is also ZMCSCG, its variance is $1-\sigma_{\Delta\mathbf{H}}^2$, where $\sigma_{\Delta\mathbf{H}}^2$ is determined by the quality of the channel estimation and decided by the dynamical variance and the channel estimation scheme.

## III. THE EFFECT OF CHANNEL ESTIMATION ERROR ON PERFORMANCE OF ZFB

The theoretical diagram of multiuser ZFB can be illustrated as following:

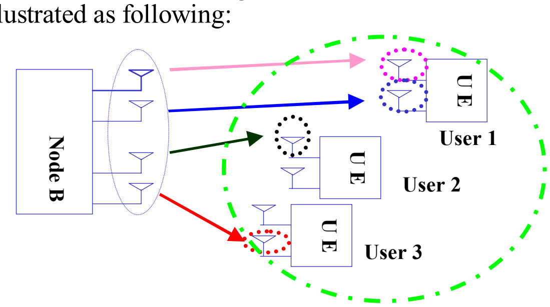


Figure 1. Zero Forcing Beamforming

All the antennas of UEs are constructed as a virtual UE, which owns all the antennas from different UEs. The MIMO channel matrix of the virtual UE is expressed as:

$$\mathbf{H} = \left[\mathbf{H}^1 \quad \ldots \quad \mathbf{H}^i \quad \ldots \quad \mathbf{H}^K\right] \quad (4)$$

Where $\mathbf{H}^i$ is MIMO channel matrix of UE $i$. Then the receiver antenna selection is executed to select partial antennas to receive at the virtual UE. To guarantee the orthogonality among the independent data streams from the Node B, the number of the selected receiving antenna should be fewer than that at the Node B.

With the MIMO channel matrix selected, zero forcing beamforming is applied. The beamforming weights for the data streams are calculated as following [3]:

$$\mathbf{F} = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\mathbf{D} \quad (5)$$

Where $\mathbf{D} = diag\left(d_1,\ldots d_k,\ldots,d_M\right)$ is the diagonal matrix which keeps the transmit power unchanged after beamforming, and $H$ means the hermit transpose.

$$d_k = 1\Big/\sqrt{\left[\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\right]_{k,k}} \quad (6)$$

If $M$ receiver antennas are selected, and $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the modulated symbol vector, the element $x_k$ is the transmitted data symbol on the data stream $k$, the transmitted signal after beamforming is:

$$\mathbf{y} = \mathbf{F}\mathbf{x} \quad (7)$$

And after the channel, the received signal at the virtual UE can be expressed as:

$$\mathbf{r} = \mathbf{H}\mathbf{y} + \mathbf{n} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n} = \mathbf{D}\mathbf{x} + \mathbf{n} \quad (8)$$

Because $\mathbf{D}$ is a diagonal matrix, the MIMO channel is decomposed into $M$ SISO channels with channel gain $d_k$. The total power of the Node B is distributed uniformly on every data streams to maximize the total system capacity.

If the power allocated to data stream $k$ is expressed as $p_k = P_T / M / N$, then the capacity is expressed as:

$$C = \sum_{k=1}^{M}\log_2(1+SNR_k) = \sum_{k=1}^{M}\log_2\left(1+\frac{P_T d_k^2}{MN\sigma^2}\right) \quad (9)$$

For multiuser MIMO system, the effect of the CEE on the system performance varies with the MIMO transmission strategies for the different utilization method. In this paper, the effect of CEE on ZFB is investigated.

The imperfect multiuser MIMO matrix selected can be modeled as following:

$$\hat{\mathbf{H}} = \mathbf{H} + \Delta\mathbf{H} \quad (10)$$

Where $\mathbf{H}$ is the perfect channel matrix, $\hat{\mathbf{H}}$ is the imperfect MIMO channel matrix obtained, and $\Delta\mathbf{H}$ is the estimation error of the MIMO channel matrix, which element is a random with distribution $CN\left(0,\sigma_{MSE}^2\right)$ and $\sigma_{MSE}^2$ is the MSE of the channel estimation. So the ZFB can be performed as following:

$$\begin{aligned}\hat{\mathbf{F}} &= \hat{\mathbf{H}}^H\left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H\right)^{-1}\hat{\mathbf{D}} \\ &= \left(\mathbf{H}+\Delta\mathbf{H}\right)\left(\left(\mathbf{H}+\Delta\mathbf{H}\right)\left(\mathbf{H}+\Delta\mathbf{H}\right)^H\right)^{-1}\hat{\mathbf{D}}\end{aligned} \quad (11)$$

Then the total received signal of all the users selected can be unified as:

$$\hat{\mathbf{r}} = \mathbf{H}\hat{\mathbf{F}}\mathbf{x} + \mathbf{n} = \mathbf{H}\hat{\mathbf{H}}^H\left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H\right)^{-1}\hat{\mathbf{D}}\mathbf{x} + \mathbf{n}$$
$$= \mathbf{H}\left(\mathbf{H}+\Delta\mathbf{H}\right)^H\left(\left(\mathbf{H}+\Delta\mathbf{H}\right)\left(\mathbf{H}+\Delta\mathbf{H}\right)^H\right)^{-1}\hat{\mathbf{D}}\mathbf{x} + \mathbf{n} \quad (12)$$
$$= \overline{\mathbf{H}}\mathbf{x} + \mathbf{n}$$

$$\overline{\mathbf{H}} = \mathbf{H}\left(\mathbf{H}+\Delta\mathbf{H}\right)^H\left(\left(\mathbf{H}+\Delta\mathbf{H}\right)\left(\mathbf{H}+\Delta\mathbf{H}\right)^H\right)^{-1}\hat{\mathbf{D}} \quad (13)$$

Where $\mathbf{x} = \left[x_1, x_2, ..., x_{N_T}\right]^T$ is the transmitted modulation symbols vector at the Node B, and $\hat{\mathbf{r}} = \left[r_1, r_2, ..., r_{N_T}\right]$ is the received signal vector of all users' $N_T$ selected receiving antennas for ZFB.

Since there exists channel estimation error $\Delta\mathbf{H}$ between the perfect channel status information and the estimated CSI, there is a error $\Delta\mathbf{F}$ between the real precoding matrix $\hat{\mathbf{F}}$ and the perfect precoding matrix $\mathbf{F}$ from the perfect CSI, and $\hat{\mathbf{F}}$ and $\mathbf{H}$ don't match any more, so after the pre-coded signal passed through the real channel, the equivalent channel response matrix $\mathbf{H}\hat{\mathbf{F}}$ of the MIMO channel is not diagonal any more:

$$\mathbf{H}\left(\mathbf{H}+\Delta\mathbf{H}\right)^H\left(\left(\mathbf{H}+\Delta\mathbf{H}\right)\left(\mathbf{H}+\Delta\mathbf{H}\right)^H\right)^{-1} \neq \mathbf{I} \quad (14)$$

So the signal $x_j$ from different transmit antenna will overlapped each other and interfere each other, and the orthogonality among the different spatial sub-channel is destroyed, and the SNR of the received signal on every spatial sub-channel of ZFB will be degraded.

The SNR of the received signal from every transmit antenna can be expressed as following:

$$SNR_i = \frac{\mathrm{E}\left|\left(\overline{\mathbf{H}}\right)_{i,i} x_i\right|^2}{\mathrm{E}\left|\sum_{\substack{j=1 \\ j\neq i}}^{N_T}\left(\overline{\mathbf{H}}\right)_{i,j} x_j\right|^2 + \sigma^2} \quad (15)$$

Assumes the signal $x_i$ on every spatial sub-channel is independent one another, which is a random with distribution $CN(0,1)$, and then the equation (14) can be simplified as following:

$$SNR_i = \frac{\mathrm{E}\left|\left(\overline{\mathbf{H}}\right)_{i,i}\right|^2}{\left|\sum_{\substack{j=1 \\ j\neq i}}^{N_T}\mathrm{E}\left(\overline{\mathbf{H}}\right)_{i,j}\right|^2 + \sigma^2} \quad (16)$$

## IV. Simulation Parameters

In our simulation, Adaptive Modulation and coding is adopted on every subcarrier group, and the Modulation and Coding Scheme (MCS) selected is as Table 1.

Table 1 The MCS and the SNR threshold

| MCS | Mod | Code Ratio | data bits | SNR threshold |
|---|---|---|---|---|
| 1 | QPSK | 1/3 | 2/3 | 0.5dB |
| 2 | QPSK | 1/2 | 1 | 3.7dB |
| 3 | QPSK | 3/4 | 3/2 | 6.3dB |
| 4 | 16QAM | 1/2 | 2 | 10dB |
| 5 | 16QAM | 3/4 | 3 | 15.2dB |

To guarantee the optimal throughput performance, the SNR threshold as Table 1 is obtained.

The frame structure from [8] is adopted in our simulation. Every 10ms is divided into two sub frames, and the structure of the sub frame is as Figure 5. Every sub frame has 7 service slots and 3 special slots. The service slot length is 0.675ms, and 6 slots can be used to transmit data. In every service time slot, 9 OFDM symbols can be contained.

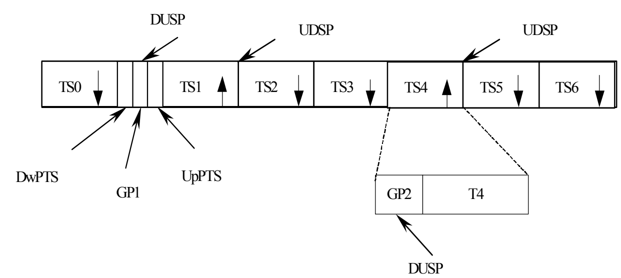


Figure 2. Frame structure of TD-SCDMA LTE

The independent multi-path Rayleigh fading MIMO channel is adopted in our simulation, and the Power Delay Profile (PDP) of Typical Urban (TU) is adopted. The variance of the channel estimation error of every element of the MIMO channel impulse response matrix is simulated from 0~0.15.

Table 2. System parameters

| Parameter | Assumption |
|---|---|
| Carrier Frequency | 2GHz |
| Band width | 10MHz |
| Sample Frequency | 1.92 MHz |
| Sub-carrier spacing | 15 kHz |
| CP length(μs/samples) | 7.29/14 |
| FFT Size | 128 |
| Occupied Subcarriers number | 16 |
| Subcarrier Group number | 75 |
| Total receiver SNR at Node B on every subcarrier | 10dB |
| PDP of channel | Typical Urban (TU) |
| User traffic | Full buffer None-Real Time service |

## V. Simulation Results

In this section, the channel estimation error on the MU-ZFB performance is presented as Figure 3 and Figure 4 for MIMO 2×4 and 4×8 respectively.

Generally, the system spectrum improved with the user number since the spatial-frequency multiuser diversity gain is achieved by MU-ZFB and spatial frequency scheduling. However, the CEE degraded the system performance since it leads to interference among the different spatial sub-channel on the same subcarrier. Compared to the case with perfect CSI, the spectrum efficiency will be reduced about

60% for both MIMO 2×4 and MIMO 4×8 case when $\sigma^2_{MSE}$ increases from 0 to 0.15

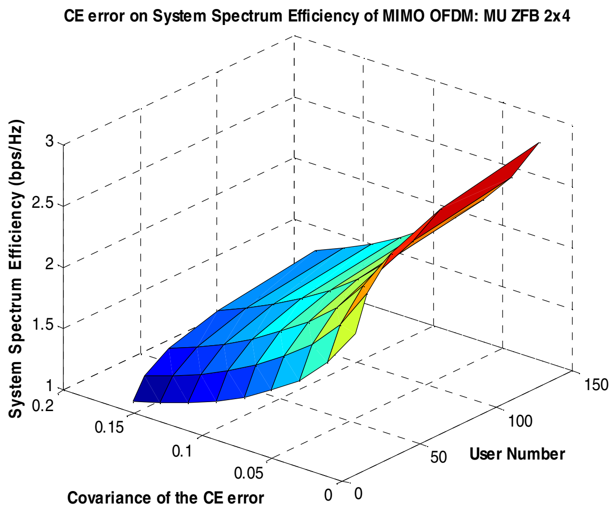


Figure 3. CEE on spectrum efficiency (MIMO 2×4)

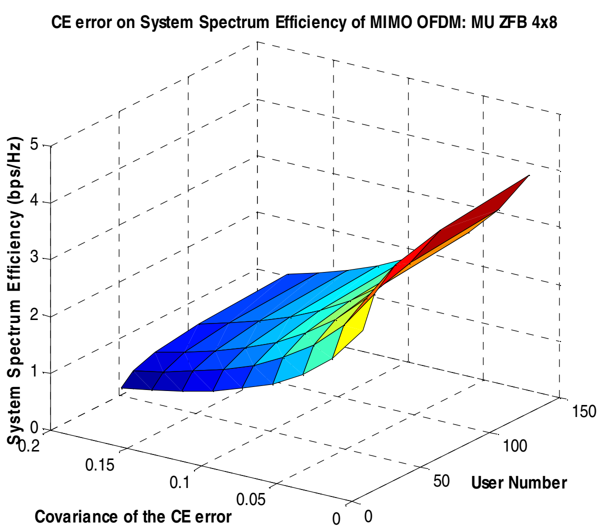


Figure 4. CEE on spectrum efficiency (MIMO 4×8)

However, in a cellular system, $\sigma^2_{MSE}$ can be achieved less than -15dB easily, the loss caused by CEE can't exceeds 17% and 24% respectively for MIMO 2×4 and MIMO 4×8. So we can draw a conclusion from this results, the MU-ZFB is little sensitive to CEE, the performance degradation caused by CEE is acceptable compared to the performance gain from the multiuser multiplexing and diversity.

## VI. Conclusion

In this paper, the impact of the channel estimation error on multiuser diversity is investigated in MIMO OFDMA downlink based on MU-ZFB. Although the multiuser diversity gain can be achieved both in spatial and frequency domain by joint spatial and frequency scheduling and the system spectrum improved with the user number, the CEE degraded the system performance since it leads to interference among the different spatial sub-channel on the same subcarrier. Compared to the case with perfect CSI, the spectrum efficiency will be reduced about 60% for both MIMO 2×4 and MIMO 4×8 case when $\sigma^2_{MSE}$ increases from 0 to 0.15. In a practical system, $\sigma^2_{MSE}$ can be achieved less than -15dB easily, and the performance degradation caused by CEE can't exceeds 17% and 24% respectively for MIMO 2×4 and MIMO 4×8. So the MU-ZFB is little sensitive to CEE, the performance degradation caused by CEE is acceptable for MU-ZFB compared to the performance gain from the multiuser multiplexing and diversity of MU-ZFB

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