ZCZ Sequences-based Frequency Synchronization for Interleaved OFDMA Uplink

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Abstract—**In this paper, we propose a zero correlation zone (ZCZ) sequences-based carrier frequency offset estimation algorithm for interleaved OFDMA uplink. By introducing *the effective carrier frequency offset* (CFO) and designing an appropriate training sequence for each user, we propose a training sequence-based CFO estimation algorithm thus avoiding the computationally complex subspace-based method proposed by Cao & Tureli[1]. Zero correlation zone sequence set is used as training sequence to mitigate multipath inference and multiuser inference. Simulation results show that the proposed algorithm is effective in multi-path fading channel**.

Keywords-orthogonal frequency division mutiple access; zero correlation zone(ZCZ) seqeunce; carrier synchronizationt

I. INTRODUCTION

Orthogonal frequency division multiplex (OFDM) has been proposed as the physical layer technology for broadband wireless multiple access system. It is referred to as orthogonal frequency division multiple access (OFDMA), such as in IEEE 802.16. In OFDMA, closely spaced and overlapped multiple carriers are divided into groups and assigned to multiple users for simultaneous transmission.

Studies have shown that OFDMA inherits the sensitive to frequency synchronization errors from OFDM [2]. Carrier frequency offset introduces not only inter-carrier interference but also multiple access interference in OFDMA. Particularly in the uplink, each user is subject to different frequency offsets. Synchronization is more challenging compared with that in single user OFDM systems.

In OFDMA uplink, sub-band based subcarrier assignment scheme has been studied in [3][4], in which signals from different users occupy non-overlapping frequency bands in a similar fashion as traditional FDMA. Guard subcarriers are suggested in [3][4] to be put in the edge of each sub-band such that the multiple access interference can be minimized. Signals from different users can thus be separated by filter banks and existing synchronization algorithms for OFDM is applicable for the signal on each sub-band.

Interleaved carrier assignment scheme is superior over the sub-based scheme in that it provides maximum frequency diversity in frequency selective fading environment. The estimation of carrier frequency offsets of multiple users in interleaved OFDMA uplink has been tackled in [1]. The algorithms in [1] investigate the signal algebraic structure on the uplink of OFDMA system adopting interleaved carrier assignment scheme and proposed a subspace based method to estimate multiple users' CFO bearing high complexity.

A new synchronization scheme is proposed for OFDMA uplink in [5], which does not rely on any specific subcarrier assignment scheme. However, it assumes that only one user is asynchronous with the uplink receiver. Hence it tackles only a single-user synchronization error problem.

In this paper, we present a simple and efficient frequency synchronization algorithm for interleaved OFDMA uplink as a modification to Cao & Tureli's method [1]. By introducing *the effective CFO* and designing a special training sequence sets, the proposed method is computationally efficient than the subspace-based algorithm. Furthermore, the ZCZ sequence set is used as the training sequences to mitigate the multipath inference and multiuser inference. Simulation results show the proposed algorithm is effective in multipath fading channels and appropriate for interleaved OFDMA uplink.

The rest of the paper is organized as the followings. Section II presents the signal structure for the interleaved OFDMA uplink and introduces *the effective CFO* to be estimated in our scheme. The design of training symbol and the estimations of multiuser CFO are described in section III. In section IV, the properties of zero correlation zone (ZCZ) sequence are presented and we constructed a ZCZ training sequences set used for test its effectiveness. Simulation results and conclusions are given in section V and VI respectively.

II. SIGNAL STRUCTURE

Suppose the interleaved OFDMA system has K users and N subcarriers, including all data bearing subcarriers, pilot subcarriers and virtual subcarriers. The N subcarriers are divided into Q sub-channel and each sub-channel has P subcarreirs ($N=P\times Q$).

Let the kth user occupy the qth sub-channel, which is composed by subcarriers with index set $\{q, q+Q, ..., q+(P-1)Q\}$, where $q\in[0,1,...,Q-1]$. Without loss of generality, we assume only the kth user transmits on the uplink during one OFDMA block and the cyclic prefix accommodates both channel delay spread and maximum time offset between base station and users. Assume the kth user is synchronized with the

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base station in time, after the removal of cyclic prefix, the received base band signal samples is

$$\Upsilon^{(k)}(n)=e^{j2\pi\varphi^{(k)}}e^{j2\pi\xi^{(k)}n/N}\sum_{p=0}^{P-1}X_p^{(k)}H_p^{(k)}e^{j2\pi(pO+q)n/N} \quad (1)$$

for n=0,1,. . . ,N-1, where $X_p^{(k)}$ and $H_p^{(k)}$ are the modulation symbol and channel frequency response of the kth user on subcarriers $\{pQ+q\}$; $\xi^{(k)}$ is the normalized CFO, which indicates the frequency offset in the percentage of subcarrier spacing ($|\xi^{(k)}|<0.5$); $\varphi^{(k)}$ is a common phase shift due to previous OFDMA blocks. Equation (1) can be simplified to

$$\Upsilon^{(k)}(n)=e^{j2\pi\theta^{k}n/P}\sum_{p=0}^{P-1}X_p^{(k)}H_p^{(k)}e^{j2\pi pn/P} \quad (2)$$

by introducing the concept of the effective CFO of the kth user, θ(k), which is given by

$$\theta^{(k)}=\frac{\xi^{(k)}+q}{Q} \quad (3)$$

From (2) and (3), we have

$$\Upsilon^{(k)}(n+\mu P)=e^{j2\pi\theta^{(k)}}\Upsilon^{(k)}(n) \quad (4)$$

Equation (4) shows the signal samples of $\{\gamma^{(k)}(n)\}$, n=0,…,N-1, has a special periodic structure with every P sample.

Assume all K users are synchronized in time, the received one OFDMA signal block by the base station in the absence of noise signal is the summation of all users' signals:

$$\Upsilon(n)=\sum_{n=0}^{N-1}\Upsilon^{(k)}(n) \quad (5)$$

III. SYNCHRONIZATION ALOGRITHM

Tufvesson proposed an approach for time synchronization and frequency offset estimation by using PN sequence [6], in which the preamble is composed of a known short repeated PN sequence and transmitted in time domain. But it cannot be directly applied for the interleaved OFDMA uplink.

Assume the training sequences for the kth user occupying the qth subchannel is $\left\{b_0^{(k)},b_1^{(k)},\ldots,b_{P-1}^{(k)}\right\}$, then we let

$$c^{(k)}(n)=\sum_{p=0}^{P-1}b_p^{(k)}e^{j2\pi(q+pQ)/N} \quad (6)$$

as the known training sequence, where n=0,1,. . ,P-1. Let $r(d)$ denote the sampled received signal , the synchronization signal for kth user is given by

$$\gamma(d)=\sum_{l=0}^{Q-2}\left[\left(\sum_{n=0}^{P-1}\left(c^{(k)}[d+n+lP]\right)^{*}\cdot r[d+n+lP]\right)^{*}\cdot\left(\sum_{n=0}^{P-1}\left(c^{(k)}[d+n+(l+1)P]\right)^{*}\cdot r\left[d+n+(l+1)P\right]\right)\right] \quad (7)$$

The frequency offset estimation is given by the phase of the synchronization signal:

$$\hat{\theta}^{(k)}=\arg\left(\gamma(d)\right)\cdot N/(2\pi P) \quad (8)$$

It can be seen from (8) that the estimated carrier frequency range is ±Q /2 subcarrier spacing. Due to the property of the *effective CFO*, if subchannel $\{q\}$ is occupied by one user, there will be one and only one *effective CFO* which falls into the range of $((q\text{-}0.5)/Q,\ (q\text{+}0.5)/Q)$. It is thus a simple mapping to match the estimated effective CFOs with corresponding users.

IV. ZERO CORRELATION ZONE (**ZCZ**) SEQUENCE

A periodic sequence with an ideal autocorrelation property is referred as a perfect sequence, or an orthogonal sequence. That is, all of the out-of-phase autocorrelation values of a perfect sequence are zero. On the other hand, a set of periodic sequences with an ideal cross-correlation property is referred to as a set of uncorrelated sequences. That is, all of the cross-correlation values between uncorrelated sequences are zero. However, no set of sequences can have both the ideal autocorrelation property and the ideal cross-correlation property simultaneously.

The out-of-phase autocorrelation and cross-correlation values of zero-correlation zone (ZCZ) sequences are zeros over the range of $|\tau|\leqslant T$. Here, τ is a time shift variable and T is an integer. Moreover, the range $|\tau|\leqslant T$ is referred to as a ZCZ.

Generally, set of ZCZ sequences are characterized by the period of sequences L, the family size, namely the number of sequences, M, the length of the ZCZ T, and the number of phases Z. The following mathematical upper bound concerning T exists:

$$T\leq\frac{L}{M}-1 \quad (9)$$

However, in the binary case, the upper bound is expected to be given by the following formula:

$$T_b\leq\frac{L}{2M} \quad (10)$$

if $L/M=2$, then the upper bounds are equal. However, in other cases, T in upper bound (9) is larger than T_b in upper bound (10).

In [7], new methods for constructing sets of ZCZ sequences having a longer ZCZ are proposed. For example, their methods can generate quadricphase ZCZ sequences sets in which the length of the ZCZ satisfies the following formula:

$$T_q = \frac{7L}{8M} \tag{11}$$

T_q is larger than T_b in upper bound (10).

Let A_0 be a perfect sequence of period l=8. Two integers, l_0 and l_1, are defined as $l=l_0l_1$, where $1 \leq l_0 < l$ and $1 < l_1 \leq l$. Using these integers, l_1 perfect sequences A_i $(0 \leq i \leq l_1-1)$ are defined as

$$\begin{aligned} A_i &= \left(a_0^i, a_1^i, \ldots, a_{l-1}^i\right) \\ &= \left(a_{il_0}^0, a_{il_0+1}^0, \ldots, a_{l-1}^0, a_0^0, \ldots, a_{il_0-1}^0\right) \end{aligned} \tag{12}$$

A_i is a perfect sequence derived by shifting A_0 cyclically to the left by il_0 places. Another two integers, l_2 and l_3, are defined as: $l_3=ll_2$ and $l_2>1$.

Let D_n be an $l_3 \times l_3$ unitary matrix. D_n can be represented as

$$D_n = \frac{1}{\sqrt{l_3}} \begin{bmatrix} d_{0,0}^n & d_{0,1}^n & \cdots & d_{0,l_3-1}^n \\ d_{1,0}^n & d_{1,1}^n & \cdots & d_{1,l_3-1}^n \\ \vdots & \vdots & \ddots & \vdots \\ d_{l_3-1,0}^n & d_{l_3-1,1}^n & \cdots & d_{l_3-1,l_3-1}^n \end{bmatrix} \tag{13}$$

Let E_0 be a sequence set with l_3 perfect sequences of period l. The sequence set E_0 is defined as

$$\begin{aligned} E_0 &= \left\{E_0^0, E_1^0, \ldots, E_i^0, \ldots, E_{l_3-1}^0\right\} \\ &= \left\{A_0, A_1, \ldots, A_{i \bmod l}, \ldots, A_{l-1}\right\}, \\ E_i^0 &= \left\{e_0^{0,i}, e_1^{0,i}, \ldots, e_j^{0,i}, \ldots, e_{l-1}^{0,i}\right\} \\ &= \left\{a_0^{i \bmod l}, a_1^{i \bmod l}, \ldots, a_j^{i \bmod l}, \ldots, a_{l-1}^{i \bmod l}\right\} \end{aligned} \tag{14}$$

where $0 \leq i \leq l_3-1, 0 \leq j \leq l-1$. Using D_n and E_0, a sequence set E_n is defined as

$$E_n = \left\{E_0^n, E_1^n, \ldots, E_i^n, \ldots, E_{l_3-1}^n\right\},$$

$$E_i^n = \left(e_0^{n,i}, e_1^{n,i}, \ldots, e_j^{n,i}, \ldots, e_{ll_3^n-1}^{n,i}\right),$$

$$e_j^{n,i} = d_{i,j \bmod l_3}^n e_{\lceil j/l_3 \rceil}^{n-1, j \bmod l_3}, \tag{15}$$

$$0 \leq i \leq l_3-1,\ 0 \leq j \leq ll_3^n-1.$$

Then the sequence set E defined by (15) is a ZCZ sequence set having parameters:

$$(L, M, T) = \left(ll_3^n, l_3, (l-2)l_3^{n-1}\right) \tag{16}$$

Let us construct the ZCZ sequence set used for our system setup by the method above. Let A_0=(00120210), where 0,1,2, 3 represent +1, $-j$,-1, j respectively. Here j=sqrt(-1) . Let l_2=2, then $l_3=l \times l_2$=16. Moreover, let D is a 16×16 Walsh matrix. Then a quadriphase ZCZ sequence set E having parameters (L,M,T)=(128,16,6) is derived. We only show the sequences E_0 and E_1:

E_0=(001202100012021001202100012021001202100012010002021000120210001021000120210001221000120210001201000102010001202000120210001202 1).

E_1=(021000120210001203222302032223021000120210001202222302032223020300120210001202102302032223020322120210001202100002032223020322 23).

The absolute value of the autocorrelation function of E_0 is given by

$|R_{E0}(\tau)|$=(128,0,0,0,0,0,0,56,64,0,0,0,0,0,0,120,0,0,0,0,0,0,48,72,0,0,0,0,0,0,112,8,0,0,0,0,0,0,40,80,0,0,0,0,0,0,104,16,0,0,0,0,0,0,32,88,0,0,0,0,0,0,96,24,0,0,0,0,0,0,24,96,0,0,0,0,0,0,88,32,0,0,0,0,0,0,16,104,0,0,0,0,0,0,80,40,0,0,0,0,0,0,8,112,0,0,0,0,0,0,72,48,0,0,0,0,0,0,120,0,0,0,0,0,0,64,56,0,0,0,0,0,0).

The other sequences belonging to the set E have the same autocorrelation property. The absolute of the cross-correlation function between E_0 and E_1 is given by

$|R_{E0,E1}(\tau)|$=(0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0,0,8,0,0,0,0,0,0).

Similarly, any sequence pairs belonging to the set E have the same cross-correlation property.

V. Simulation results

The simulation parameters are as the followings. The sampling frequency Fs is 16MHz and the FFT size is N=2048. Hence the subcarrier spacing, Δf, is 7.8125kHz and sample interval Ts is 0.0625us. The guard interval (CP) is composed of 256 samples. The simulations are performed in additive white Gaussian noise (AWGN) channels and Rayreigh fading channel respectively. The Rayleigh fading channel adopted is ITU vehicular channel A which is composed of six paths with path delays of 0, 310, 710, 1090, 1730, 2510 ns respectively, corresponding to 0, 4, 11, 17, 27, 40 samples.

In our simulations, the 2048 subcarriers are divided into Q=16 subchannels, the maximum normalized frequency offset is set 0.3. The mean squared errors of carrier frequency offset estimations are calculated respectively for different number of active users.

To compare the estimator performance using the ZCZ training sequences set with that using the PN training sequences set, the mean squared errors of both estimators in AWGN channel are calculated respectively and the results are shown in Fig.1 and Fig.2. Here, a gold sequences set with the length of 128 and the family number of 16 are used as our PN sequence sets. From fig.2, it can be clearly seen that when the number of active users exceeds 4, the performance of PN-estimator degrades quickly because of multi-user interference.

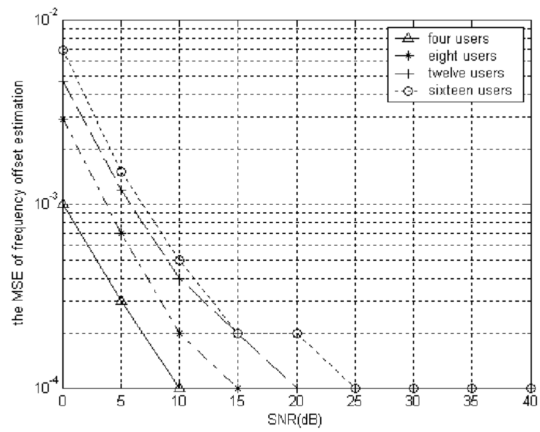

Figure 1. The mean squared errors of ZCZ sequences-based estimator for the number of active users K=4,8,12 and 16 in AWGN channel

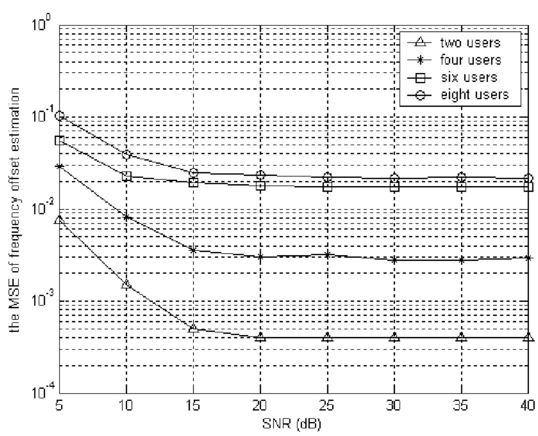

Figure 2. The mean squared errors of PN sequences-based estimator for the number of active users K=2,4,6 and 8 in AWGN channel

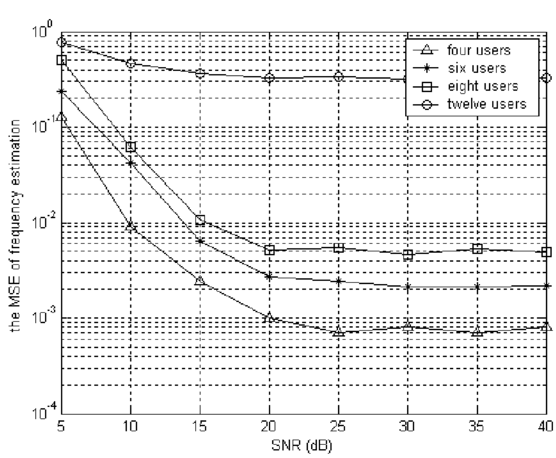

Figure 3. The mean squared errors of ZCZ sequences-based estimator for the number of active users K=4,6 ,8 and 12 in multipath fading channel

The mean squared error of ZCZ sequence-based estimator is calculated in multi-path fading channel and the results is shown in Fig.3. It can be seen that the performance is degraded in comparison with that in AWGN channels. The reason for that is in multipath fading channels, the frequency selective fading destroys the ZCZ sequences' correlation property and with the number of active users increases, the performance degradation become more serious as in CDMA systems. However, when the number of active users not exceeds half of the number of subchannels, the performance is desirable. The performance can be further improved by mulituser detection.

VI. CONCLUSIONS

A training symbol-based carrier frequency offset estimation algorithm for interleaved OFDMA uplink system is presented. By utilizing the signal inner structure in the interleaved OFDMA uplink and introducing the *effective carrier frequency offset*, we design a special training sequence sets for all involved users. Then the carrier frequency offset for each user can be estimated. The ZCZ sequence set is used to mitigate multipath inference and multiuser inference. Simulation results show that the proposed algorithm is potential for interleaved OFDMA uplink.

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